The present value of the investment is $8718.76.

$$\frac{8718.76}{3} = 2906.25$$

Each sibling will need to contribute $2906.25 to the GIC.

14. e.g., In an investment, you agree to loan a sum of money to another entity (like a company); the amount you loan is called the *present value* of the principal. The *interest rate* dictates the amount of money they pay you for the loan, for a given time period, called the *term*. *Simple interest* pays you a percentage of the loaned amount at the
end of the term. With *compound interest,* the interest is paid out more often, defined by the *compounding frequency*. You don't get the *compound interest* immediately, but effectively loan the entity the interest as well, until the end of the term. The *present value* plus the interest you earn is called the *future value*. A higher *interest rate* and a higher *compounding frequency* will earn you more interest.

15. I will use a present value of $100 so my future value will be $300.
The present value is $100.
The annual interest rate is unknown.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 12.
The future value is $300.
I used the financial application on my calculator: An interest rate of 9.26% will allow the investment to triple every 12 years.

16. a) The present value is $1000.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 1 year.
The future value is unknown.
The future value of the investment is $1050.
b) i) The present value is $1000.
The annual interest rate is unknown.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.94%.
ii) The present value is $1000.
The annual interest rate is unknown.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.91%.
iii) The present value is $1000.
The annual interest rate is unknown.
The compounding period is monthly, or 12 times per year.
The term (in years) is 1 year.
The future value is $1050.
The annual interest rate is 4.89%.

c) e.g., By choosing a lower interest rate with more frequent compounding, you can take advantage of the power of compound interest and earn the same interest as you could at a higher interest rate with less frequent compounding. This is useful when interest rates are low.

Mid-Chapter Review, page 45

1. $A = P + Prt$
A is $477.56; P is $450; r *is* 2.04% or 0.0204
$477.56 = 450 + (450)(0.0204)(t)$
$27.56 = 9.18t$
$t = 3.002\ldots$
Paula held the investment for 3 years.

2. a) $A = P + Prt$
A is $7200; P is $6000; r *is* 6.4% or 0.064
$7200 = 6000 + (6000)(0.064)(t)$
$1200 = 384t$
$t = 3.125$
It will take 3.125 years for the investment to earn $1200 in interest.
b) If paid annually, the interest will be paid out at the end of the next full year, or in 4 years.
c) If paid quarterly, the interest will be paid out at the end of the next full quarter, or in 3.25 years (3 years and 4 months).

3. a) Katherine: P is $5000; r is 4.867 75% or 0.048 677 5; t is 1, 20 times
$A = P(1 + (0.048\ 677\ 5)(1))$
Calculate the interest, using the value of A as the new value of P for each new year. Use a table to organise the answers. (Some values have been omitted.)

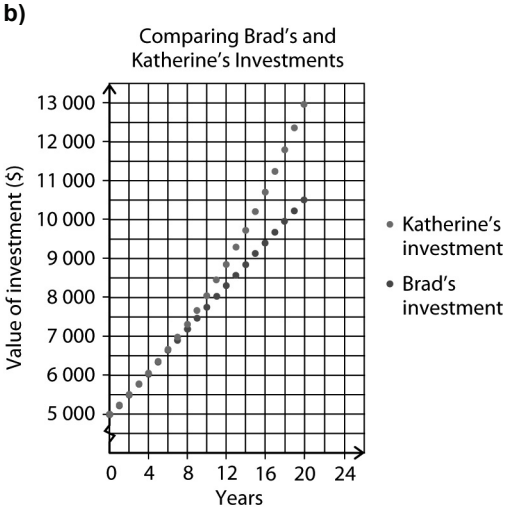
Year	Principal ($)	Year-end Value ($)
1	5000.00	5243.39
5	6046.97	6341.32
10	7669.16	8042.47
15	9726.52	10199.98
20	12 335.79	12 936.27

Katherine's account will be worth $12 936.27 after 20 years.
Brad: P is $5000; r is 5.5% or 0.055; t is 1, 5, 10, 15, 20
$A = P(1 + rt)$

Year	Principal ($)	Year-end Value ($)
1	5000	5275
5	5000	6375
10	5000	7750
15	5000	9125
20	5000	10500

Brad's account will be worth $10 500 after 20 years.

b)



Time (years)	Value of Investment ($)	
	Katherine	Brad
1	5243.39	5275
5	6341.32	6375
10	8042.47	7750
15	10199.98	9125
20	12 936.27	10500

c) The intersection point on the graph represents the point at which both investments earn the same amount of interest.

