b) $P(F') = 0.3$
$P(F' \cap F') = 0.3 \cdot 0.3$
$P(F' \cap F') = 0.09$
The probability Jarrod will make both shots is 0.09, or 9%.
c) In this case, there are only 3 possibilities. These are make both shots, make one shot, and make neither shot.
Let O represent Jarrod making exactly one shot.
$P(O) = 100\% - P(F \cap F) - P(F' \cap F')$
$P(O) = 100\% - 49\% - 9\%$
$P(O) = 42\%$
The probability Jarrod will make one shot is 0.42 or 42%.
d) Let A represent Jarrod making at least one shot.
$P(A) = 100\% - P(F' \cap F')$
$P(A) = 100\% - 9\%$
$P(A) = 91\%$
The probability Jarrod will make at least one shot is 0.91 or 91%

8. Let H represent two hearts being drawn from the deck, and let O represent all possible drawings. There are 6 hearts in every euchre deck. Therefore, the number of ways to draw two hearts from a Euchre deck is ${}_6C_2$.

$$n(H) = {}_6C_2$$
$$n(H) = \frac{6!}{(6-2)!\cdot 2!}$$
$$n(H) = \frac{6!}{4!\cdot 2!}$$
$$n(H) = \frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}$$
$$n(H) = \frac{6\cdot 5}{2}$$
$$n(H) = 15$$

The number of ways to draw 2 cards from a Euchre deck is ${}_{24}C_2$.

$$n(O) = {}_{24}C_2$$
$$n(O) = \frac{24!}{(24-2)!\cdot 2!}$$
$$n(O) = \frac{24!}{22!\cdot 2!}$$
$$n(O) = \frac{24\cdot 23\cdot 22!}{22!\cdot 2\cdot 1}$$
$$n(O) = \frac{24\cdot 23}{2}$$
$$n(O) = 276$$

Now determine the probability.

$$P(H) = \frac{n(H)}{n(O)}$$
$$P(H) = \frac{15}{276}$$
$$P(H) = \frac{5}{92}$$

The probability that George draws two hearts is $\frac{5}{92}$, or about 0.0543 or 5.43%.

9. Let S represent a set alarm clock, and let N represent an alarm clock that is not set. Let L represent Miguel being late.

$$P(S \cap L) = P(S)\cdot P(L|S)$$
$$P(S \cap L) = 0.72\cdot 0.10$$
$$P(S \cap L) = 0.072$$
$$P(N \cap L) = P(N)\cdot P(L|N)$$
$$P(N \cap L) = 0.28\cdot 0.70$$
$$P(N \cap L) = 0.196$$
$$P(L) = 0.072 + 0.196$$
$$P(L) = 0.268$$
$$P(S|L) = \frac{P(S\cap L)}{P(L)}$$
$$P(S|L) = \frac{0.072}{0.268}$$
$$P(S|L) = 26.865...\%$$

The probability that Miguel remembered to set his alarm is about 0.269 or 26.9%.

Chapter Review, page 367

1. a) Outcome Table

Coin Flips	**Winner**
HHHH	Chloe
HHHT	Tie
HHTH	Tie
HHTT	Camila
HTHH	Tie
HTHT	Camila
HTTH	Camila
HTTT	Tie
THHH	Tie
THHT	Camila
THTH	Camila
THTT	Tie
TTHH	Camila
TTHT	Tie
TTTH	Tie
TTTT	Chloe

$$P(\text{Chloe wins}) = \frac{2}{16}$$
$$P(\text{Chloe wins}) = \frac{1}{8}$$
$$P(\text{Camila wins}) = \frac{6}{16}$$
$$P(\text{Camila wins}) = \frac{3}{8}$$

This game is not fair, e.g., Camila has a better chance of winning.

b) It is equally like that a 1 or a 2 will come up than a 3 or a 4. Therefore, the game is fair, Cooper and Alyssa will have an equal chance at winning.

2. The probability is 0, because when the tension is at 100% you can no longer turn the pedal. At this point the tension is at its maximum. Turning the tension up to 130% implies that tension can be increased beyond its maximum setting. What Bob's instructor probably means is to adjust the tension so that it is 1.3 times greater than it currently is, which may be possible.

