

$$\text{period} = \frac{360^\circ}{b}$$

$$\text{period} = \frac{360^\circ}{2}$$

$$\text{period} = 180^\circ$$

Horizontal translation =  $60^\circ$  to the right

Equation of the midline:

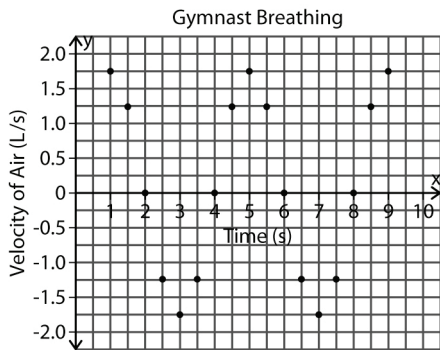
$$y = d$$

$$y = -4$$

The range of this graph is  $\{y \mid -6 \leq y \leq -2, y \in \mathbb{R}\}$ , and its amplitude is 2. The period is  $180^\circ$  and the horizontal translation is  $60^\circ$  to the right. The equation of the midline is  $y = -4$ .

5. To determine the equation of the sinusoidal regression function, plot the data using a graphing calculator and use the data to determine the equation. The equation of the sinusoidal regression function for this data is  $y = 19.557... \sin(0.476...x - 1.762...) + 6.141...$

6. a)



The equation of the sinusoidal regression function for this data is

$$y = 1.751... \sin(1.570...x)$$

b) Period = second maximum – first maximum

$$\text{Period} = 5 - 1$$

$$\text{Period} = 4$$

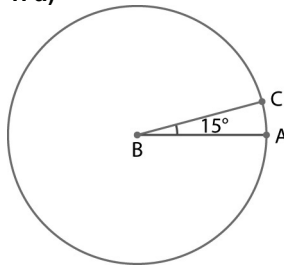
The period of this graph is 4 s.

c) e.g., Positive and negative velocities correspond to exhalations and inhalations (or vice versa).

e) The velocity of air being 0 L/s corresponds to the x-intercepts of the graph. Between 9 s and 19 s, the velocity of the air will be 0 at 10 s, 12 s, 14 s, 16 s, and 18 s.

## Chapter Review, page 581

1. a)



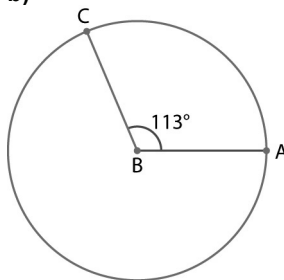
e.g.,  $15^\circ$  is one quarter of  $60^\circ$ .

$60^\circ$  is about 1 radian.

One quarter of 1 radian is 0.25 radians.

It is about 0.3 radians.

b)



e.g.,  $113^\circ$  is slightly less than  $180^\circ - 60^\circ$ , or  $120^\circ$ .

$180^\circ$  is about 3.2 radians.

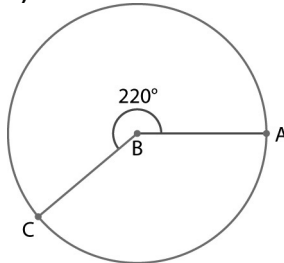
$60^\circ$  is about 1 radian.

$$3.2 - 1 = 2.2$$

$120^\circ$  is about 2.2 radians.

It is about 2.1 radians.

c)



e.g.,  $220^\circ$  is slightly greater than  $180^\circ + 30^\circ$ , or  $210^\circ$ .

$180^\circ$  is about 3.2 radians.

$30^\circ$  is one half of  $60^\circ$ .

$60^\circ$  is about 1 radian.

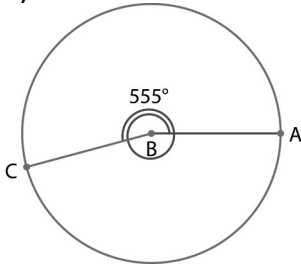
One half of 1 radian is 0.5 radians.

$$3.2 + 0.5 = 3.7$$

$210^\circ$  is about 3.7 radians.

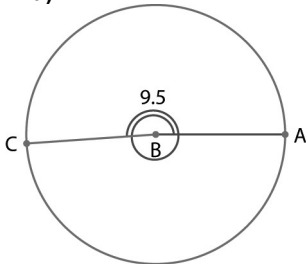
It is about 3.9 radians.

d)



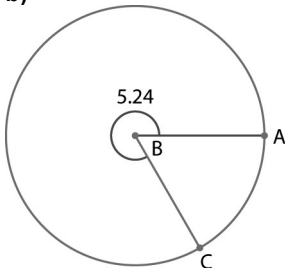
e.g.,  $555^\circ = 360^\circ + 180^\circ + 15^\circ$ .  
 $360^\circ$  is about 6.3 radians.  
 $180^\circ$  is about 3.2 radians.  
 $15^\circ$  is one quarter of  $60^\circ$ .  
 $60^\circ$  is about 1 radian.  
 One quarter of 1 radian is 0.25 radians.  
 $6.3 + 3.2 + 0.25 = 9.75$   
 It is about 9.8 radians.