4. a) The principal is $3000.
The annual interest rate is 5.6%.
The compounding period is annual, or once per year.
The term (in years) is 10.
The future value is unknown.
The future value of the GIC is $5173.21.
b) $3000 will double twice to become $12 000.

$$\frac{72}{5.6} \times 2 = 25.714...$$

It will take about 25.7 years for the investment to be worth at least $12 000.
c) i) e.g., If the compounding frequency increases to monthly, the future value of the investment will increase. More frequent compounding results in more interest being earned.
The principal is $3000.
The annual interest rate is 5.6%.
The compounding period is monthly or 12 times per year.
The term (in years) is 10.
The future value is unknown.
The future value of the GIC is $5245.18. This is more than the original future value of $$5173.21.
ii) e.g., If the interest rate changed to 2.8% compounded semi-annually, the future value of the investment would decrease. A lower interest rate means less interest will be earned on the investment. The change from annual to semi-annual compounding will not affect the future value as much as the interest rate being reduced by half.
The principal is $3000.
The annual interest rate is 2.8%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 10.
The future value is unknown.
The future value of the GIC is $3961.69. This is less than the original future value of $5173.21.
d) The principal is $3000.
The annual interest rate is unknown.
The compounding period is daily, or 365 times per year.
The term (in years) is 10.
The future value is $5273.21.
An interest rate of 5.64% is needed to have a future value of $5273.21.
5. The principal is $50 000.
The annual interest rate is 7.75%.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 3.
The future value is unknown.
The future value of the investment is $62 947.39
62 947.39 – 50 000 = 12 947.39
There will be $12 947.39 available for sports equipment at the end of 3 years.

6. Initial Investment: The principal is $1000.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 5.
The future value is unknown.
The future value of the initial investment is $1276.28.
Option A: The principal is $1050.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 5.
The future value is unknown.
The future value of option A is $1340.10.
Option B: The principal is $1000.
The annual interest rate is 6%.
The compounding period is annual, or once per year.
The term (in years) is 5.
The future value is unknown.
The future value of option B is $1338.23.
Option C: The principal is $1000.
The annual interest rate is 5%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 5.
The future value is unknown.
The future value of the option C is $1283.36.
Option D: The principal is $1000.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 6.
The future value is unknown.

The future value of the initial investment is $1340.10.

	Future Value ($)
Initial Investment	1276.28
Option C: Increase compounding frequency to monthly	1283.36
Option B: Increase interest rate by 1%	1338.23
Option A: increase principal by $50	1340.10
Option D: Increase term by 1 year	1340.10

Option A and option D have the greatest, and equal, impact on the future value. Option C has the least impact.

7. a) *The present value is unknown.*
The annual interest rate is 6%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 3 years.
The future value is $10 000.
Desiree should invest $8356.45 to have $10 000 in three years.
b) *The present value is unknown.*
The annual interest rate is 6%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 3 years.
The future value is $10 000.
Desiree should invest $8374.84 to have $10 000 in three years.

8. a) i) *The present value is unknown.*
The annual interest rate is 7.2%.
The compounding period is annual, or once per year.
The term (in years) is 22 years.
The future value is $11 000.
The original investment was $2382.91.
ii) *The present value is unknown.*
The annual interest rate is 7.2%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 22 years.
The future value is $11 000.
The original investment was $2320.40.
b) The ratio would be higher because the compounding is more frequent.
The principal is $11 000.
The annual interest rate is 7.2%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 22 years.
The future value is unknown.
The future value of the investment is $53 365.38
Original investment:
i) $\frac{A}{P} = \frac{11\,000.00}{2382.91}$ **ii)** $\frac{A}{P} = \frac{11\,000.00}{2320.40}$
$\frac{A}{P} = 4.616...$ $\frac{A}{P} = 4.740...$
New investment:
$\frac{A}{P} = \frac{53\,365.38}{11\,000.00}$
$\frac{A}{P} = 4.851...$

The new investment has a higher ratio than the original investment.

9. The principal is $400.
The annual interest rate is unknown.
The compounding period is monthly, or 12 times per year.
The term (in years) is 10.
The future value is $625.
The annual interest rate is 4.47%.

10. $250 will double twice to become $1000.
$\frac{72}{6} \times 2 = 24$
It will take about 24 years for the $250 investment to grow to $1000.
The principal is $250.
The annual interest rate is 6%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is unknown.
The future value is $1000.
It will take 23.45 years, or 23.5 years for the $250 investment to grow to $1000.

Lesson 1.5: Investments Involving Regular Payments, page 55

1. a) The regular payment amount is $200.
The payment frequency is monthly, or 12 times per year.
The number of payments is 600.
The payments are made at the end of each payment period.
The annual interest rate is 4.8%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
The future value of the investment is $498 526.60.
b) The regular payment amount is $1750.
The payment frequency is semi-annual, or 2 times per year.
The number of payments is 40.
The payments are made at the end of each payment period.
The annual interest rate is 5.6%.
The compounding frequency is semi-annual, or 2 times per year.
The future value is unknown.
The future value of the investment is $126 127.32
c) The regular payment amount is $50.
The payment frequency is quarterly or 4 times per year.
The number of payments is 160.
The payments are made at the end of each payment period.
The annual interest rate is 8.4%.
The compounding frequency is quarterly or 4 times per year.
The future value is unknown.
The future value of the investment is $63 820.79.
d) The regular payment amount is $5500.
The payment frequency is semi-annual, or 2 times per year.