d) e.g.: **i)** $\frac{29}{30} = 0.966...$ or 96.7%

ii) $1 - \frac{29}{30} = 0.033...$ or 3.3%

e) For example, they were close but not the same.

Mid-Chapter Review, page 259

1. The number of subs to choose from, *S*, is based on the number of buns (*b*), the number of cold cuts (*cc*), the number of cheeses (*c*), the number of toppings (*t*), and the number of sauces (*s*):
S = (# of *b*) · (# of *cc*) · (# of *c*) · (# of *t*) · (# of *s*)
S = 3 · 5 · 3 · 12 · 3
S = 1620
So, Mario can choose from 1620 different subs.

2. e.g., You can use one of K, W, and C for the first character, one of the 26 uppercase letters for the second and third characters, and one of the 26 uppercase letters or a blank for the last character. From this, I get the following calculation:
# of station names = 3 · 26 · 26 · 27
# of station names = 54 756
Therefore, 54 756 station names are possible.

3. Event *A*: Rolling a 2 OR
Event *B*: Rolling a 10

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

From the table above, there is one way to roll a sum of 2 with a pair of dice and three ways to roll a sum of 10 with a pair of dice.
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 1 + 3$
$n(A \cup B) = 4$
There are 4 ways that a sum of 2 or a sum of 10 can be rolled with a pair of dice.

4. 10 · 9 · 8 = 720
There are 720 ways to select 3 horses to come first, second, third in a 10-horse race.

5. a) 8! = 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
8! = 40 320

b) 6! · 3! = (6 · 5 · 4 · 3 · 2 · 1) · (3 · 2 · 1)
6! · 3! = (720) · (6)
6! · 3! = 4320

c)
$$\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$\frac{9!}{6!} = 9 \cdot 8 \cdot 7 \cdot \frac{6}{6} \cdot \frac{5}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$$
$$\frac{9!}{6!} = 9 \cdot 8 \cdot 7 \cdot \frac{6!}{6!}$$
$$\frac{9!}{6!} = 9 \cdot 8 \cdot 7 \cdot 1$$
$$\frac{9!}{6!} = 504$$

d)
$$\frac{10 \cdot 9!}{5 \cdot 8!} = \frac{10 \cdot (9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{5 \cdot (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}$$
$$\frac{10 \cdot 9!}{5 \cdot 8!} = \frac{10}{5} \cdot 9 \cdot \frac{8}{8} \cdot \frac{7}{7} \cdot \frac{6}{6} \cdot \frac{5}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$$
$$\frac{10 \cdot 9!}{5 \cdot 8!} = \frac{10}{5} \cdot 9 \cdot \frac{8!}{8!}$$
$$\frac{10 \cdot 9!}{5 \cdot 8!} = \frac{10}{5} \cdot 9 \cdot 1$$
$$\frac{10 \cdot 9!}{5 \cdot 8!} = 2 \cdot 9$$
$$\frac{10 \cdot 9!}{5 \cdot 8!} = 18$$

6. There are nine players on the team so there are 9 different positions. Let *L* represent the number of permutations:
L = 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
L = 9!
L = 9 · 8 · 7 · 6 · 5 · 4 · 3 · 2 · 1
L = 72 · 42 · 20 · 6
L = 362 880
There are 362 880 different lineups that can be formed by nine players on a softball team.

7. a)
$$(n+5)(n+4)!$$
$$= (n+5)\left[(n+4)(n+3)(n+2)...(3)(2)(1)\right]$$
$$= (n+5)(n+4)(n+3)(n+2)...(3)(2)(1)$$
$$= (n+5)!$$

b)
$$\frac{(n+4)!}{(n+2)!} = \frac{(n+4)(n+3)(n+2)(n+1)(n)...(3)(2)(1)}{(n+2)(n+1)(n)...(3)(2)(1)}$$
$$\frac{(n+4)!}{(n+2)!} = \frac{(n+4)(n+3)(n+2)!}{(n+2)!}$$
$$\frac{(n+4)!}{(n+2)!} = (n+4)(n+3)$$
$$\frac{(n+4)!}{(n+2)!} = n^2 + 4n + 3n + 12$$
$$\frac{(n+4)!}{(n+2)!} = n^2 + 7n + 12$$

c) $\dfrac{(n-4)!}{(n-5)!}=\dfrac{(n-4)(n-5)(n-6)(n-7)...(3)(2)(1)}{(n-5)(n-6)(n-7)...(3)(2)(1)}$

$\dfrac{(n-4)!}{(n-5)!}=\dfrac{(n-4)(n-5)!}{(n-5)!}$

$\dfrac{(n-4)!}{(n-5)!}=n-4$

d) $\dfrac{(n+2)!}{n!}=\dfrac{(n+2)(n+1)(n)(n-1)(n-2)...(3)(2)(1)}{(n)(n-1)(n-2)...(3)(2)(1)}$

$\dfrac{(n+2)!}{n!}=\dfrac{(n+2)(n+1)(n!)}{n!}$

$\dfrac{(n+2)!}{n!}=(n+2)(n+1)$

$\dfrac{(n+2)!}{n!}=n^2+n+2n+2$

$\dfrac{(n+2)!}{n!}=n^2+3n+2$

8. a) $\dfrac{n!}{(n-2)!}=72$

$\dfrac{(n)(n-1)(n-2)(n-3)...(3)(2)(1)}{(n-2)(n-3)...(3)(2)(1)}=72$

$\dfrac{(n)(n-1)(n-2)!}{(n-2)!}=72$

$(n)(n-1)=72$

$n^2-n=72$

$n^2-n-72=0$

$(n+8)(n-9)=0$

$n+8=0$ or $n-9=0$

$n=-8$ $\quad n=9$

Check n = –8

LS	RS
$\dfrac{(-8)!}{(-8-2)!}$	72
$\dfrac{(-8)!}{(-10)!}$ is undefined	

Check n = 9

LS	RS
$\dfrac{9!}{(9-2)!}$	72
$\dfrac{9!}{7!}$	
$\dfrac{9\cdot 8\cdot 7!}{7!}$	
$9\cdot 8$	
72	

