18. e.g., I would use permutations in a problem where the order of the items was important, and use combinations in a problem where order was not important. Permutations: Determine the probability that two items are next to each other in a lineup of seven different items that has been placed in a random order. Combinations: Determine the probability that, given eight books, four of which are about math, if I choose five of the eight books, I choose three math books.

19. Let *A* represent a route that passes the pool, and let *O* represent all routes. The route Tuyet takes consists of 5 'rights' and 4 'downs'. Therefore, the total number of possible routes is the total number of moves (9!), dividing out repetition (5! · 4!)

$$n(O) = \frac{9!}{5! \cdot 4!}$$

$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2}$$

$$n(O) = \frac{3024}{24}$$

$$n(O) = 126$$

To pass by the pool, Tuyet must first take 3 'rights' and 2 'downs', then must take 2 'rights' and 2 'downs' in any order. Again, repetition must be divided out.

$$n(A) = \frac{5!}{3! \cdot 2!} \cdot \frac{4!}{2! \cdot 2!}$$

$$n(A) = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2 \cdot 1}$$

$$n(A) = \frac{5 \cdot 4}{2} \cdot \frac{4 \cdot 3}{2}$$

$$n(A) = 5 \cdot 2 \cdot 2 \cdot 3$$

$$n(A) = 60$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{60}{126}$$

$$P(A) = \frac{10}{21}$$

The probability that Tuyet passes the pool is $\frac{10}{21}$, or 47.6%.

20. I know from chapter 4 that the formula to use is $1 - \frac{{}_{365}P_n}{365^n}$, where n is the number of people. I tried a large number, because I thought that there would need to be a lot of people for the percentage to be as high as 80%. I tried *n* = 100.

$1 - \frac{{}_{365}P_{100}}{365^{100}}$ The calculator gave an error message. This number was too high. I tried *n* = 30.

$1 - \frac{{}_{365}P_{30}}{365^{30}} = 0.706...$. This answer was about 71%, so I thought n would be a little more. I tried n = 35.

$1 - \frac{{}_{365}P_{35}}{365^{35}} = 0.814...$ This is a little over 80%. There would need to be 35 people in a room for the probability that two of them have the same birthday is 80%.

History Connection, page 324

Let us now follow in Pascal's footsteps and analyze correctly the chances of winning in these two games.
Single die Rolling a single die once leads to precisely one of 6 possible outcomes: Exactly one of the numbers 1, 2, 3, 4, 5, or 6 will be on top. The die is called fair, if each of these outcomes is equally likely. Players of dice games usually assume they are using fair dice, so I will assume this too. If I roll a die 4 times, then the total number of all possible outcomes is 6 · 6 · 6 · 6 = 1296
Out of these there are 5 · 5 · 5 · 5 = 625 outcomes with **no** 6 in them. Thus, if I bet on getting at least one 6 when rolling a die 4 times, there are 625 possibilities of losing, and 1296 – 625 = 671 possibilities of winning. So, my chances of winning are higher than my chances of losing.
Two dice Let us now turn to the two-dice game. Rolling two dice once leads to one of 36 possible outcomes, namely all possible outcomes of rolling die number 1 combined with all possible outcomes of rolling die number two. Thus, if we roll two dice 24 times, then the total number of possible outcomes is 36 · 36 · … · 36 (36 multiplied with itself 24 times), which is approximately $2.245\ldots \cdot 10^{37}$. Out of these there are 35 · 35 · … · 35 (35 multiplied with itself 24 times), which is approximately $1.141\ldots \cdot 10^{37}$ outcomes with **no** double 6. Thus, if I gamble on getting at least one double 6 when rolling two dice 24 times, there are approximately $1.141\ldots \cdot 10^{37}$ possibilities of losing, and $1.103\ldots \cdot 10^{37}$ possibilities of winning. This means that the chances of winning with this game are lower than the chances of losing—as the Chevalier De Mere learned the hard way.

