Total if A is the first letter:
21 + 12 = 33
Therefore, if the first letter is A, there are 33 possible arrangements.
If the first letter is L, S, or K, there are three possibilities for the second letter: A, and 2 of L, S, and K (the ones that are not the first letter).
If A is the second letter:
3 possibilities for the third letter: A, and 2 of L, S, and K
The A has 3 possibilities for the fourth letter and the other two letters have 2. 3 + 2(2) = 7
If A is not the second letter:
2 possibilities for the third letter
The A has 2 possibilities for the fourth letter and the other letter has 1. 2 + 1 = 3
Total for both second letters that are not A:
3(2) = 6
Total for one of three times where first letter is L, S, or K:
7 + 6 = 13
Total when first letter is L, S, or K:
3(13) = 39
Total arrangements:
39 + 33 = 72
Therefore, 72 four-letter arrangements can be made using all of the letters in the word ALASKA.

20. If I have an O as the first letter, there are 4 possibilities for the second letter, each of which has 3 possibilities for the third letter. 4(3) = 12
Therefore, there are 12 possible arrangements when O is the first letter.
If the first letter is B, K, or S:
There are 3 possibilities for the second letter: O, and two of B, K, and S (the ones that are not the first letter). O has 3 possibilities for the third letter while the other 2 have 2. 3 + 2(2) = 7
Total if the first letter is B, K, or S:
3(7) = 21
Total arrangements:
21 + 12 = 33
Therefore, 33 three-letter arrangements can be made using all of the letters in the word BOOKS.

History Connection, page 290

A. Yes. Each number from 0 to 127 is assigned a different character or symbol on the keyboard. Since the numbers already have an established order, the characters and symbols assigned to these numbers do, as well.
B. Yes. Each number in ASCII (pronounced "askey") must be converted into a string of 0s and 1s to create the binary code, so order matters. Each 0 or 1 is associated with a position in the string. A different permutation of 0s and 1s represents a different number in the ASCII code system.
C. There are 128 numbers in ASCII that must be represented by a string of 0s and 1s. You need to determine the length of the string needed to create 128 different arrangements of 0s and 1s. You can begin by thinking about a string of length of 5.
A box diagram, □□□□□, can help you determine the number of ASCII numbers you can represent.
Within each box you can place a 0 or a 1. There are two choices for each box, since repetition of 0s and 1s is allowed. So for a string length of 5, there are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ or 32 ASCII numbers that can be represented. Obviously, the string must be longer for 128 numbers. If n represents the string length, and 128 numbers must be represented, then $2^n = 128$. By trial and error, $n = 7$.
A binary string of length 7 is needed to represent each ASCII code.

Chapter Self-Test, page 291

1. a) Let N represent the number of different serial numbers:
$N = 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 3$
$N = 2\ 028\ 000$
There are 2 028 000 different serial numbers possible, if repetition of characters is allowed.
b) Let N represent the number of different serial numbers:
$N = 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 3$
$N = 1\ 296\ 000$
There are 1 296 000 different serial numbers possible, if no repetition is allowed.

2. Event A: Drawing a spade
Event B: Drawing a diamond
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 13 + 13$
$n(A \cup B) = 26$
Therefore, there are 26 ways to draw 1 card that is a spade or a diamond.

3. a) $n + 9 \geq 0$
$n \geq -9$
$(n + 10)(n + 9)!$ is defined for $n \geq -9$, where $n \in I$.
$(n + 10)(n + 9)! = (n + 10)[(n + 9)(n + 8)\ldots(3)(2)(1)]$
$(n + 10)(n + 9)! = (n + 10)(n + 9)(n + 8)\ldots(3)(2)(1)$
$(n + 10)(n + 9)! = (n + 10)!$
b) $n - 2 \geq 0$ AND $n \geq 0$
$n \geq 2$
$\frac{(n-2)!}{n!}$ is defined for $n \geq 2$, where $n \in I$.
$$\frac{(n-2)!}{n!} = \frac{(n-2)(n-3)\ldots(3)(2)(1)}{n(n-1)(n-2)(n-3)\ldots(3)(2)(1)}$$
$$\frac{(n-2)!}{n!} = \frac{1}{n(n-1)}$$
$$\frac{(n-2)!}{n!} = \frac{1}{n^2 - n}$$

4. a) 5! = 120
Therefore, there are 120 different ways that the 5 cars can be parked side by side.
b) Let B represent the number of arrangements:
$B = {}_2P_2 \cdot {}_4P_4$
$B = 2! \cdot 4!$
$B = 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$B = 48$
Therefore, there are 48 different ways the cars can be parked so the 2 black cars are next to each other.

