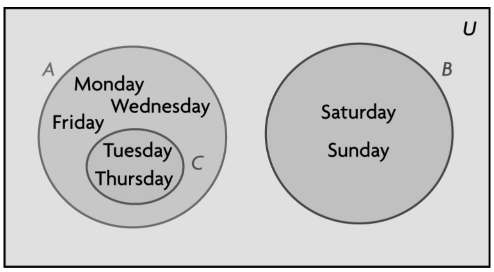
D. One strategy is to observe which card your opponent collects and to think about why he or she wants this card. Does your opponent want it to make a set or a run? If so, you might want to prevent your opponent from getting cards that would allow him or her to complete the set. Another strategy is to collect a card you do not really need to fool your opponent into thinking that you are looking for this type of card.

3–5 Cumulative Review, page 373

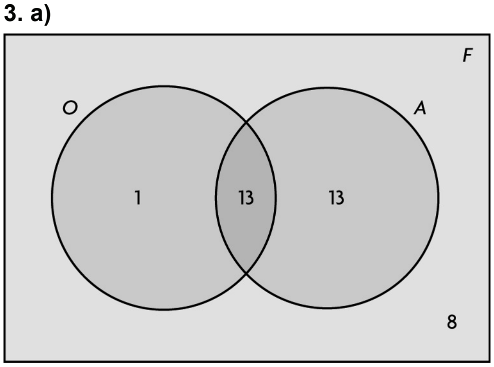
1. a) e.g., Manitoba, Québec, PEI, New Brunswick, Nova Scotia
b) e.g., x is an integer, so $x = \{\ldots, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$. b is 10 times more than x.
$10 \cdot 1 = 10$, $10 \cdot 2 = 20$, $10 \cdot 3 = 30$, $10 \cdot 4 = 40$,
$10 \cdot 5 = 50$. So, 10, 20, 30, 40, 50 are five elements.

2. a)



b) A and B have no common elements, so they are disjoint sets.
c) i) false; e.g., A and B are disjoint sets, so A is not a subset of B.
ii) true; C is entirely inside A, so it a subset of A.
iii) true; B contains all the elements that are not in A, so B is the inverse of A.
iv) false;
$n(A) + n(B) + n(C) = 5 + 2 + 2$
$n(A) + n(B) + n(C) = 9$
but $n(U) = 7 \neq 9$.
This is because A includes C, so $n(C)$ is counted twice.

3. a)



b) There are 35 students in all, and 8 like neither fruit, so $35 - 8 = 27$ students like oranges or apples.
c) $27 - 14 = 13$, so 13 students like only apples.
$27 - 26 = 1$, so 1 student likes only oranges.
$27 - 13 - 1 = 13$, so 13 students like oranges and apples. 14 students like only apples or only oranges.

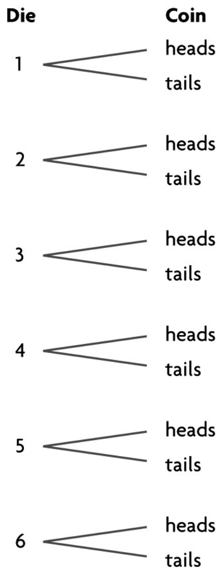
4. a) e.g., odd whole numbers less than 100 and even whole numbers less than 100
b) e.g., odd whole numbers less than 100 and prime numbers less than 100

5. Let S = {campers who learned singing},
D = {campers who learned dancing},
A = {students who learned acting}
Then: $n(S) = 35$, $n(D) = 38$, $n(A) = 33$
$n(S \cap D) = 21$, $n(D \cap A) = 23$, $n(S \cap A)$
$n(D \cup S \cup A) = 56$

By the Principle of Exclusion and Inclusion:
$n(D \cup S \cup A)$
$= n(S) + n(D) + n(A) - n(S \cap D) - n(D \cap A) - n(S \cap A)$
$+ n(D \cap S \cap A)$
$56 = 35 + 38 + 33 - 21 - 23 - 18 + n(D \cap S \cap A)$
$12 = n(D \cap S \cap A)$
So, 12 campers learned singing, dancing, and acting.

