I wrote a table of values for the first few guesses.

n	***q***
1000	0
$1000\left(\frac{1}{2}\right)$	1
$\left[1000\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right)=1000\left(\frac{1}{2}\right)^2$	2
$\left[1000\left(\frac{1}{2}\right)^2\right]\left(\frac{1}{2}\right)=1000\left(\frac{1}{2}\right)^3$	3

I noticed that the exponent on the base $\left(\frac{1}{2}\right)$ is equal to *q*, the number of questions asked. So, I set up an equation expressing *n* in terms of *q* to model the situation.

$$n=1000\left(\frac{1}{2}\right)^q$$

• I substituted *n* = 1 into my equation.

$$1=1000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1000}=\left(\frac{1}{2}\right)^q$$

$$q=\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1000}\right)$$

$$q=9.965...$$

You would need to ask 10 questions to find a number from 1 to 1000. This answer matches my experimental answer.

• To determine the number of questions needed to guess a number from 1 to one billion, I changed 1000 to 1 000 000 000 in my equation.

$$n = 1\ 000\ 000\ 000\left(\frac{1}{2}\right)^q$$

Then I substituted *n* = 1 and solved for *q*:

$$1=1\ 000\ 000\ 000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1\ 000\ 000\ 000}=\left(\frac{1}{2}\right)^q$$

$$q=\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1\ 000\ 000\ 000}\right)$$

$$q=29.897...$$

You would need to ask 30 questions to guess a number from 1 to 1 000 000 000.

• Answers will vary. e.g., One possibility is to allow the guesser to divide the list of remaining numbers into three groups, instead of two. Questions would now be like this: “Is your number between *a* and *b*, *b* and *c*, or *c* and *d*?” This would make the game go much more quickly, since each turn reduces the number of remaining numbers by $\frac{2}{3}$.

To guess a number from 1 to 1000, the equation that would model the situation would now be

$$n=1000\left(\frac{1}{3}\right)^q$$

$$1=1000\left(\frac{1}{3}\right)^q$$

$$\frac{1}{1000}=\left(\frac{1}{3}\right)^q$$

$$q=\log_{\left(\frac{1}{3}\right)}\left(\frac{1}{1000}\right)$$

$$q=6.287...$$

Chapter Self-Test, page 501

1. To match the functions with the graphs, look at the *x*- or *y*-intercepts and directions of the functions and of the graphs. This is because each function and each graph has a unique *y*-intercept.

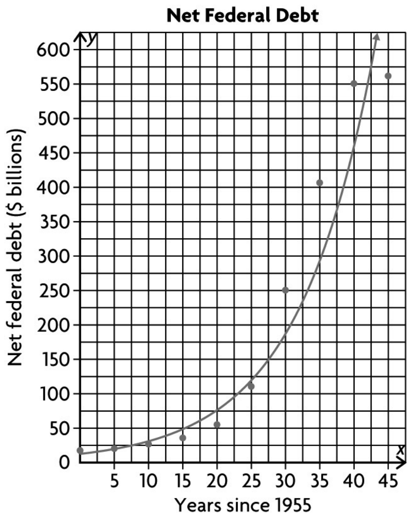
a) It is a decreasing exponential function with *y*-intercept of 0.2 so it must match with ii.

b) It is a decreasing logarithmic function, so it must match with iv.

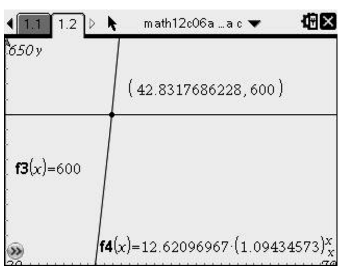
c) It is an increasing exponential function with *y*-intercept of 2, so it must match with iii.

d) It is an increasing logarithmic function, so it must match with i.

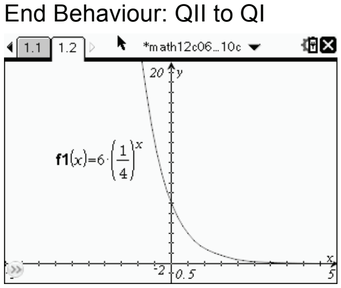
2. a) The exponential regression function is $y = 12.620\ldots(1.094\ldots)^x$, where y is the debt in billions of dollars and x is the number of years after 1955.



b) 1988 – 1955 = 33
$y = 12.620\ldots(1.094\ldots)^{33}$
$y = 12.620\ldots(19.592\ldots)$
$y = \$247.282\ldots$ billion
The net federal debt was $247.28 billion.
c) Assuming the same growth rate and that these figures were taken at the end of the year, the net federal debt should have reached $600 billion in 1998.



3. Number of x-intercepts: 0
y-intercept: $y = 6$
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y > 0, y \in R\}$
End Behaviour: QII to QI



4. x-intercept: 1
y-intercept: none
End behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Function: decreasing

5. a) Let x represent the energy released. Let y represent the magnitude of the earthquake.
The regression equation is
$y = -1.199\ldots + 0.289\ldots\ln x$.
b) $x = 1.1 \cdot 10^{16}$
$y = -1.199\ldots + 0.289\ldots\ln x$
$y = -1.199\ldots + 0.289\ldots\ln(1.1 \cdot 10^{16})$
$y = 9.493\ldots$
The Valdivia earthquake had a magnitude of about 9.5.

Chapter Review, page 504

1. a) e.g., The end behaviours of exponential functions extend from QII to QI. Polynomials have varying end behaviours.
b) e.g., The domain is always $\{x \mid x \in R\}$, the range is always $\{y \mid y > 0, y \in R\}$, and they all extend from QII to QI. a is the y-intercept.

2. a) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
y-intercept: $y = 9$; end behaviour: QII to QI
This is a decreasing function.
b) To make the function into an increasing exponential function, the b should be changed so Its value is greater than 1.
3. a) i) number of x-intercepts: 0
ii) y-intercept: 125
iii) end behaviour: QII to QI
iv) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
v) The function decreases.
b) i) number of x-intercepts: 0
ii) y-intercept: 0.12
iii) end behaviour: QII to QI
iv) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
v) The function decreases.
c) i) number of x-intercepts: 0
ii) y-intercept: 1
iii) end behaviour: QII to QI
iv) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
v) The function increases.
d) i) number of x-intercepts: 0
ii) y-intercept: 0.85
iii) end behaviour: QII to QI
iv) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$
v) The function increases.

4. To match the functions with the graphs, look at the y-intercepts of the functions and of the graphs. This is because each function and each graph has a unique y-intercept.
a) It is an increasing exponential function with y-intercept of 5, so it must match with i.
b) It is a decreasing exponential function with y-intercept of 2 so it must match with ii.