There is one solution, n = 9.

b) $\dfrac{(n-1)!}{(n-3)!}=30$

$\dfrac{(n-1)(n-2)(n-3)(n-4)...(3)(2)(1)}{(n-3)(n-4)...(3)(2)(1)}=30$

$\dfrac{(n-1)(n-2)(n-3)!}{(n-3)!}=30$

$(n-1)(n-2)=30$

$n^2-2n-n+2=30$

$n^2-3n+2=30$

$n^2-3n-28=0$

$(n+4)(n-7)=0$

$n+4=0$ or $n-7=0$

$n=-4$ $\quad n=7$

Check n = –4

LS	RS
$\dfrac{(-4-1)!}{(-4-3)!}$	30
$\dfrac{(-5)!}{(-7)!}$ is undefined	

Check n = 7

LS	RS
$\dfrac{(7-1)!}{(7-3)!}$	30
$\dfrac{6!}{4!}$	
$\dfrac{6\cdot 5\cdot 4!}{4!}$	
$6\cdot 5$	
30	

There is one solution, n = 7.

9. a) ${}_9P_2=\dfrac{9!}{(9-2)!}$

${}_9P_2=\dfrac{9!}{7!}$

${}_9P_2=\dfrac{9\cdot 8\cdot 7!}{7!}$

${}_9P_2=9\cdot 8$

${}_9P_2=72$

b) ${}_{12}P_8=\dfrac{12!}{(12-8)!}$

${}_{12}P_8=\dfrac{12!}{4!}$

${}_{12}P_8=\dfrac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4!}{4!}$

${}_{12}P_8=12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5$

${}_{12}P_8=19\,958\,400$

c) $_5P_5 = 5!$

$_5P_5 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$_5P_5 = 120$

d) $_{12}P_{10} = \dfrac{12!}{(12-10)!}$

$_{12}P_{10} = \dfrac{12!}{2!}$

$_{12}P_{10} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$

$_{12}P_{10} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

$_{12}P_{10} = 239\,500\,800$

10. a) a**:** $n+5 \geq 0$ AND $n+4 \geq 0$

$n \geq -5$ $\quad$ $n \geq -4$

The expression is defined for $n \geq -4$, where $n \in$ I.

b**:** $n+4 \geq 0$ AND $n+2 \geq 0$

$n \geq -4$ $\quad$ $n \geq -2$

The expression is defined for $n \geq -2$, where $n \in$ I.

c**:** $n-4 \geq 0$ AND $n-5 \geq 0$

$n \geq 4$ $\quad$ $n \geq 5$

The expression is defined for $n \geq 5$, where $n \in$ I.

d**:** $n+2 \geq 0$ AND $n \geq 0$

$n \geq -2$

The expression is defined for $n \geq 0$, where $n \in$ I.

b) a**:** $n \geq 0$ AND $n-2 \geq 0$

$n \geq 2$

The expression is defined for $n \geq 2$, where $n \in$ I.

b**:** $n-1 \geq 0$ AND $n-3 \geq 0$

$n \geq 1$ $\quad$ $n \geq 3$

The expression is defined for $n \geq 3$, where $n \in$ I.

11. $n = 20$ and $r = 6$

$_{20}P_6 = \dfrac{20!}{(20-6)!}$

$_{20}P_6 = \dfrac{20!}{14!}$

$_{20}P_6 = \dfrac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14!}{14!}$

$_{20}P_6 = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15$

$_{20}P_6 = 27\,907\,200$

Rennie can load his CD player in 27 907 200 different ways.

12. $n = 14$ and $r = 2$

$_{14}P_2 = \dfrac{14!}{(14-2)!}$

$_{14}P_2 = \dfrac{14!}{12!}$

$_{14}P_2 = \dfrac{14 \cdot 13 \cdot 12!}{12!}$

$_{14}P_2 = 14 \cdot 13$

$_{14}P_2 = 182$

There are 182 ways that Manny and 2 other players can line up to receive the championship trophy.

13. Agree. e.g., The number of ways to choose a president and a vice-president from a group of five students is $\dfrac{5!}{(5-2)!} = 20$. I could also use the Fundamental Counting Principle because there are five choices for president and four choices remaining for vice-president: $5 \cdot 4 = 20$.

Lesson 4.4: Permutations When Objects Are Identical, page 266

1. a) $\dfrac{7!}{3!2!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1) \cdot (2 \cdot 1)}$

$\dfrac{7!}{3!2!} = \dfrac{7 \cdot 3 \cdot 2 \cdot 5 \cdot 4 \cdot 3}{(3 \cdot 2 \cdot 1)}$

$\dfrac{7!}{3!2!} = 7 \cdot 5 \cdot 4 \cdot 3$

$\dfrac{7!}{3!2!} = 420$

b) $\dfrac{8!}{2!2!2!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2 \cdot 1 \cdot 2!2 \cdot 1}$

$\dfrac{8!}{2!2!2!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 3$

$\dfrac{8!}{2!2!2!} = 5040$

c) $\dfrac{10!}{4!3!2!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

$\dfrac{10!}{4!3!2!} = 10 \cdot 9 \cdot 7 \cdot 5 \cdot 4$

$\dfrac{10!}{4!3!2!} = 12600$

d) $\dfrac{12!}{2!4!5!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$\dfrac{12!}{2!4!5!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 7$

$\dfrac{12!}{2!4!5!} = 83160$