6. a) Yes, it is true; a negative number must be less than zero.
b) If a number is less than zero, then it is negative. Yes. e.g., All numbers less than zero must be negative.
c) If a number is not negative, then it is not less than zero. Yes. e.g., All numbers that are not negative are either zero or positive, and all of these numbers are not less than zero.
d) If a number is not less than zero, then it is not negative. Yes. e.g., All numbers not less than zero are either zero or positive, and they are all negative.
e) Yes. e.g., Both the statement itself and and the converse are true so the statement can be written as a biconditional.

7. a)



b) Rolling a die has 6 outcomes: 1, 2, 3, 4, 5, 6; tossing a coin has 2 outcomes: heads, tails. By the Fundamental Counting Principle, there are 6 · 2 = 12 combined outcomes, which is how many there are in the tree diagram.

8. a) A licence plate would be of the form *LLL NN*, where *L* represents a letter, and *N* represents a number. Any one of the 26 letters could be used in a spot, and so could any one of the 10 numbers.
Total licence plates = 26 · 26 · 26 · 10 · 10
Total licence plates = 1 757 600

b) Total licence plates without repetition
= 26 · 25 · 24 · 10 · 9
= 1 404 000

Total licence plates with repetition
= total licence plates – total licence plates without repetition
= 1 757 600 – 1 404 000
= 353 600
Therefore, 353 600 different plates use the same letter or same number more than once.

9. a) 12! = 479 001 600

b)
$$_8P_8 = \frac{8!}{(8-8)!}$$
$$_8P_8 = \frac{8!}{0!}$$
$$_8P_8 = 8!$$
$$_8P_8 = 40\ 320$$

c)
$$\frac{10!}{6!3!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6!}{6!3!}$$
$$\frac{10!}{6!3!} = \frac{10\cdot 9\cdot 8\cdot 7}{3\cdot 2\cdot 1}$$
$$\frac{10!}{6!3!} = 840$$

d)
$$_9P_5 = \frac{9!}{(9-5)!}$$
$$_9P_5 = \frac{9!}{4!}$$
$$_9P_5 = \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!}$$
$$_9P_5 = 9\cdot 8\cdot 7\cdot 6\cdot 5$$
$$_9P_5 = 15\ 120$$

e)
$$_9C_5 = \frac{9!}{(9-5)!5!}$$
$$_9C_5 = \frac{9!}{4!5!}$$
$$_9C_5 = \frac{9\cdot 8\cdot 7\cdot 6\cdot 5!}{4!5!}$$
$$_9C_5 = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2}$$
$$_9C_5 = 9\cdot 2\cdot 7$$
$$_9C_5 = 126$$

f)
$$\binom{12}{8} = \frac{12!}{(12-8)!8!}$$
$$\binom{12}{8} = \frac{12!}{4!8!}$$
$$\binom{12}{8} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8!}{4!8!}$$
$$\binom{12}{8} = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2}$$
$$\binom{12}{8} = 11\cdot 5\cdot 9$$
$$\binom{12}{8} = 495$$

10. a) There are 15! ways, or 1 307 674 368 000 ways.
b) Out of 15 members, 3 can be chosen, with regard for order, in $_{15}P_3$ = 2730 ways.
c) Out of 15 members, Jill, and 2 other members can be chosen in $_{14}C_2$ = 91 ways.

11. There were 13 + 5 = 18 games played, so there are $_{18}C_{13}$ = 8568 ways in which 13 wins could have occurred.

12. a) There are 7 squares to the east, and 3 squares south. 7 + 3 = 10, so there are $\binom{10}{7} = \binom{10}{3} = 120$ different routes.

b) In the first grid, there are 3 squares to the east, and 2 south. In the one square, there is one square to the east and one to the south. In the bottom grid, there are 3 squares to the east and 3 to the south. So, there are $\binom{5}{3} \cdot 2 \cdot \binom{6}{3} = 400$ different routes.