Mid-Chapter Review, page 327

1. a) Outcome table

Dice roll	Product	Sum	Who wins?
(1,1,1)	1	3	Erik
(1,1,2)	2	4	Erik
(1,1,3)	3	5	Erik
(1,1,4)	4	6	Erik
(1,2,1)	2	4	Erik
(1,2,2)	4	5	Erik
(1,2,3)	6	6	Ethan
(1,2,4)	8	7	Ethan
(1,3,1)	3	5	Erik
(1,3,2)	6	6	Ethan
(1,3,3)	9	7	Ethan

(1,3,4)	12	8	Ethan
(1,4,1)	4	6	Erik
(1,4,2)	8	7	Ethan
(1,4,3)	12	8	Ethan
(1,4,4)	16	9	Ethan
(2,1,1)	2	4	Erik
(2,1,2)	4	5	Erik
(2,1,3)	6	6	Ethan
(2,1,4)	8	7	Ethan
(2,2,1)	4	5	Erik
(2,2,2)	8	6	Ethan
(2,2,3)	12	7	Ethan
(2,2,4)	16	8	Ethan
(2,3,1)	6	6	Ethan
(2,3,2)	12	7	Ethan
(2,3,3)	18	8	Ethan
(2,3,4)	24	9	Ethan
(2,4,1)	8	7	Ethan
(2,4,2)	16	8	Ethan
(2,4,3)	24	9	Ethan
(2,4,4)	32	10	Ethan
(3,1,1)	3	5	Erik
(3,1,2)	6	6	Ethan
(3,1,3)	9	7	Ethan
(3,1,4)	12	8	Ethan
(3,2,1)	6	6	Ethan
(3,2,2)	12	7	Ethan
(3,2,3)	18	8	Ethan
(3,2,4)	24	9	Ethan
(3,3,1)	9	7	Ethan
(3,3,2)	18	8	Ethan
(3,3,3)	27	9	Ethan
(3,3,4)	36	10	Ethan
(3,4,1)	12	8	Ethan
(3,4,2)	24	9	Ethan
(3,4,3)	36	10	Ethan
(3,4,4)	48	11	Ethan
(4,1,1)	4	6	Erik
(4,1,2)	8	7	Ethan
(4,1,3)	12	8	Ethan
(4,1,4)	16	9	Ethan
(4,2,1)	8	7	Ethan
(4,2,2)	16	8	Ethan
(4,2,3)	24	9	Ethan
(4,2,4)	32	10	Ethan
(4,3,1)	12	8	Ethan
(4,3,2)	24	9	Ethan
(4,3,3)	36	10	Ethan
(4,3,4)	48	11	Ethan
(4,4,1)	16	9	Ethan
(4,4,2)	32	10	Ethan
(4,4,3)	48	11	Ethan
(4,4,4)	64	12	Ethan

$P(\text{sum is higher}) = \frac{13}{64}$

$P(\text{sum is not higher}) = \frac{51}{64}$

This game is not fair. Ethan has the advantage. There are more products that are greater than the sums.

b) Fair. e.g., Coins have an equal chance of landing on heads and landing on tails. Therefore, since each player has the same requirements to win, but with the opposite coin face, this game is fair.

2. a) Outcome Table:

	Spinner 1					
	SUM	**1**	**2**	**3**	**4**	**5**
Spinner 2	**1**	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9

Total number of outcomes = 20

b) i) $P(\text{sum of 5}) = \frac{4}{20}$

$P(\text{sum of 5}) = \frac{1}{5}$

The probability of getting a sum of 5 is $\frac{1}{5}$, 0.2 or 20%.

ii) $P(\text{sum of 8 or 9}) = \frac{3}{20}$ The probability of getting a sum of 8 or 9 is $\frac{3}{20}$, 0.15 or 15%.

iii) $P(\text{sum of 4}) = \frac{3}{20}$ The probability of getting a sum of 4 is $\frac{3}{20}$, 0.15 or 15%.

3. a) There are 26 black cards in a standard 52-card deck.

$P(\text{black card}) = \frac{26}{52}$

$P(\text{black card}) = \frac{1}{2}$

The probability of drawing a black card is $\frac{1}{2}$, 0.5 or 50%.

b) Since 26 cards are black, then 26 cards are red. Therefore, the odds in favour of the card being black are 26 : 26, or 1 : 1.

c) There are 13 diamonds in a standard deck, so there are 39 cards that are not diamonds. Therefore, the odds against the card being a diamond are 39 : 13, or 3 : 1.

d) Forty cards in each deck are not face cards.

