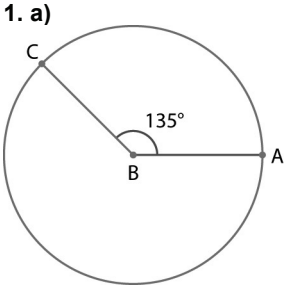
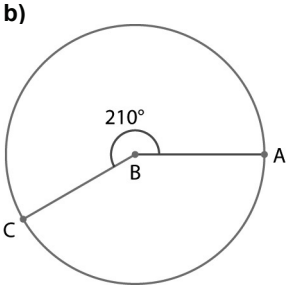
Mid-Chapter Review, page 545

1. a)



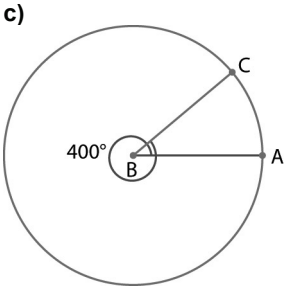
e.g., 135° = 180° – 45°
180° is about 3.2 radians.
45° is about a quarter of 180° so 45° is about $\frac{3.2}{4}$, or 0.8 radians.
3.2 – 0.8 = 2.4 radians.
Therefore 135° is about 2.4 radians.

b)



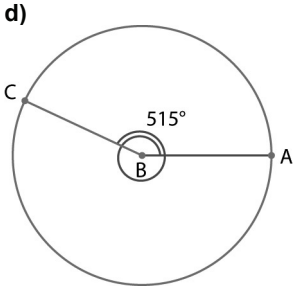
e.g., 210° = 180° + 30°
180° is about 3.2 radians.
30° is about one half of 60° so 30° is about $\frac{1}{2}$, or 0.5 radians.
3.2 + 0.5 = 3.7 radians.
Therefore 210° is about 3.7 radians.

c)



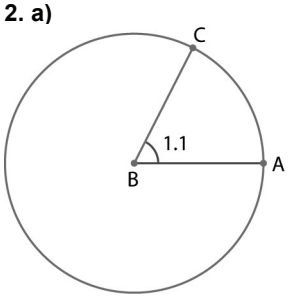
e.g., 400° is slightly greater than 360° + 30°, or 390°.
360° is about 6.3 radians.
30° is about one half of 60° so 30° is about $\frac{1}{2}$, or 0.5 radians.
6.3 + 0.5 = 6.8 radians.
390° is about 6.8 radians.
Therefore 400° is about 7.0 radians.

d)



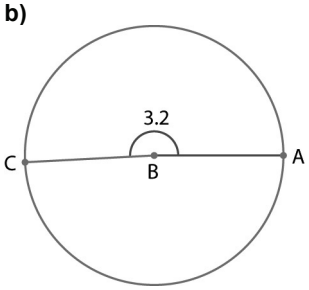
e.g., 515° is slightly greater than 360° + 180° – 30°, or 510°.
360° is about 6.3 radians.
180° is about 3.2 radians.
30° is about one half of 60° so 30° is about $\frac{1}{2}$, or 0.5 radians.
6.3 + 3.2 – 0.5 = 9.0 radians.
510° is about 9.0 radians.
Therefore 515° is about 9.1 radians.

2. a)



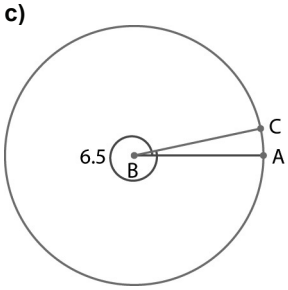
e.g., 1.1 radians is slightly greater than 1 radian.
1 radian is about 60°.
Therefore 1.1 radians is about 65°.

b)



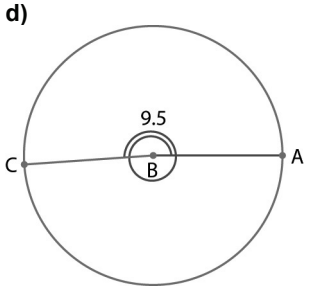
e.g., 3.2 radians is about 180°.

c)



e.g., 6.5 radians is slightly greater than 6.3 radians.
6.3 radians is about 360°.
Therefore 6.5 radians is about 370°.

d)



e.g., 9.5 = 6.3 + 3.2
6.3 radians is about 360°.
3.2 radians is about 180°.
360° + 180° = 540°
Therefore 9.5 radians is about 540°.

3. To arrange the angle measures in ascending order, compare the angles as either degrees or radians. e.g., Here they are compared as degrees:
$\pi = 180°$

1.5 radians:
1.5 radians is slightly less than 1.6 radians.
1.6 radians is about 90°.
Therefore 1.5 radians is about 85°.

0.5 radians:
0.5 radians is one half of 1 radian.
1 radian is about 60°.
One half of 60° is 30°.
0.5 radians is about 30°.

0.1 radians:
$0.1 = \frac{1}{10}$
1 radian is about 60°.
$\frac{60}{10} = 6$
0.1 radians is about 6°.
The angle measures in ascending order are:
0.1, 20°, 0.5, 1.5, π, 190°, 200°, 310°, and 400°.

4. a) The range of $y = \sin x$ is $\{y \mid -1 \le y \le 1, y \in R\}$, and the amplitude is 1. The period is 2π or 360°, and the equation of the midline is $y = 0$.
b) Their range, amplitude, period and equation of the midline are the same. Their y-intercept is different.

5. a) The x-intercepts of the graph $y = \sin x$ are 0°, 180°, 360°, 540°, and 720°. This is when the value of the function is equal to zero over the interval from 0° to 720°.
b) The values of x for when the function is at a maximum is 90°, 450°. The values of x when the function is at a minimum is 270° and 630°.

6. a) Range: $\{y \mid -5 \le y \le 1, y \in R\}$
Maximum value = 1, Minimum value = –5

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{1-(-5)}{2}$$
$$\text{Amplitude} = \frac{6}{2}$$
$$\text{Amplitude} = 3$$
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 202.5° - 22.5°$$
$$\text{Period} = 180°$$

Equation of the midline:
$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{1+(-5)}{2}$$
$$y = \frac{-4}{2}$$
$$y = -2$$

The range of this graph is $\{y \mid -5 \le y \le 1, y \in R\}$, and the amplitude is 3. The equation of the midline is $y = -2$, and the period is 180°.
b) Range: $\{y \mid -3 \le y \le 1, y \in R\}$
Maximum value = 1, Minimum value = –3

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{1-(-3)}{2}$$
$$\text{Amplitude} = \frac{4}{2}$$
$$\text{Amplitude} = 2$$
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 390° - 30°$$
$$\text{Period} = 360°$$

Equation of the midline:
$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{1+(-3)}{2}$$
$$y = \frac{-2}{2}$$
$$y = -1$$

The range of this graph is $\{y \mid -3 \le y \le 1, y \in R\}$, and the amplitude is 2. The equation of the midline is $y = -1$, and the period is 360°.

7. a) Range: $\{y \mid -4 \le y \le 6, y \in R\}$
Maximum value = 6, Minimum value = –4

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{6 - (-4)}{2}$$

$$\text{Amplitude} = \frac{10}{2}$$

$$\text{Amplitude} = 5$$

Equation of Midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{6 + (-4)}{2}$$

$$y = \frac{2}{2}$$

$$y = 1$$

Period = second max – first max
Period = 8.7 – 1.7
Period = 7
The range of this graph is $\{y \mid -4 \le y \le 6, y \in R\}$ and the amplitude is 5. The equation of midline is $y = 1$ and the period is 7.

b) Range: $\{y \mid 0.5 \le y \le 3.5, y \in R\}$
Maximum value = 3.5, Minimum value = 0.5

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{3.5 - 0.5}{2}$$

$$\text{Amplitude} = \frac{3}{2}$$

$$\text{Amplitude} = 1.5$$

Equation of Midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{3.5 + (0.5)}{2}$$

$$y = \frac{4}{2}$$

$$y = 2$$

Period = second max – first max
Period = 0.5 – 0
Period = 0.5
The range of this graph is $\{y \mid 0.5 \le y \le 3.5, y \in R\}$ and the amplitude is 1.5. The equation of midline is $y = 2$ and the period is 0.5.

8. a) Period = second max – first max
Period = 0.41 – 0.09
Period = 0.32 s
The period is about 0.32 seconds.
b) Maximum value = 4.5
Minimum value = –4.5

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{4.5 + (-4.5)}{2}$$

$$y = 0$$

The equation of the midline of the function is $y = 0$ amperes.
c)

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{4.5 - (-4.5)}{2}$$

$$y = \frac{9}{2}$$

$$y = 4.5$$

The amplitude of the function is 4.5 amperes.

Lesson 8.4: The Equations of Sinusoidal Functions, page 558

1. The equations in order from the least amplitude to the most amplitude, are **c)**, **a)**, and **b)**. I ordered the functions based on the values of a since the amplitude of an equation is given by a. For equation **a)**, $a = 3$. For equation **b)**, $a = 4$. For equation **c)**, $a = 2$. Therefore,

2. The equations, in order from the smallest range to the greatest range, are **c)**, **a)**, and **b)**. Since the range is defined by the function's amplitude, so I ordered the functions based on the value of a.

The range of an equation is given by a and d.
Equation **a)**:

$$\text{Minimum value} = d - a$$

$$\text{Minimum value} = 0 - 8$$

$$\text{Minimum value} = -8$$

$$\text{Maximum value} = d + a$$

$$\text{Maximum value} = 0 + 8$$

$$\text{Maximum value} = 8$$

$$\text{Maximum value} - \text{minimum value} = 8 - (-8)$$

$$= 16$$

The range of equation **a)** is 16.

Equation **b)**:

$$\text{Minimum value} = d - a$$

$$\text{Minimum value} = -1 - 9$$

$$\text{Minimum value} = -10$$

$$\text{Maximum value} = d + a$$

$$\text{Maximum value} = -1 + 9$$

$$\text{Maximum value} = 8$$

$$\text{Maximum value} - \text{minimum value} = 8 - (-10)$$

$$\text{Maximum value} - \text{minimum value} = 18$$

The range of equation **b)** is 18.