13.

$$
\begin{aligned}
{}_nC_4 &= {}_nP_3 \\
\frac{n!}{(n-4)!4!} &= \frac{n!}{(n-3)!} \\
\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!4!} &= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \\
\frac{n(n-1)(n-2)(n-3)}{4!} &= n(n-1)(n-2) \\
n(n-1)(n-2)(n-3) &= 24n(n-1)(n-2) \\
n-3 &= 24 \\
n &= 27
\end{aligned}
$$

14. a) There are 7 + 9 = 16 people in total.
If 3 are parents and 3 are students, then:
${}_7C_3 \cdot {}_9C_3 = 35 \cdot 84$ or 2940 ways in this can happen.

b) If there is at least 1 student, then there can be:
• 5 parents and 1 student:
${}_7C_5 \cdot {}_9C_1 = 21 \cdot 9$ or 189 ways

• 4 parents and 2 students:
${}_7C_4 \cdot {}_9C_2 = 35 \cdot 36$ or 1260 ways

• 3 parents and 3 students:
${}_7C_3 \cdot {}_9C_3 = 35 \cdot 84$ or 2940 ways

• 2 parents and 4 students:
${}_7C_2 \cdot {}_9C_4 = 21 \cdot 126$ or 2646 ways

• 1 parent and 5 students:
${}_7C_1 \cdot {}_9C_5 = 7 \cdot 126$ or 882 ways

• 0 parents and 6 students:
${}_9C_6 = 84$ ways

Total number of ways
= 189 + 1260 + 2940 + 2646 + 882 + 84
= 8001
There are 8001 ways in which the committee can have at least 1 student.

c) There will be more parents than students if there are
• 6 parents and no students:
${}_7C_6 = 7$ ways

• 5 parents and 1 student:
${}_7C_5 \cdot {}_9C_1 = 21 \cdot 9$ or 189 ways

• 4 parents and 2 students:
${}_7C_4 \cdot {}_9C_2 = 35 \cdot 36$ or 1260 ways
Total number of ways
= 7 + 189 + 1260
= 1456
There are 1456 ways in which the committee can have more parents than students.

15. There are 4 aces in a deck, so the number of ways in which an ace can be in the hand is ${}_4C_1 = 4$.
There are 4 tens in the deck, so the number of ways in which an ace can be in the hand is ${}_4C_1 = 4$.
There are 50 other cards in the deck, and the ways in one of them can be in the hand is ${}_{50}C_1 = 50$.
Multiply these figures.
$4 \cdot 4 \cdot 50 = 800$
There are 800 ways in which a three-card hand can have (at least) one ace and (at least) one ten.

16. I made table of the possible results. I put an "A" in the spots where Amber would win, a "J" in the spots where Jackson would win, and a "/" in the spots where neither would win.

Die 1 / Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	J	/	J	/	J	/
2	/	A	/	A	/	A
3	J	/	J	/	J	/
4	/	A	/	A	/	A
5	J	/	J	/	J	/
6	/	A	/	A	/	A

Amber would win 9 out 36 times. Jackson would win 9 out of 36 times. Neither will win 18 out of 36 times. The game is fair, because both players have an equal chance of winning.

17. a) To determine the odds in favour when the odds against are known, switch the terms in the ratio. The odds against are 1 : 25, so the odds in favour are 25 : 1.
b) To determine the probability in favour of an event when the odds in favour are known, add the terms, and use the sum for the ratio. The odds in favour are 25 : 1, so the probability is $\frac{25}{1+25} = \frac{25}{26}$ or about 0.96.
18. Since each student is equally likely to win, then Sonya is just as likely to finish in the top three (the top half) as in the bottom three (the bottom half). The probability Sonya will finish in the top three is 0.5.

