Rn-222:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{3.823}}$$

where *x* represents time in days

Ra-226:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{1600}}$$

Th-227:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{18.5}}$$

where *x* represents time in days

Th-230:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{8.0\times10^4}}$$

Th-231:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{25.5}}$$

where *x* represents time in hours

Th-234:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{2.41}}$$

where *x* represents time in days

Applying Problem-Solving Strategies, page 469

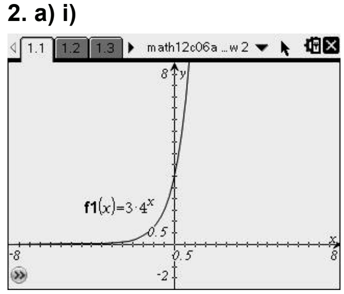
A.–G. Answers will vary, based on the decisions that players make.

H. Possible variations:
- Increase the size of the playing square, or use another shape.
- If a player hits a square, she or he gets to go again immediately.
- Once a function is announced, the opposing player must tell the person if the function passes above, below, or between her or his squares.
- Assign points to each hit, or start out with a given number of points and deduct points for misses or questions (for example, "Does the function pass above your squares?").

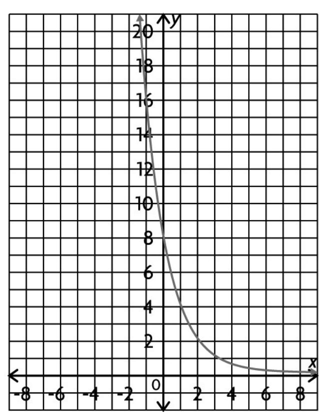
Mid-Chapter Review, page 472

1. a) This graph does not represent an exponential function because it does not have a horizontal asymptote. In addition, its end behaviour is unlike that of an exponential equation since it extends from QII to QIV.
b) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function. It is constantly decreasing at an unsteady rate like an exponential decay function. It has a *y*-intercept but no *x*-intercept. It has a domain of $\{x \mid x \in R\}$ and $a > 0$ and $0 < b < 1$. Also, it decreases from QII to QI.
c) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function It is constantly increasing at an unsteady rate like an exponential growth function. It has a *y*-intercept but no *x*-intercept. It has a domain of $\{x \mid x \in R\}$ with $a < 0$ and $0 < b < 1$. Also, it decreases from QII to QI.
d) This graph does represent an exponential function because it has all of the basic characteristics of an exponential function. It has only one asymptote that is horizontal. It is constantly decreasing at an unsteady rate like an exponential decay function. It has a *y*-intercept but no *x*-intercept. it has a domain of $\{x \mid x \in R\}$, with $a < 0$ and $b > 1$. Also, it decreases from QII to QI.

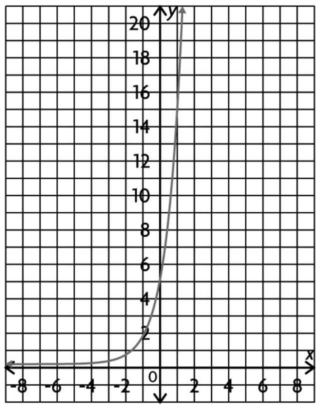
2. a) i)



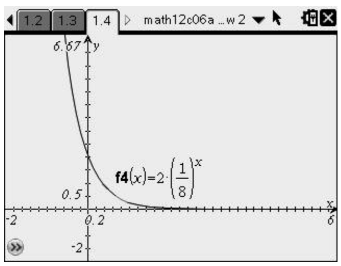
ii)



iii)

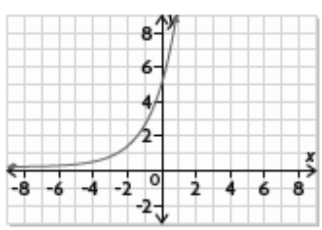


iv)

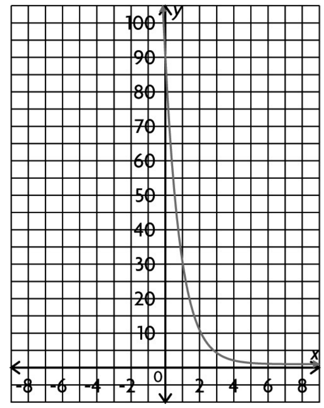


b) i) number of x intercepts: 0; y-intercept: $y = 3$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y < 0, y \in \mathrm{R}\}$
end behaviour: QII to QI
ii) number of x-intercepts: 0; y-intercept: $y = 8$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
end behaviour: QII to QI
iii) number of x-intercepts: 0; y-intercept: $y = 5$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
end behaviour: QII to QI
iv) number of x-intercepts: 0; y-intercept: $y = 2$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
end behaviour: QII to QI