5. a) ${}_9C_4 = \frac{9!}{4!5!}$
${}_9C_4 = 126$
There are 126 different four-book selections that can be made.
b) ${}_9P_4 = \frac{9!}{(9-4)!}$
${}_9P_4 = \frac{9!}{5!}$
${}_9P_4 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!}$
${}_9P_4 = 9 \cdot 8 \cdot 7 \cdot 6$
${}_9P_4 = 3024$
There are 3024 different four-book selections can be arranged in order of preference.
c) e.g., The order matters in part b). There are still 126 ways to choose the four books from the nine options, but there are also 4! = 24 ways to arrange the books.(126 · 24 = 3024)

6.
$${}_nP_4 = 84({}_nC_2)$$
$$\frac{n!}{(n-4)!} = 84\left[\frac{n!}{2!(n-2)!}\right]$$
$$\frac{n!}{(n-4)!} = \frac{42n!}{(n-2)!}$$
$$n(n-1)(n-2)(n-3) = 42n(n-1)$$
$$(n-2)(n-3) = 42$$
$$n^2 - 3n - 2n + 6 = 42$$
$$n^2 - 5n - 36 = 0$$
$$(n+4)(n-9) = 0$$
$n+4=0$ or $n-9=0$
$n=-4$ $\quad n=9$

Check n = –4

LS	RS
${}_{-4}P_4$	$84({}_{-4}C_2)$
$\frac{(-4)!}{(-4-4)!}$	$84\left[\frac{(-4)!}{2!(-4-2)!}\right]$
$\frac{(-4)!}{(-8)!}$ is undefined	

Check n = 9

LS	RS
${}_9P_4$	$84({}_9C_2)$
$\frac{9!}{(9-4)!}$	$84\left[\frac{9!}{2!(9-2)!}\right]$
$\frac{9!}{5!}$	$84\left[\frac{9!}{2! \cdot 7!}\right]$
$\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!}$	$84\left[\frac{9 \cdot 8 \cdot 7!}{2 \cdot 1 \cdot 7!}\right]$
$9 \cdot 8 \cdot 7 \cdot 6$	$84\left[\frac{9 \cdot 8}{2 \cdot 1}\right]$
3024	$84[9 \cdot 4]$
	3024

There is one solution, n = 9.

7. a) ${}_6C_2 \cdot {}_8C_3 = \frac{6!}{2!4!} \cdot \frac{8!}{3!5!}$
${}_6C_2 \cdot {}_8C_3 = 840$
There are 840 different ways that a 5-person committee can be selected if there must be 2 boys and 3 girls.
b) Case 1: 2 boys and 3 girls:
${}_6C_2 \cdot {}_8C_3 = \frac{6!}{2!4!} \cdot \frac{8!}{3!5!}$
${}_6C_2 \cdot {}_8C_3 = 840$

Case 2: 3 boys and 2 girls:
${}_6C_3 \cdot {}_8C_2 = \frac{6!}{3!3!} \cdot \frac{8!}{2!6!}$
${}_6C_3 \cdot {}_8C_2 = 560$

Case 3: 4 boys and 1 girl:
${}_6C_4 \cdot {}_8C_1 = \frac{6!}{4!2!} \cdot \frac{8!}{1!7!}$
${}_6C_4 \cdot {}_8C_1 = 120$

Case 4: 5 boys and 0 girls:
${}_6C_5 \cdot {}_8C_0 = \frac{6!}{5!1!} \cdot \frac{8!}{0!8!}$
${}_6C_5 \cdot {}_8C_0 = 6$
Let C represent the number of 5-person committees with at least 2 boys:
C = 840 + 560 + 120 + 6
C = 1526
There are 1526 different ways that a 5-person committee can be selected if there must be at least 2 boys.
c) ${}_{12}C_3 = \frac{12!}{3!9!}$
${}_{12}C_3 = 220$