19. a) There are ${}_{52}C_{13}$ = 635 013 559 600 ways in which 13 cards can be selected from a deck of 52 cards. There is only one way in which all of these 13 cards can be hearts. The probability that a player is dealt all hearts is $\frac{1}{635\ 013\ 559\ 600}$.

b) $\frac{\binom{\text{all kings}}{\text{other players's cards}}}{\text{all possible hands}} = \frac{\binom{48}{39}}{\binom{52}{13}}$

$= \frac{1\ 677\ 106\ 640}{635\ 013\ 559\ 600}$

The probability is $\frac{1\ 677\ 106\ 640}{635\ 013\ 559\ 600}$.

20. a) e.g., choosing an apple or a pear from a bowl of fruit

b) e.g., choosing 2 blue marbles from a bag containing 7 blue and 3 red marbles, without replacement

c) e.g., rolling a standard die and getting 4, tossing a coin and getting heads

21. I made a table of the sums. There are 36 possible results.

Die 1 / Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a) The sum is 3 or 12 on three occasions.

The probability of this is $\frac{3}{36}$ or about 0.0833.

b) The sum is odd or the sum is greater than 7 on 27 occasions. The probability of this is $\frac{27}{36}$ or 0.75.

c) There are 6 occasions on which the first die is 1. Of these, there are three occasions on which the sum is odd. The probability of this is $\frac{3}{6}$ or 0.5.

d) There are 2 occasions out of 36 when the first die is less than 3 and the second is greater than 5. The probability of this is $\frac{2}{36}$ or about 0.0556.

22. a) Jan can win at least one game in three ways:

• She wins both games:

$P(\text{win both games})$

$= P(\text{win game 1}) \cdot P(\text{win game 2} \mid \text{won game 1})$

$= \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{8}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{5}{8}\right)$

$= \frac{5}{16}$ or 0.3125

• She wins game 1 and loses game 2:

$P(\text{win game 1, lose game 2})$

$= P(\text{win game 1}) \cdot P(\text{lose game 2} \mid \text{won game 1})$

$= \frac{1}{2} \cdot \left(1 - \frac{5}{8}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{3}{8}\right)$

$= \frac{3}{16}$ or 0.1875

• She lose game 1 and wins game 2:

$P(\text{lose game 1, win game 2})$

$= P(\text{lose game 1}) \cdot P(\text{win game 2} \mid \text{lost game 1})$

$= \frac{1}{2} \cdot \left(\frac{1}{2} - \frac{1}{4}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{1}{4}\right)$

$= \frac{1}{8}$ or 0.125

The total of these probabilities is
0.3125 + 0.1875 + 0.125 = 0.625
The probability Jan will win at least one game is 0.625.

b) $P(\text{Stan wins both games})$

$= 1 - P(\text{Jan wins at least one game})$

$= 1 - 0.625$

$= 0.375$

23. a) There are 24 cards, of which 4 are queens. The probability of drawing a queen on the first draw is $\frac{4}{24} = \frac{1}{6}$. If the card is replaced, the probability of drawing a queen on the second draw is the same, $\frac{1}{6}$.

So, P(2 queens) = $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ or about 0.0278.

b) The probability of drawing a queen on the first draw remains the same, $\frac{1}{6}$. If this card is not replaced, then there is one less queen to draw and one less card to draw from. The probability of drawing a queen on the second draw is $\frac{4-1}{24-1}=\frac{3}{23}$. So,

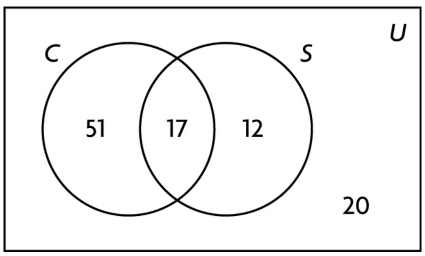
P(2 queens) = $\frac{4}{24}\cdot\frac{3}{23}=\frac{12}{552}$ or about 0.0217.