3. a) P(female) = 60%; P(not female) = 40%
The odds in favour of this person being female are 60 : 40, or 3 : 2
b) The odds against this person being female are 2 : 3.

4. The odds against rain tomorrow are (100% – 70%) : 70%. This is equal to 30 : 70, or 3 : 7.
5. a) If Keir fell twice, then he didn't fall the other five times. Therefore, the odds in favour of him falling are 2 : 5.
b) The odds against Keir falling are 5 : 2.
6. No, she is not correct. e.g., To calculate the odds, it should be odds against : odds for. If Ariana has scored 6 goals on 30 shots, then she did not score 24 times. Therefore, the odds against her scoring a goal are 24 : 6, or 4 : 1, not 4 : 5.
7. Let F represent Averill getting an A in the first course, and let S represent Averill getting an A in the second course.

$P(F)=\frac{3}{3+8}$ $\quad$ $P(S)=\frac{6}{6+11}$

$P(F)=\frac{3}{11}$ $\quad$ $P(S)=\frac{6}{17}$

$P(F)=27.272...\%$ $\quad$ $P(S)=35.294...\%$

Averill should take the second course, because the probability of getting an A is higher.
8. Let C represent Cameron and Wyatt being chosen for president and secretary, and let O represent all possible choices. Because order is important, there are 2 ways that Cameron and Wyatt can be chosen for president and secretary. The total number of ways that 2 people can be chosen is ${}_9P_2$.

$n(O)={}_9P_2$

$n(O)=\frac{9!}{(9-2)!}$

$n(O)=\frac{9!}{7!}$

$n(O)=\frac{9\cdot 8\cdot 7!}{7!}$

$n(O)=9\cdot 8$

$n(O)=72$

Now determine the probability.

$P(C)=\frac{n(C)}{n(O)}$

$P(C)=\frac{2}{72}$

$P(C)=\frac{1}{36}$

The probability Cameron and Wyatt will be chosen for president and secretary is $\frac{1}{36}$, or about 0.0278 or 2.78%.

9. Let M represent a team containing Marina and MacKenzie, and let O represent all possible teams. The number of ways that Marina and MacKenzie can be placed on the team is ${}_4P_2$, and the number of ways to fill the other 2 spots on the team is ${}_9P_2$. Therefore, the total number of teams containing Marina and MacKenzie is ${}_4P_2\cdot{}_9P_2$.

$n(M)={}_4P_2\cdot{}_9P_2$

$n(M)=\frac{4!}{(4-2)!}\cdot\frac{9!}{(9-2)!}$

$n(M)=\frac{4!}{2!}\cdot\frac{9!}{7!}$

$n(M)=\frac{4\cdot 3\cdot 2!}{2!}\cdot\frac{9\cdot 8\cdot 7!}{7!}$

$n(M)=4\cdot 3\cdot 9\cdot 8$

$n(M)=864$

The total number of possible teams is ${}_{11}P_4$.

$n(O)={}_{11}P_4$

$n(O)=\frac{11!}{(11-4)!}$

$n(O)=\frac{11!}{7!}$

$n(O)=\frac{11\cdot 10\cdot 9\cdot 8\cdot 7!}{7!}$

$n(O)=11\cdot 10\cdot 9\cdot 8$

$n(O)=7920$

Now determine the probability.

$P(M)=\frac{n(M)}{n(O)}$

$P(M)=\frac{864}{7920}$

$P(M)=\frac{6}{55}$

The probability Marina and MacKenzie will be chosen to be on the relay team is $\frac{6}{55}$, or about 0.109 or 10.9%.

10. Let A represent a password containing the letters A, R, and T, and let O represent all possible passwords.

a) There are 3! ways of arranging the letters A, R, and T, and there are ${}_{10}P_4$ ways of arranging the four digits. Therefore, the number of passwords that contain the letters A, R, and T is $3! \cdot {}_{10}P_4$.