There are 220 different ways that a 5-person committee can be selected if David and Susan must be on the committee.

d) Case 1: 2 boys and 3 girls

$${}_6C_2 \cdot {}_8C_3 = \frac{6!}{2!4!} \cdot \frac{8!}{3!5!}$$
$${}_6C_2 \cdot {}_8C_3 = 840$$

Case 2: 1 boy and 4 girls

$${}_6C_1 \cdot {}_8C_4 = \frac{6!}{1!5!} \cdot \frac{8!}{4!4!}$$
$${}_6C_1 \cdot {}_8C_4 = 420$$

Case 3: 0 boys and 5 girls

$${}_6C_0 \cdot {}_8C_5 = \frac{6!}{0!6!} \cdot \frac{8!}{5!3!}$$
$${}_6C_0 \cdot {}_8C_5 = 56$$

Let C represent the number of 5-person committees with more girls than boys:
C = 840 + 420 + 56
C = 1316
There are 1316 different ways that a 5-person committee can be selected if there must be more girls than boys.

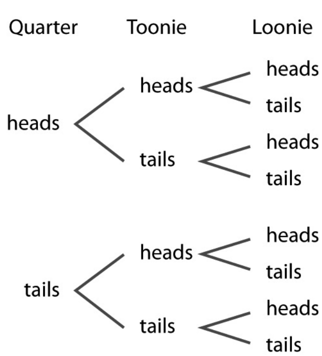
8. $\frac{5!}{2!2!} = 30$
There are 30 different arrangements of the letters in the word TEETH.

9. 5! · 4! = 2880
There are 2880 different arrangements possible.

Chapter Review, page 293

1. e.g., The Fundamental Counting Principle is used when a counting problem has different tasks related by the word AND. For example, you can use it to figure out how many ways you can roll a 3 with a die and draw a red card from a deck of cards.

2.



The tree diagram shows there are 8 possible ways that the three coins can land.

3. Let A represent the number of sets of answers:
$A = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
$A = 4^{10}$
A = 1 048 576
The student can give 1 048 576 different sets of answers.

4. a) $n + 2 \ge 0$ AND $n \ge 0$
$n \ge -2$

$\frac{(n+2)!}{n!} = 20$ is defined for $n \ge 0$, where $n \in$ I.

$$\frac{(n+2)!}{n!} = 20$$
$$\frac{(n+2)(n+1)(n)(n-1)...(3)(2)(1)}{(n)(n-1)...(3)(2)(1)} = 20$$
$$(n+2)(n+1) = 20$$
$$n^2 + n + 2n + 2 - 20 = 0$$
$$n^2 + 3n - 18 = 0$$
$$(n+6)(n-3) = 0$$

$n + 6 = 0$ or $n - 3 = 0$
$n = -6$ $\quad n = 3$

The root n = –6 is outside the restrictions on the variable in the equation, so it cannot be a solution. There is one solution, n = 3.

b) The simplified version of the equation is
$\frac{(n+1)!}{(n-1)!} = 132$.

$n + 1 \ge 0$ AND $n - 1 \ge 0$
$n \ge -1$ $\quad n \ge 1$

$\frac{(n+1)!}{(n-1)!} = 132$ is defined for $n \ge 1$, where $n \in$ I.

$$\frac{(n+1)!}{(n-1)!} = 132$$
$$\frac{(n+1)(n)(n-1)(n-2)...(3)(2)(1)}{(n-1)(n-2)...(3)(2)(1)} = 132$$
$$(n+1)(n) = 132$$
$$n^2 + n = 132$$
$$n^2 + n - 132 = 0$$
$$(n+12)(n-11) = 0$$

$n + 12 = 0$ or $n - 11 = 0$
$n = -12$ $\quad n = 11$

The root n = –12 is outside the restrictions on the variable in the equation, so it cannot be a solution. There is one solution, n = 11.