$P(\text{not face card}) = \frac{40}{52}$

$P(\text{not face card}) = \frac{10}{13}$

The probability the card drawn is not a face card is $\frac{10}{13}$, or about 0.769 or 76.9%.

4. $P(\text{win}) = \frac{5}{23+5}$

$P(\text{win}) = \frac{5}{28}$

The probability he will win is $\frac{5}{28}$, or about 0.179 or 17.9%.

5. a) The probability that two of our prime ministers will have the same birthday is slightly less than 50%, or $\frac{1}{2}$. Therefore, the odds in favour of two prime ministers having the same birthday are slightly less than 1 : 1.

b) The odds against two prime ministers having the same birthday are slightly more than 1 : 1.

c) Yes, Sir John A. Macdonald and Jean Chrétien both share a birthday on January 11. However, Macdonald's official date of birth recorded in the General Register Office in Edinburgh, Scotland is January 10. But, Macdonald celebrated his birthday on January 11.

6. Let *A* represent Frodo, Sam, and Aragorn being chosen for president, treasurer and secretary. Let *O* represent all possible selections. The number of ways that Frodo, Sam, and Aragorn can be chosen is ${}_3P_3$ or 3!. The number of ways that the 14 people can be chosen for the 3 positions is ${}_{14}P_3$.

$n(A) = {}_{14}P_3$

$n(A) = \frac{14!}{(14-3)!}$

$n(A) = \frac{14!}{11!}$

$n(A) = \frac{14 \cdot 13 \cdot 12 \cdot 11!}{11!}$

$n(A) = 14 \cdot 13 \cdot 12$

$n(A) = 2184$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{3!}{2184}$

$P(A) = \frac{3 \cdot 2 \cdot 1}{2184}$

$P(A) = \frac{6}{2184}$

$P(A) = \frac{1}{364}$

The probability Frodo, Sam, and Aragorn will be chosen is $\frac{1}{364}$, or about 0.002 75 or 0.275%.

7. a) Let *A* represent a hand containing the ace, king, queen and jack of the same suit, and let *O* represent all euchre hands. There are 4 different ways to place the A, K, Q, and J of the same suit in a hand (one for each suit). There are 20 ways of placing the fifth card (one for each remaining card). So, there are 80 ways to have the ace, king, queen, and jack of the same suit in a hand. The total number of hands is ${}_{24}C_5$.

$n(O) = {}_{24}C_5$

$n(O) = \frac{24!}{(24-5)! \cdot 5!}$

$n(O) = \frac{24!}{19! \cdot 5!}$

$n(O) = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{19! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$n(O) = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{5 \cdot 4 \cdot 3 \cdot 2}$

$n(O) = \frac{5\ 100\ 480}{120}$

$n(O) = 42\ 504$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{80}{42504}$

$P(A) = \frac{10}{5313}$

The probability that a dealt hand will contain an ace, king, queen and jack of the same suit is $\frac{10}{5313}$, or about 0.001 882 or 0.1882%.

b) Let *F* represent a hand containing five cards of the same colour, and let *O* represent all euchre hands. There are 12 red cards and 12 black cards. The number of ways a hand can have either colour is ${}_{12}C_5$. Considering both colours, the number of ways a dealt hand can have five cards of the same colour is $2 \cdot {}_{12}C_5$.

$n(F) = 2 \cdot {}_{12}C_5$

$n(F) = 2 \cdot \frac{12!}{(12-5)! \cdot 5!}$

$n(F) = 2 \cdot \frac{12!}{7! \cdot 5!}$

$n(F) = \frac{2 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3}$

$n(F) = \frac{95\ 040}{60}$

$n(F) = 1584$

The total number of hands is ${}_{24}C_5$, or 42 504.

$$P(F)=\frac{n(F)}{n(O)}$$
$$P(F)=\frac{1584}{42504}$$
$$P(F)=\frac{6}{161}$$
The probability that a dealt hand contains five cards of the same colour is $\frac{6}{161}$, or about 0.037 267 or 3.727%.

c) Let *F* represent a hand containing a four of a kind, and let *O* represent all euchre hands.
There are 6 different ways to have four of a kind in euchre (one for each rank of card). There are 20 ways to place the fifth card. Therefore, there are 120 ways to have a four of a kind in a dealt hand.
The total number of hands is ${}_{24}C_5$, or 42 504.
$$P(F)=\frac{n(F)}{n(O)}$$
$$P(F)=\frac{120}{42504}$$
$$P(F)=\frac{5}{1771}$$
The probability that a dealt hand will have four of a kind is $\frac{5}{1771}$, or about 0.002 823 or 0.282%.

8. Let *A* represent a playlist in which Emanuella's six favourite songs are played together. Let *O* represent all playlists.
The number of ways to arrange Emanuella's favourite songs so that they are together is ${}_6P_6 \cdot 25$, or $6! \cdot 25$. The number of ways to arrange the other 24 songs is ${}_{24}P_{24}$, or 24!. The number of ways to arrange 30 songs is ${}_{30}P_{30}$, or 30!. Therefore, the total number of playlists that contain the six favourite songs together is $25 \cdot 6! \cdot 24!$, and the total number of playlists is 30!.
$$P(A)=\frac{n(A)}{n(O)}$$
$$P(A)=\frac{6!\cdot 25\cdot 24!}{30!}$$
$$P(A)=\frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 25\cdot 24!}{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25\cdot 24!}$$
$$P(A)=\frac{6\cdot 5\cdot 4\cdot 3\cdot 2}{30\cdot 29\cdot 28\cdot 27\cdot 26}$$
$$P(A)=\frac{6\cdot 5}{30}\cdot\frac{4}{28}\cdot\frac{3}{27}\cdot\frac{2}{26}\cdot\frac{1}{29}$$
$$P(A)=\frac{1}{7\cdot 9\cdot 13\cdot 29}$$
$$P(A)=\frac{1}{23751}$$
The probability that all 6 of Emanuella's favourite songs will be played together is $\frac{1}{23\,751}$, or about 0.000 042 1 or 0.004 21%.

9. Let *C* represent Stella dropping 2 loonies and one other coin. Let *O* represent all of the combinations of 3 coins possible.
Stella has 6 loonies and 6 other coins. The number of ways that 2 loonies and 1 other coin could be dropped is ${}_6C_2 \cdot {}_6C_1$.
$$n(C)={}_6C_2\cdot{}_6C_1$$
$$n(C)=\frac{6!}{(6-2)!\cdot 2!}\cdot\frac{6!}{(6-1)!\cdot 1!}$$
$$n(C)=\frac{6!}{4!\cdot 2!}\cdot\frac{6!}{5!\cdot 1!}$$
$$n(C)=\frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}\cdot\frac{6\cdot 5!}{5!\cdot 1}$$
$$n(C)=\frac{6\cdot 5}{2}\cdot 6$$
$$n(C)=3\cdot 5\cdot 6$$
$$n(C)=90$$
The total number of ways that 3 coins can be dropped is ${}_{12}C_3$.
$$n(O)={}_{12}C_3$$
$$n(O)=\frac{12!}{(12-3)!\cdot 3!}$$
$$n(O)=\frac{12!}{9!\cdot 3!}$$
$$n(O)=\frac{12\cdot 11\cdot 10\cdot 9!}{9!\cdot 3\cdot 2\cdot 1}$$
$$n(O)=\frac{12\cdot 11\cdot 10}{3\cdot 2}$$
$$n(O)=\frac{1320}{6}$$
$$n(O)=220$$
Now determine the probability.
$$P(C)=\frac{n(C)}{n(O)}$$
$$P(C)=\frac{90}{220}$$
$$P(C)=\frac{9}{22}$$
The probability that exactly two of the dropped coins are loonies is $\frac{9}{22}$, or about 0.409 or 40.9%.

Lesson 5.4: Mutually Exclusive Events, page 338

1. a) Let *A* represent rolling a sum of 2. Let *B* represent rolling a sum of 8.
A = {1, 1}, *B* = {4, 4}.