$n(A) = 3! \cdot {}_{10}P_4$

$n(A) = 3! \cdot \frac{10!}{(10-4)!}$

$n(A) = 3! \cdot \frac{10!}{6!}$

$n(A) = 3 \cdot 2 \cdot 1 \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!}$

$n(A) = 3 \cdot 2 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

The total number of passwords containing three capital letters followed by four digits, without repetition, is ${}_{26}P_3 \cdot {}_{10}P_4$.

$n(O) = {}_{26}P_3 \cdot {}_{10}P_4$

$n(O) = \frac{26!}{(26-3)!} \cdot \frac{10!}{(10-4)!}$

$n(O) = \frac{26!}{23!} \cdot \frac{10!}{6!}$

$n(O) = \frac{26 \cdot 25 \cdot 24 \cdot 23!}{23!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!}$

$n(O) = 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{3 \cdot 2 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7}$

$P(A) = \frac{3 \cdot 2}{26 \cdot 25 \cdot 24}$

$P(A) = \frac{1}{26 \cdot 25 \cdot 4}$

$P(A) = \frac{1}{2600}$

The probability that a password chosen at random will contain the letters A, R, and T is $\frac{1}{2600}$, or about 0.000 385 or 0.0385%.

b) There are 3! ways of arranging the letters A, R, and T, and there are 10^4 ways of arranging the four digits. Therefore, the number of passwords that contain the letters A, R, and T is $3! \cdot 10^4$.

$n(A) = 3! \cdot 10^4$

$n(A) = 3 \cdot 2 \cdot 1 \cdot 10\,000$

$n(A) = 60\,000$

The total number of passwords containing three capital letters followed by four digits, allowing repetition, is $26^3 \cdot 10^4$

$n(O) = 26^3 \cdot 10^4$

$n(O) = 175\,760\,000$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{60\,000}{175\,760\,000}$

$P(A) = \frac{3}{8788}$

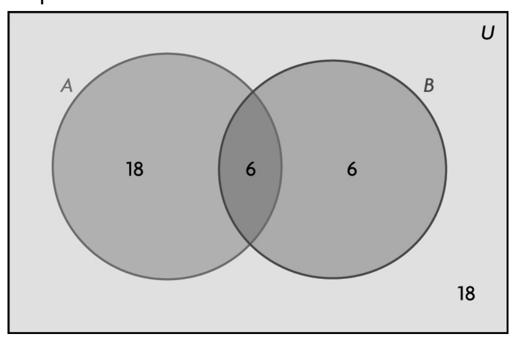
The probability that a password chosen at random will contain the letters A, R, and T is $\frac{3}{8788}$, or about 0.000 341 or 0.0341%.

11. a) These events are not mutually exclusive; e.g., There are prime numbers that are also odd numbers.

b) These events are mutually exclusive; e.g., You cannot roll a 6 and an 8 at the same time.

c) These events are mutually exclusive; e.g., You cannot eat a peach and an apple at the same time.

12. a) Let *A* represent a face card, and let *B* represent a spade.



b) These events are not mutually exclusive. This is because there are 6 cards that are both face cards and spades (2 jacks, 2 queens and 2 kings).

c) $P(A) = \frac{24}{48}$ $P(B) = \frac{12}{48}$

$P(A \cap B) = \frac{6}{48}$

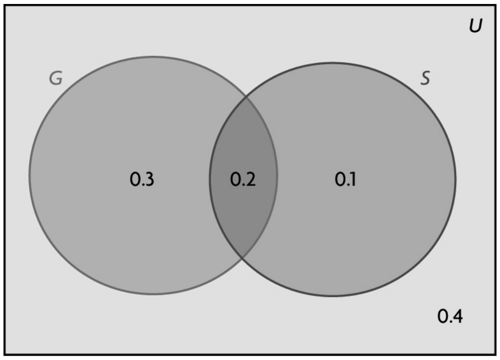
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{24}{48} + \frac{12}{48} - \frac{6}{48}$

$P(A \cup B) = \frac{30}{48}$

The probability that Hunter will draw a face card or a spade is $\frac{30}{48}$, 0.625 or 62.5%.