2. a)



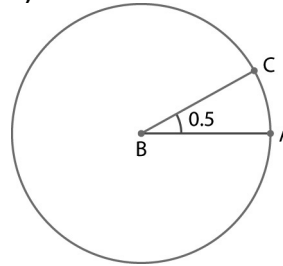
e.g.,  $9.5 = 6.3 + 3.2$   
 $6.3$  radians is about  $360^\circ$ .  
 $3.2$  radians is about  $180^\circ$ .  
 $360^\circ + 180^\circ = 540^\circ$   
 $9.5$  radians is about  $540^\circ$ .

b)



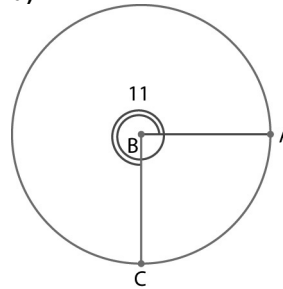
e.g., 5.24 radians is slightly less than  $6.3 - 1$ , or 5.3 radians.  
 $6.3$  radians is about  $360^\circ$ .  
 1 radian is about  $60^\circ$ .  
 $360^\circ - 60^\circ = 300^\circ$ .  
 $5.3$  radians is about  $300^\circ$ .  
 $5.24$  radians is about  $290^\circ$ .

c)



e.g., 0.5 radians is one half of 1 radian.  
 1 radian is about  $60^\circ$ .  
 One half of  $60^\circ$  is  $30^\circ$ .  
 0.5 radians is about  $30^\circ$ .

d)



e.g.,  $11.0 = 6.3 + 4.7$   
 $6.3$  radians is about  $360^\circ$ .  
 $4.7$  radians is about  $270^\circ$ .  
 $360^\circ + 270^\circ = 630^\circ$   
 $11.0$  radians is about  $630^\circ$ .

3. a) The values of  $x$  where  $\sin x < 0$  over the interval from  $0^\circ$  to  $720^\circ$  are

$\{x \mid 180^\circ < x < 360^\circ, x \in \mathbb{R}\}$  or  
 $\{x \mid 540^\circ < x < 720^\circ, x \in \mathbb{R}\}$ .

b) The values of  $x$  where  $\cos x < 0$  over the interval from  $0^\circ$  to  $720^\circ$  are

$\{x \mid 0^\circ \leq x < 90^\circ, x \in \mathbb{R}\}$  or  
 $\{x \mid 270^\circ < x < 450^\circ, x \in \mathbb{R}\}$  or  
 $\{x \mid 630^\circ < x \leq 720^\circ, x \in \mathbb{R}\}$ .

4. a) Range:  $\{y \mid -4 \leq y \leq 0, y \in \mathbb{R}\}$

Maximum =  $d + a$

Maximum =  $-2 + 2$

Maximum = 0

Minimum =  $d - a$

Minimum =  $-2 - 2$

Minimum =  $-4$

Amplitude =  $\frac{\max - \min}{2}$

Amplitude =  $\frac{0 - (-4)}{2}$

Amplitude =  $\frac{4}{2}$

Amplitude = 2

$y = \frac{360^\circ}{b}$

$y = \frac{360^\circ}{1}$

$y = 360^\circ$

Equation of the midline:

$$y = d$$

$$y = -2$$

The range of this graph is  $\{y \mid -4 \leq y \leq 0, y \in \mathbb{R}\}$ , and its amplitude is 2. The equation of the midline is  $y = -2$ , and the graph's period is  $360^\circ$ .

b) Range:  $\{y \mid -2.5 \leq y \leq -0.5, y \in \mathbb{R}\}$

$$\text{Maximum} = d + a$$

$$\text{Maximum} = -1.5 + 1$$

$$\text{Maximum} = -0.5$$

$$\text{Minimum} = d - a$$

$$\text{Minimum} = -1.5 - 1$$

$$\text{Minimum} = -2.5$$

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{-0.5 - (-2.5)}{2}$$

$$\text{Amplitude} = \frac{2}{2}$$

$$\text{Amplitude} = 1$$

$$y = \frac{360^\circ}{b}$$

$$y = \frac{360^\circ}{4}$$

$$y = 90^\circ$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{-0.5 + (-2.5)}{2}$$

$$y = \frac{-3}{2}$$

$$y = -1.5$$

The range of this graph is

$\{y \mid -2.5 \leq y \leq -0.5, y \in \mathbb{R}\}$ , and its amplitude is 1.

The equation of the midline is  $y = -1.5$ , and the graph's period is  $90^\circ$ .

