

History Connection, page 423

A.

x	$f(x)$
0	41
1	43
2	47
3	53
4	61
5	71
6	83
7	97
8	113
9	131
10	151
11	173
12	197
13	223
14	251
15	281
16	313
17	347
18	383
19	421
20	461
21	503
22	547
23	593
24	641
25	691
26	743
27	797
28	853
29	911
30	971
31	1033
32	1097
33	1163
34	1231
35	1301
36	1373
37	1447
38	1523
39	1601

B. $f(x) = x^2 + x + 41$

$$f(40) = 40^2 + 40 + 41$$

$$f(40) = 1681$$

I can tell that $f(40)$ is not prime because I can factor the expression.

$$f(40) = 40^2 + 40 + 41$$

$$f(40) = 40(40 + 1) + 41$$

$$f(40) = 40(41) + 41$$

$$f(40) = 41(40 + 1)$$

$$f(40) = (41)(41)$$

C. The quadratic function has a positive leading coefficient and a constant term of 41. This means that the graph is a parabola that opens up and has a y -intercept of 41. These characteristics are important because 41 is prime. If 41 were not prime, then the very first number generated by the function would not be prime. Opening up is also important. If the parabola opened down, then the function would reach zero very quickly and it would not likely generate many prime numbers.

D. $f(x) = x^2 + x + 5$ generates 4 consecutive prime numbers.

$f(x) = x^2 + x + 11$ generates 9 consecutive prime numbers.

$f(x) = x^2 - x + 11$ generates 10 consecutive prime numbers.

Chapter Self-Test, page 424

1. a) i) e.g., $y = -x + 3$

ii) e.g., $y = x - 6$

b) i) e.g., $y = -2x^3 - 7x^2 + 24x + 45$

ii) e.g., $y = 4x^3 + 12x^2 - 5x - 6$

2. a) Possible number of x -intercepts: 0, 1, or 2
 y -intercept: -1

End behaviour: graph extends from quadrant III to quadrant IV.

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \leq 3, y \in \mathbb{R}\}$

Possible number of turning points: 1

b) Possible number of x -intercepts: 1

y -intercept: 1.5

End behaviour: graph extends from quadrant III to quadrant I.

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

Possible number of turning points: 0

c) Possible number of x -intercepts: 1, 2, or 3

y -intercept: -42

End behaviour: graph extends from quadrant III to quadrant I.

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

Possible number of turning points: 0 or 2

d) Possible number of x -intercepts: 1, 2, or 3

y -intercept: 2

End behaviour: graph extends from quadrant II to quadrant IV.

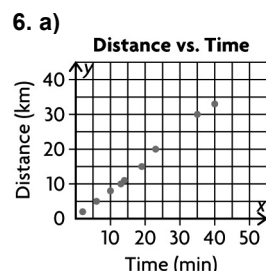
Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

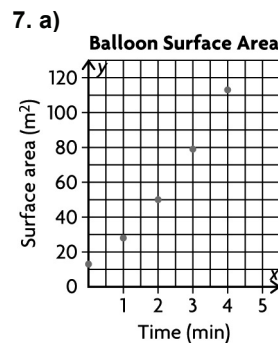
Possible number of turning points: 0 or 2