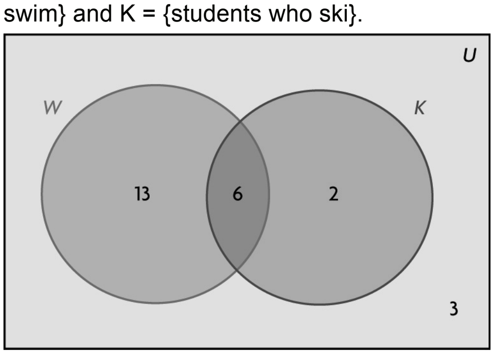
13. a) Let *G* represent exercising on Sunday, and let *S* represent shopping on Sunday.



b) The two events are not mutually exclusive, because the probability that Mya will both exercise and shop on Sunday is not equal to 0 (it is equal to 0.2).

c) $P(G) = 0.5$
$P(S) = 0.3$
$P(G \cap S) = 0.2$
$P(G \cup S) = P(G) + P(S) - P(G \cap S)$
$P(G \cup S) = 0.5 + 0.3 - 0.2$
$P(G \cup S) = 0.6$
The probability that Mya will do one of these activities on Sunday is 0.6, or 60%.

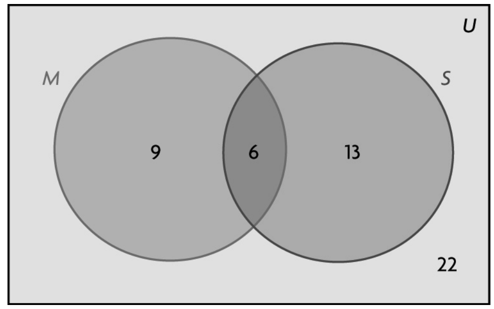
14. e.g., Suppose 6 students can both ski and swim. What is the probability that a randomly selected student cannot ski or swim. Let W = {students who swim} and K = {students who ski}.



Solution: The total number of students who can ski, swim or ski and swim is 13 + 6 + 2 = 21. There are 3 students who cannot ski or swim. The probability that a randomly selected student cannot ski or swim is

$\frac{3}{24} = \frac{1}{8}$, 0.125 or 12.5%.

15. e.g., Out of 50 retail outlets, 19 are holding sales this month. 15 outlets sell only sustainably manufactured items, and 6 of these are holding sales this month. What is the probability that a retail outlet sells some non-sustainably manufactured items and is having a sale?
M = {outlets selling sustainably manufactured items}
S = {outlets having sales}



Solution: Let R represent retail outlets having sales that sell some non-sustainably manufactured items. From the Venn diagram we see 13 retail outlets that sell some non-sustainably manufactured items are having sales.

$P(R) = \frac{13}{50}$ The probability is $\frac{13}{50}$, 0.26 or 26%.

16. Let *B* represent a black sock, and let *W* represent a white sock. Let *F* represent Parker pulling a black sock then a white sock, and let *G* represent Parker pulling a white sock then a black sock. There are two ways to draw a white sock and a black sock.

$P(B) = \frac{8}{18}$ $\quad P(F) = P(B) \cdot P(W|B)$

$P(B) = \frac{4}{9}$ $\quad P(F) = \frac{4}{9} \cdot \frac{10}{17}$

$P(W|B) = \frac{10}{17}$ $\quad P(F) = \frac{40}{153}$

$P(W) = \frac{10}{18}$ $\quad P(G) = P(W) \cdot P(B|W)$

$P(W) = \frac{5}{9}$ $\quad P(G) = \frac{5}{9} \cdot \frac{8}{17}$

$P(B|W) = \frac{8}{17}$ $\quad P(G) = \frac{40}{153}$

$P(B \cap W) = P(F) + P(G)$

$P(B \cap W) = \frac{40}{153} + \frac{40}{153}$

$P(B \cap W) = \frac{80}{153}$

The probability that Parker will pull out a pair of mismatched socks is $\frac{80}{153}$, or about 0.523 or 52.3%.