5. a) Maximum = 14 m

Minimum = 2 m

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{14 + 2}{2}$$

$$y = \frac{16}{2}$$

$$y = 8$$

The equation of the midline is  $y = 8$  m. The midline represents the position of the swing at rest or her initial distance from the motion sensor. She started at 8 m from the sensor.

$$\text{b) Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{14 - 2}{2}$$

$$\text{Amplitude} = \frac{12}{2}$$

$$\text{Amplitude} = 6$$

The amplitude of the function is 6 m.

c) Period = second max – first max

$$\text{Period} = 7 - 3$$

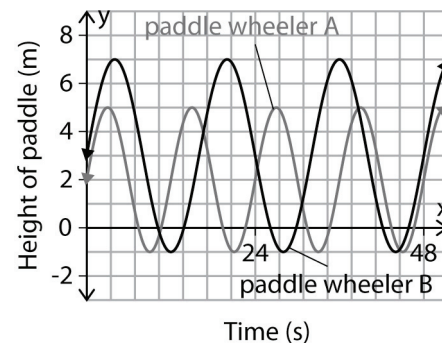
$$\text{Period} = 4 \text{ s}$$

The period of the function is 4 seconds. The period represents the time it takes to swing back and forth once.

d) The closest Olivia came to the motion detector is 2 m.

e) e.g., Yes, since she is at her furthest distance from the detector.  $t = 7 \text{ s}$  corresponds to a maximum, so Olivia is her maximum distance, 14 m, away from the motion detector.

6. The radius of each wheel corresponds to the amplitude of the graph. The height of the axle relative to the water corresponds to the equation of the midline. The time taken to complete one revolution corresponds to the period of the graph.



a) Paddle Wheeler A:

Maximum = 5 m

Minimum = -1 m

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{5 - (-1)}{2}$$

$$\text{Amplitude} = \frac{6}{2}$$

$$\text{Amplitude} = 3$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{5 + (-1)}{2}$$

$$y = \frac{4}{2}$$

$$y = 2$$

Period: The curve repeats at about 3.5 squares on the graph, which is about  $\frac{24}{7} \cdot 3.5 = 12$  s.

Period = 12 s

The radius of the wheel is 3 m, the height of the axle relative to the water is 2 m, and the time taken to complete one revolution is 12 s.

Paddle Wheeler B:

Maximum = 7 m Minimum = -1 m

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{7 - (-1)}{2}$$

$$\text{Amplitude} = \frac{8}{2}$$

Amplitude = 4

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{7 + (-1)}{2}$$

$$y = \frac{6}{2}$$

y = 3

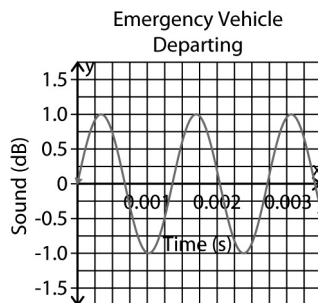
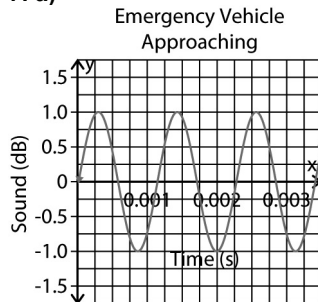
Period: The curve repeats at about 4.7 squares on the graph, which is about  $\frac{24}{7} \cdot 4.7 = 16$  s.

Period = 16 s

The radius of the wheel is 4 m, the height of the axle relative to the water is 3 m, and the time taken to complete one revolution is 16 s.

b) They are travelling at about the same speed (75 m in 48 s).

7. a)



The siren has a higher frequency as it drives towards you, since the period is shorter.

b) Towards you:

$$\text{Frequency} = \frac{1}{\text{period}}$$

$$\text{Frequency} = \frac{1}{\left(\frac{2\pi}{b}\right)}$$

$$\text{Frequency} = \frac{5510.35}{2\pi}$$

$$\text{Frequency} = 876.999\dots$$

Away from you"

$$\text{Frequency} = \frac{1}{\text{period}}$$

$$\text{Frequency} = \frac{1}{\left(\frac{2\pi}{b}\right)}$$

$$\text{Frequency} = \frac{4618.14}{2\pi}$$

$$\text{Frequency} = 734.999\dots$$

The frequency as the emergency vehicle drives towards you and away from you is 877 Hz and 735 Hz respectively.

8. a)  $\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$$\text{Amplitude} = \frac{3 - (-7)}{2}$$

$$\text{Amplitude} = \frac{10}{2}$$

$$\text{Amplitude} = 5$$

The amplitude of this graph is 5, which eliminates equation i).

The rest of the equations are equivalent except for the horizontal translations.

Graph A crosses the midline and is increasing at  $30^\circ$ , so the horizontal translation of the sine function is  $30^\circ$  to the right.

Equation iv) corresponds to Graph A.

b) The amplitude of this graph is the same as Graph A,  $a = 5$ , which eliminates equation i).

The first maximum of Graph B occurs at  $30^\circ$ , so the horizontal translation of the cosine function is  $30^\circ$  to the right.

Equation ii) corresponds to Graph B.

9. a)  $a = 4, b = 6, c = 2.5, d = 3$

$$\text{Maximum} = d + a$$

$$\text{Maximum} = 3 + 4$$

$$\text{Maximum} = 7$$

$$\text{Minimum} = d - a$$

$$\text{Minimum} = 3 - 4$$

$$\text{Minimum} = -1$$

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Amplitude} = \frac{7 - (-1)}{2}$$

$$\text{Amplitude} = \frac{8}{2}$$

$$\text{Amplitude} = 4$$

$$y = \frac{\text{max} + \text{min}}{2}$$

$$y = \frac{7 + (-1)}{2}$$

$$y = \frac{6}{2}$$

$$y = 3$$

Range:  $\{y \mid -1 \leq y \leq 7, y \in \mathbb{R}\}$

$$\text{Period} = \frac{2\pi}{b}$$

$$\text{Period} = \frac{2\pi}{6}$$

$$\text{Period} = \frac{\pi}{3}$$

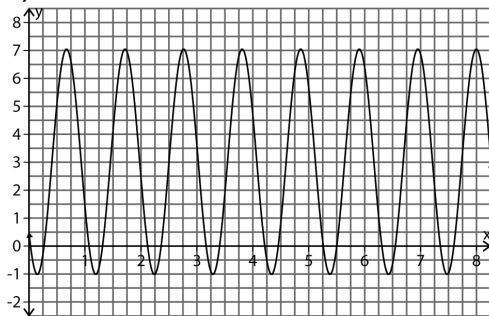
Horizontal translation = 2.5 to the right

The amplitude of this function is 4. The equation of the midline is  $y = 3$ . The range of this equation

is  $\{y \mid -1 \leq y \leq 7, y \in \mathbb{R}\}$ . The period is  $\frac{\pi}{3}$  or

1.047.... The horizontal translation is 2.5 to the right.

**b)**



10.  $\text{Frequency} = \frac{1}{\text{period}}$

$$\text{Frequency} = \frac{1}{\left(\frac{2\pi}{b}\right)}$$

$$\text{Frequency} = \frac{b}{2\pi}$$

$$\text{Frequency} = \frac{15.08}{2\pi}$$

$$\text{Frequency} = 2.400\dots$$

$$\text{Frequency} = \frac{1}{\text{period}}$$

$$\text{Frequency} = \frac{1}{\left(\frac{2\pi}{b}\right)}$$

$$\text{Frequency} = \frac{b}{2\pi}$$

$$\text{Frequency} = \frac{31.42}{2\pi}$$

$$\text{Frequency} = 5.000\dots$$

The frequency of the WiFi signals are 2.4 cycles per nanosecond and 5.0 cycles per nanosecond respectively.

11. e.g., To predict the average temperature in Humboldt on December 8, plot the data using graphing technology and determine the equation of the sinusoidal regression function.

The equation of the sinusoidal regression function is

$$y = 18.976\dots \sin(0.466\dots x - 1.688\dots) - 0.798\dots$$

Using the data points Jan = 1, Feb = 2, etc., and assuming that the average temperature is for the middle of the month, December 8 would be 11.75.

Substitute  $x = 11.75$ .

$$y = 18.976\dots \sin(0.466\dots x - 1.688\dots) - 0.798\dots$$

e.g., I predict the average temperature on December 8 to be about  $-12.2^\circ\text{C}$ .

12. e.g., To predict the monthly low temperatures, assume that the average temperatures occur in the middle of the month and each day number is for those days. Plot the data using graphing technology and determine the sinusoidal regression functions.

The equation of the sinusoidal regression function for Edmonton is

$$y = 15.181\dots \sin(0.015\dots x - 1.453\dots) - 5.412\dots$$

The equation of the sinusoidal regression function for Rio de Janeiro is

$$y = 2.567\dots \sin(0.017\dots x + 1.002\dots) + 20.949\dots$$

Substitute  $x = 179$  (June 28)

Edmonton:

$$y = 15.181\dots \sin(0.015\dots x - 1.453\dots) - 5.412\dots$$

$$y = 9.268\dots$$

Rio de Janeiro:

$$y = 2.567\dots \sin(0.017\dots x + 1.002\dots) + 20.949\dots$$

$$y = 18.861\dots$$

$$\text{Difference: } 18.861\dots - 9.268\dots = 9.593\dots$$

The difference is about  $9.6^\circ\text{C}$ .