- 3. a)** e.g., Since the function must have two turning points, the function must be cubic. Since the function must extend from quadrant II to quadrant IV, the leading coefficient must be negative. Since the function must have a y -intercept of -6 , the constant term must be -6 . Therefore a function that satisfies these characteristics is $y = -x^3 - 6x^2 - 11x - 6$.
- b)** e.g., Since the function must have degree 2, it must be quadratic. Since the function extends from quadrant III to quadrant IV, the leading coefficient must be negative. Since the function must have x -intercepts of -2 and 4 , the equation of the function in factored form must be $y = a(x + 2)(x - 4)$, where $a < 0$. By letting $a = -1$ and expanding this equation, a function that satisfies these characteristics is $y = -x^2 + 2x + 8$.
- 4. a)** The degree of this function is 2, and the leading coefficient is 2. Thus, the graph extends from quadrant II to quadrant I. The y -intercept is 12, and the possible number of x -intercepts is 0, 1, or 2. The possible number of turning points is 1. The domain is $\{x \mid x \in \mathbb{R}\}$ and the range is $\{y \mid y \geq 4, y \in \mathbb{R}\}$.
- b)** The degree of this function is 3, and the leading coefficient is -1 . This means that the graph extends from quadrant II to quadrant IV. The y -intercept is -1 , and the possible number of x -intercepts is 1, 2, or 3. The possible number of turning points is 0 or 2. The domain is $\{x \mid x \in \mathbb{R}\}$ and the range is $\{y \mid y \in \mathbb{R}\}$.

5. e.g., The line of best fit will have a slope of about -1 , and the y -intercept will be about 120. About the same number of points lie above and below the line.



- b)** The linear regression equation is $y = 0.844...x - 0.308...$
- c) i)** $y = 0.844...x - 0.308...$
 $25 = 0.844...x - 0.308...$
 $x = 29.975...$
 It took 30 min to drive 25 km.
- ii)** $y = 0.844...x - 0.308...$
 $y = 0.844...(45) - 0.308...$
 $y = 37.685...$
 Toby could've travelled 38 km in 45 min.
- iii)** From the linear regression equation, the average speed is $0.844... \text{ km/min}$.
 $0.844... \text{ km/min} = (0.844... \text{ km/min})(60 \text{ min/h})$
 $= 50.659... \text{ km/h}$
 His average speed for the trip was 50.7 km/h .



- b)** The quadratic regression equation is $y = 3.214...x^2 + 12.242...x + 12.828...$
- c)** e.g., Interpolate: when $x = 1.5 \text{ min}$:
 $y = 3.214...x^2 + 12.242...x + 12.828...$
 $y = 3.214...(1.5)^2 + 12.242...(1.5) + 12.828...$
 $y = 38.425$
 The surface area of the balloon at 1.5 min is about 38.4 m^2 .
- Extrapolate: when $x = 5 \text{ min}$:
 $y = 3.214...x^2 + 12.242...x + 12.828...$
 $y = 3.214...(5)^2 + 12.242...(5) + 12.828...$
 $y = 154.4$
 The surface area of the balloon at 5 min is about 154.4 m^2 .

Chapter Review, page 427

- 1. a)** This is a polynomial function since the graph extends from quadrant II to quadrant IV, it has 1 y -intercept, 0 turning points and 1 x -intercept. It appears to be a cubic function.
- b)** This is not a polynomial function, because it is not a function at all. For a curve to be a function, every x -value must have only one corresponding y -value, which is not true for this curve.
- c)** This is not a polynomial function because it has no x -intercepts but it is not a constant function.
- d)** This is a polynomial function since the graph extends from quadrant III to quadrant I, it has 1 y -intercept, no turning points and 1 x -intercept. It appears to be a linear function.
- e)** This is a polynomial function since the graph extends from quadrant III to quadrant IV, it has 1 y -intercept, 1 turning point and 2 x -intercepts. It appears to be a quadratic function.
- f)** This is not a polynomial function since the graph has infinitely many turning points.
- 2. a)** This is not a polynomial function, because there is a variable in the denominator of a fraction.
- b)** Possible number of x -intercepts: 1, 2, or 3
 y -intercept: -4
 Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y \in \mathbb{R}\}$
 Possible number of turning points: 0 or 2
- c)** Possible number of x -intercepts: 0, 1, or 2
 y -intercept: -32
 Domain: $\{x \mid x \in \mathbb{R}\}$; Range: $\{y \mid y \leq 0, y \in \mathbb{R}\}$
 Possible number of turning points: 1
- d)** This is not a polynomial function, because there is a variable under a square root sign.