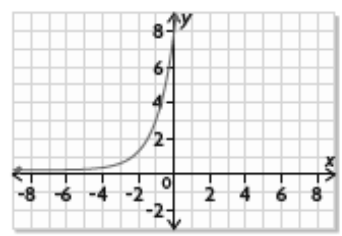
3. a) Number of x-intercepts: 0; y-intercept: $y = 5$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
The function is increasing.



b) Number of x-intercepts: 0; y-intercept: $y = 90$
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
The function is decreasing.

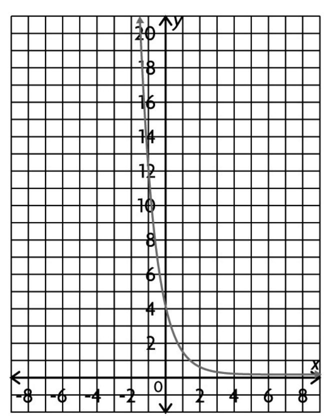


c) number of x-intercepts: 0
y-intercept: 8
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y > 0, y \in \mathrm{R}\}$
The function is increasing.

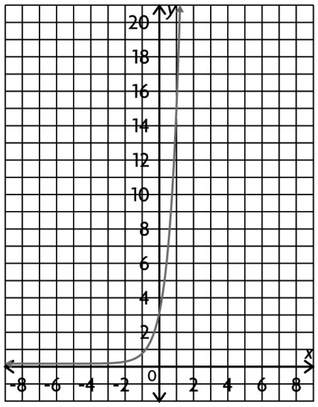


4. a) b) $a = 2$; c) $a = 5$; d) $a = 3$
b) b) It is between $0 < b < 1$ because the function is decreasing.
c) It is between $0 < b < 1$ because the function is decreasing.
d) b is greater than 1 because the function is increasing.
c) b) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
c) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$
d) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$

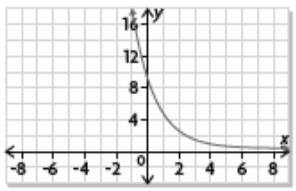
5. a) e.g., It is a decreasing function because the base is less than 1.



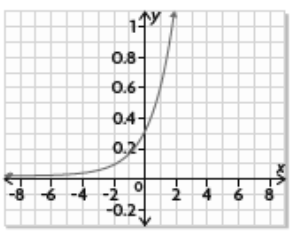
b) e.g., It is an increasing function because the base is greater than 1.



c) e.g., It is a decreasing function because the base is less than 1.



d) e.g., It is an increasing function because the base is greater than 1.



6. a) Assuming that Lauren does not count as a student who has received a handout, the following is a table of values that models this situation:

Round (x)	Number of Students Who Received Handouts (y)
0	7
1	49
2	343
3	2401
4	16807

b) Yes. $y = 7(7)^x$; e.g., The ratio of consecutive numbers of students who received handouts in each round is constant.

7. Using a graphing calculator, the function is $y = (36.871\ldots)(0.663\ldots)^x$.

8. a) Using a graphing calculator, the function is $y = (7.628\ldots)(1.742\ldots)^x$.

b) i) $y = (7.628\ldots)(1.742\ldots)^{0.5}$
$y = (7.628\ldots)(1.320)$
$y = \$10.069\ldots$
The card had a value of about $10.07 6 months after purchase.

ii) $y = (7.628\ldots)(1.742\ldots)^{1.5}$
$y = (7.628\ldots)(2.300\ldots)$
$y = \$17.546\ldots$
The card had a value of about $17.55 18 months after purchase.

iii) $y = (7.628\ldots)(1.742\ldots)^{2.5}$
$y = (7.628\ldots)(4.008\ldots)$
$y = \$30.575\ldots$
The card had a value of about $30.58 30 months after purchase.

iv) $y = (7.628\ldots)(1.742\ldots)^{3.5}$
$y = (7.628\ldots)(6.984\ldots)$
$y = \$53.279\ldots$
The card had a value of about $53.28 42 months after purchase.