17. Let L represent a plane leaving from Winnipeg on time, and let A represent a plane arriving in Calgary on time.

$P(L) = 0.70$

$P(L \cap A) = 0.56$

$P(A \mid L) = \frac{P(L \cap A)}{P(L)}$

$P(A \mid L) = \frac{0.56}{0.70}$

$P(A \mid L) = 0.8$

The probability that a plane will arrive in Calgary on time, given it left Winnipeg on time, is 0.8, or 80%.

18. Let R represent the first card drawn being red, and let B represent the second card drawn being black.

$P(R) = \frac{20}{40}$ $\quad$ $P(R \cap B) = P(R) \cdot P(B|R)$

$P(R) = \frac{1}{2}$ $\quad$ $P(R \cap B) = \frac{1}{2} \cdot \frac{20}{39}$

$P(B|R) = \frac{20}{39}$ $\quad$ $P(R \cap B) = \frac{20}{78}$

The probability the first card is red and the second card is black is $\frac{20}{78}$, or about 0.256 or 25.6%.

19. Let S represent using a stair machine, and let B represent going to a body sculpt class.

$P(S) = \frac{1}{3}$ $\quad$ $P(B) = \frac{1}{2}$

$P(S \cap B) = P(S) \cdot P(B)$

$P(S \cap B) = \frac{1}{3} \cdot \frac{1}{2}$

$P(S \cap B) = \frac{1}{6}$

The probability that Peyton will use a stair machine and take a body sculpting class when she next visits the gym is $\frac{1}{6}$, or about 0.167 or 16.7%.

20. If the events are independent, then the product of the individual probabilities will equal the probability of both the events occurring.

$P(A \cap B) = P(A) \cdot P(B)$

$0.3 = 0.5 \cdot 0.6$

$0.3 = 0.3$

Yes, these events are independent.

21. Let F represent passing French, and let C represent passing chemistry.

a) $P(F) = 0.7$

$P(C) = 0.6$

$P(F \cap C) = P(F) \cdot P(C)$

$P(F \cap C) = 0.7 \cdot 0.6$

$P(F \cap C) = 0.42$

The probability Tanya will pass both French and chemistry is 0.42, or 42%

b) $P(C') = 0.4$

$P(F \cap C') = P(F) \cdot P(C')$

$P(F \cap C') = 0.7 \cdot 0.4$

$P(F \cap C') = 0.28$

The probability Tanya will pass French but fail chemistry is 0.28, or 28%.

c) $P(F') = 0.3$

$P(F' \cap C') = P(F') \cdot P(C')$

$P(F' \cap C') = 0.3 \cdot 0.4$

$P(F' \cap C') = 0.12$

The probability Tanya will fail both French and chemistry is 0.12, or 12%.

Chapter Task, page 369

A. I chose Gin Rummy.

B. For one round, I have A♥, 2♥, 3♥, 6♥, 6♦, 8♣, 9♣, 10♦, J♠, and K♥ in my hand. I know that my opponent has picked up the K♣ and the 7♠. I also know that he has discarded the 9♦ and the J♥. The J♥ is currently face up. There are still 20 unturned cards in the deck.

C. There are, at most, seven hearts left in the deck, including the unknown cards in my opponent's hand. I could pick up the J♥, hoping to get either another jack or the Q♥ during a later play; or I could draw a face-down card. The conditional probability of drawing the 4♥ or 5♥ is, at most, $\frac{2}{20}$ or $\frac{1}{10}$. This would strengthen my hearts, potentially getting a longer run; but if I drew the 5♥ and did not get the 4♥ during a later turn, my hand would not be strengthened. The probability of drawing one of these cards now and one on the next turn would be, at most, $\left(\frac{1}{10}\right) \cdot \left(\frac{1}{19}\right)$ or $\frac{1}{190}$ (I used 19 because I assumed that my opponent would pick up my discarded card. I would use 18 if he drew a card from the deck.) If I pick up the J♥, then there are possibly three cards that would strengthen my hand remaining in the deck: the Q♥, the J♣, and the J♦. The probability that I will get one of those cards in my next draw is, at most, $\frac{3}{19}$. It makes sense to pick up the J♥. I still need to discard one of the cards in my hand. Since my opponent threw away the 9♦, I am going to guess that he may not have cards near the 9♦ to make a run, so it makes sense to discard the 10♦.