Chapter 5 Diagnostic Test, TR page 343

1. Let U be the universal set.

$C \setminus S$ = 68 – 17 = 51

$S \setminus C$ = 29 – 17 = 12



2. a) The number of ways to roll a 5 is 1; the number of outcomes for the die is 6. The number of ways to toss a tail is 1; the number of outcomes is 2.

$$\left(\frac{1}{6}\right)\cdot\left(\frac{1}{2}\right)=\frac{1}{12}$$

b) The number of ways to roll an even number is 3; the number of outcomes is 6. The number of ways to toss a head is 1; the number of outcomes is 2.

$$\left(\frac{3}{6}\right)\cdot\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right) \text{ or } \frac{1}{4}$$

3. The total volume of the cleaning compound is volume of concentrate and volume of water: 2 + 5. The ratio of volume of concentrate to total volume of the cleaning compound is 2 : 7.

4. There are ten letters in STATISTICS.
S is repeated three times, T is repeated three times, and I is repeated twice. The letters can be arranged 10! ways. To eliminate arrangements that would be the same because of the repeating letters, divide by 3!, 3! and 2!.

$$\frac{10!}{3!\cdot 3!\cdot 2!}=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{3!\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1}$$

$$\frac{10!}{3!\cdot 3!\cdot 2!}=\frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4}{3\cdot 2\cdot 2}$$

$$\frac{10!}{3!\cdot 3!\cdot 2!}=\frac{604\ 800}{12}$$

$$\frac{10!}{3!\cdot 3!\cdot 2!}=50\ 400$$

The letters can be arranged in 50 400 ways.

5. $${}_9P_5=\frac{9!}{(9-5)!}$$

$${}_9P_5=\frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!}$$

$${}_9P_5=9\cdot 8\cdot 7\cdot 6\cdot 5$$

$${}_9P_5=15\ 120$$

Remi can design the pendant in 15 120 ways.

6. $${}_{24}C_5=\frac{24!}{(24-5)!\cdot 5!}$$

$${}_{24}C_5=\frac{24\cdot 23\cdot 22\cdot 21\cdot 20\cdot 19!}{19!\cdot 5\cdot 4\cdot 3\cdot 2}$$

$${}_{24}C_5=\frac{24\cdot 23\cdot 22\cdot 21\cdot 20}{5\cdot 4\cdot 3\cdot 2}$$

$${}_{24}C_5=42\ 504$$

The students can be selected in 42 504 ways.

7. Agree. e.g., The outcome of tossing the coin has no bearing on the outcome of tossing the die. This means these are independent events.

Review of Terms and Connections, TR page 345

1. a) ix) If one thing can be done in five ways and another thing can be done in three ways, then both things can be done in 5 · 3 or 15 ways, according to the **Fundamental Counting Principle**.

b) ii) Two sets that have no elements in common are **mutually exclusive**.

c) v) The value $\frac{3}{10}$, written as a **ratio**, is 3 : 10.

d) iii) An arrangement of objects in a definite order is a **permutation** of the objects.

e) vii) If the product 5 · 4 · 3 · 2 · 1 is written as 5!, it is written in **factorial notation**.

f) iv) A group of objects in which order does not matter is a **combination** of the objects.

g) i) The set of all possible outcomes is called the **sample space**.

h) vi) Brenda tossed a coin 10 times. It turned up tails 8 times. She used these results to determine that the probability of tossing tails is $\frac{8}{10}$. This is an example of **experimental** probability.

i) viii) Serge says that the probability of tossing heads with a fair coin is $\frac{1}{2}$, because the likelihood that the coin will land heads is equal to the likelihood that it will land tails. This is an example of **theoretical** probability.