9. a) e.g., First divide amounts in consecutive rows to determine the base, then use the initial amount and the base to write an exponential formula.

b)

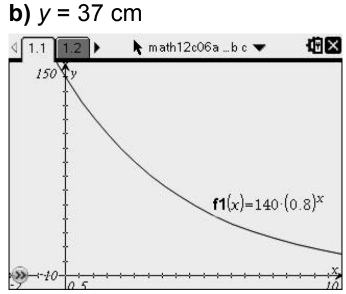
Time (years)	Amount ($)	Increase in Amount (%)
0	1500.00	
1	1560.00	4
2	1622.40	4
3	1687.30	4.000…
4	1754.79	3.999…

The annual interest rate is 4%.

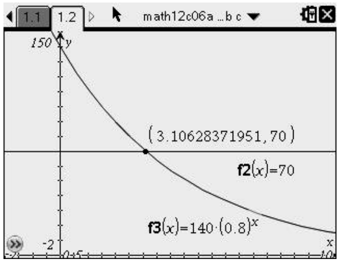
c) The function, which we must use, is
$y = 1500(1.04)^x$.
$y = 1500(1.04)^{10}$
$y = 1500(1.480\ldots)$
$y = \$2220.366\ldots$
Paula will have $2220.37 in her account after 10 years.

10. a) $y = 140\ (0.80)^0$
$y = 140(1)$
$y = 140$ cm
The ball was first dropped from a height of 140 cm.

b) $y = 37$ cm

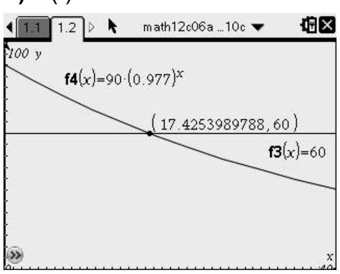


c) $x = 3.106\ldots$ when $y = 70$ cm

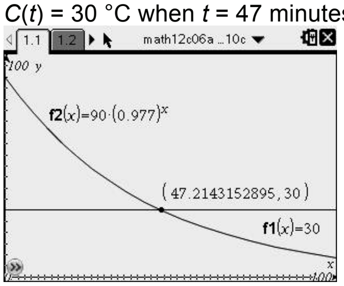


The height was less than half the initial drop height on the 4th bounce.

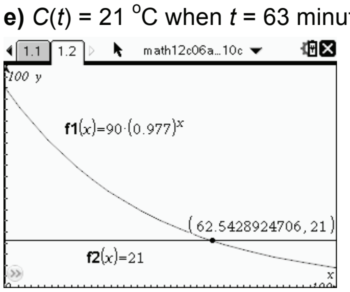
11. a) The function is decreasing. It has one y-intercept and no x-intercepts.
b) The domain is time, so it must be positive: $\{t \mid t > 0, x \in R\}$
The range is all the possible temperatures from 90 °C to 21 °C: $\{C(t) \mid 21 < C(t) < 90, C(t) \in R\}$
c) $C(t) = 90(0.977)^t$
$C(10) = 90(0.977)^{10}$
$C(10) = 71.316\ldots$
The temperature of the coffee after 10 min is 71 °C.
d) $C(t) = 60$ °C when $t = 17$ minutes.



$C(t) = 30$ °C when $t = 47$ minutes.



e) $C(t) = 21$ °C when $t = 63$ minutes.



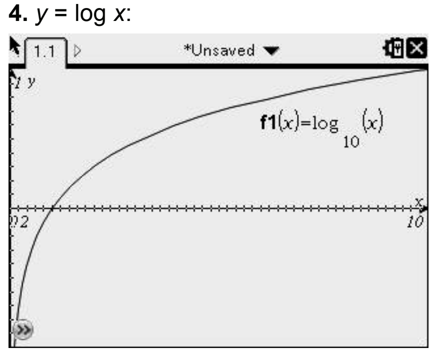
Lesson 7.4: Characteristics of Logarithmic Functions with Base 10 and Base *e*, page 482

1. x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$

2. a) No. e.g., no x-intercept and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept. It is an exponential function.
b) No. e.g., two x-intercepts and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept. It is a quadratic function.
c) Yes. e.g., one x-intercept and no y-intercept.
d) No. e.g., one x-intercept and one y-intercept whereas logarithmic functions have x-intercepts of 1.
e) Yes. e.g., one x-intercept and no y-intercept.
f) No. e.g., no x-intercept and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept.

3. c: x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
a: $a > 0$ e.g., since the graph is increasing
e: x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
a: $a < 0$ e.g., since the graph is decreasing

4. $y = \log x$:



x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Increasing or decreasing: increasing
$y = -\log x$: