7. a) Range: $\{y \mid -4 \le y \le 6, y \in R\}$
Maximum value = 6, Minimum value = –4

$$\text{Amplitude} = \frac{\max - \min}{2}$$
$$\text{Amplitude} = \frac{6 - (-4)}{2}$$
$$\text{Amplitude} = \frac{10}{2}$$
$$\text{Amplitude} = 5$$

Equation of Midline:

$$y = \frac{\max + \min}{2}$$
$$y = \frac{6 + (-4)}{2}$$
$$y = \frac{2}{2}$$
$$y = 1$$

Period = second max – first max
Period = 8.7 – 1.7
Period = 7
The range of this graph is $\{y \mid -4 \le y \le 6, y \in R\}$ and the amplitude is 5. The equation of midline is $y = 1$ and the period is 7.

b) Range: $\{y \mid 0.5 \le y \le 3.5, y \in R\}$
Maximum value = 3.5, Minimum value = 0.5

$$\text{Amplitude} = \frac{\max - \min}{2}$$
$$\text{Amplitude} = \frac{3.5 - 0.5}{2}$$
$$\text{Amplitude} = \frac{3}{2}$$
$$\text{Amplitude} = 1.5$$

Equation of Midline:

$$y = \frac{\max + \min}{2}$$
$$y = \frac{3.5 + (0.5)}{2}$$
$$y = \frac{4}{2}$$
$$y = 2$$

Period = second max – first max
Period = 0.5 – 0
Period = 0.5
The range of this graph is $\{y \mid 0.5 \le y \le 3.5, y \in R\}$ and the amplitude is 1.5. The equation of midline is $y = 2$ and the period is 0.5.

8. a) Period = second max – first max
Period = 0.41 – 0.09
Period = 0.32 s
The period is about 0.32 seconds.
b) Maximum value = 4.5
Minimum value = –4.5
Equation of the midline:

$$y = \frac{\max + \min}{2}$$
$$y = \frac{4.5 + (-4.5)}{2}$$
$$y = 0$$

The equation of the midline of the function is $y = 0$ amperes.
c)

$$y = \frac{\max + \min}{2}$$
$$y = \frac{4.5 - (-4.5)}{2}$$
$$y = \frac{9}{2}$$
$$y = 4.5$$

The amplitude of the function is 4.5 amperes.

Lesson 8.4: The Equations of Sinusoidal Functions, page 558

1. The equations in order from the least amplitude to the most amplitude, are **c)**, **a)**, and **b)**. I ordered the functions based on the values of *a* since the amplitude of an equation is given by *a*. For equation **a)**, $a = 3$. For equation **b)**, $a = 4$. For equation **c)**, $a = 2$. Therefore,

2. The equations, in order from the smallest range to the greatest range, are **c)**, **a)**, and **b)**. Since the range is defined by the function's amplitude, so I ordered the functions based on the value of *a*.

The range of an equation is given by *a* and *d*.
Equation **a)**:

$$\text{Minimum value} = d - a$$
$$\text{Minimum value} = 0 - 8$$
$$\text{Minimum value} = -8$$
$$\text{Maximum value} = d + a$$
$$\text{Maximum value} = 0 + 8$$
$$\text{Maximum value} = 8$$
$$\text{Maximum value} - \text{minimum value} = 8 - (-8)$$
$$= 16$$

The range of equation **a)** is 16.

Equation **b)**:

$$\text{Minimum value} = d - a$$
$$\text{Minimum value} = -1 - 9$$
$$\text{Minimum value} = -10$$
$$\text{Maximum value} = d + a$$
$$\text{Maximum value} = -1 + 9$$
$$\text{Maximum value} = 8$$
$$\text{Maximum value} - \text{minimum value} = 8 - (-10)$$
$$\text{Maximum value} - \text{minimum value} = 18$$

The range of equation **b)** is 18.

Equation **c)**:

$\text{Minimum value} = d - a$

$\text{Minimum value} = 2 - 6$

$\text{Minimum value} = -4$

$\text{Maximum value} = d + a$

$\text{Maximum value} = 2 + 6$

$\text{Maximum value} = 8$

$\text{Maximum value} - \text{minimum value} = 8 - (-4)$

$\text{Maximum value} - \text{minimum value} = 12$

The range of equation **c)** is 12.

3. The period is given by $\frac{360°}{b}$ or $\frac{2\pi}{b}$.

For equation **a)**, the period is $\frac{360°}{4}$, or 90°.

For equation **b)**, the period is $\frac{360°}{2}$, or 180°.

For equation **c)**, the period is $\frac{360°}{0.25}$, or 1440°.

Therefore, the equations in order from the least period to the greatest period are **a)**, **b)** and **c)** since the magnitude of the period is inversely proportional to *b*.

4. The horizontal translation is represented by c in the form $y = a \sin b(x - c) + d$ or $y = a \sin b(x - c) + d$.

a) $c = -45°$, the horizontal translation is 45° to the left.

b) $c = 180°$, the horizontal translation is 180° to the right.

c) $c = 45°$, the horizontal translation is 45° to the right.

5. a) Amplitude = 7

$\text{Minimum value} = d - a$

$\text{Minimum value} = 0 - 7$

$\text{Minimum value} = -7$

$\text{Maximum value} = d + a$

$\text{Maximum value} = 0 + 7$

$\text{Maximum value} = 7$

The amplitude of this function is 7, and the range is $\{y \mid -7 \le y \le 7, y \in R\}$.

b) Amplitude = 13

$\text{Minimum value} = d - a$

$\text{Minimum value} = 0 - 13$

$\text{Minimum value} = -13$

$\text{Maximum value} = d + a$

$\text{Maximum value} = 0 + 13$

$\text{Maximum value} = 13$

The amplitude of this function is 13, and the range is $\{y \mid -13 \le y \le 13, y \in R\}$.

6. a) Equation of the midline: $y = 5$

Amplitude = 8

$\text{Minimum value} = d - a$

$\text{Minimum value} = 5 - 8$

$\text{Minimum value} = -3$

$\text{Maximum value} = d + a$

$\text{Maximum value} = 5 + 8$

$\text{Maximum value} = 13$

The equation of the midline of this function is $y = 5$. The amplitude is 8, and the range is $\{y \mid -3 \le y \le 13, y \in R\}$.

b) Equation of the midline: $y = -7$

Amplitude = 6

$\text{Minimum value} = d - a$

$\text{Minimum value} = -7 - 6$

$\text{Minimum value} = -13$

$\text{Maximum value} = d + a$

$\text{Maximum value} = -7 + 6$

$\text{Maximum value} = -1$

The equation of the midline of this function is $y = -7$. The amplitude is 6, and the range is $\{y \mid -13 \le y \le -1, y \in R\}$.

7. a) Equation of the midline: $y = 2$

$\text{Maximum value} = d + a$

$\text{Maximum value} = 2 + 5$

$\text{Maximum value} = 7$

$\text{Minimum value} = d - a$

$\text{Minimum value} = 2 - 5$

$\text{Minimum value} = -3$

The equation of the midline of this function is $y = 2$. The maximum value is 7, and the minimum value is –3.

b) Equation of the midline: $y = -3$

$\text{Maximum value} = d + a$

$\text{Maximum value} = -3 + 3$

$\text{Maximum value} = 0$

$\text{Minimum value} = d - a$

$\text{Minimum value} = -3 - 3$

$\text{Minimum value} = -6$

The equation of the midline of this function is $y = -3$. The maximum value is 0, and the minimum value is –6.

8. a) $y = \sin x$ would have to be horizontally translated 30° right.

b) $y = \cos x$ would have to be horizontally translated 100° left.

c) $y = \sin x$ would have to be horizontally translated 4.5 right.

d) $y = \cos x$ would have to be horizontally translated 3 left.

9. a) Period $= \dfrac{360°}{b}$

Period $= \dfrac{360°}{4}$

Period $= 90°$

Horizontal translation: c = 45°
The period of this function is 90°, and the horizontal translation is 45° right.

b) Period $= \dfrac{2\pi}{b}$

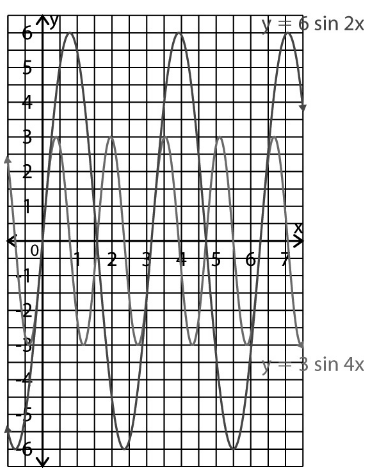
Period $= \dfrac{2\pi}{0.5}$

Period $= 4\pi$

Horizontal translation: c = 1
The period of this function is 4π, and the horizontal translation is 1 right.

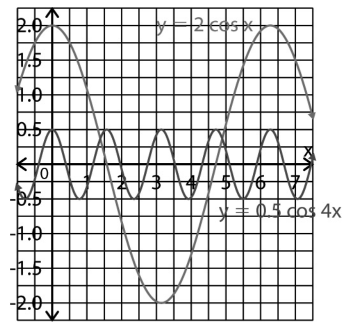
10. a) If the equation is in the form $y = a \sin b\,(x + c) + d$, the amplitude is a, the midline is $y = d$, it is translated to the right by c degrees or radians and the period is $\dfrac{360°}{b}$ or $\dfrac{2\pi}{b}$. Thus, in order to double the amplitude and period, a must be double the value of the original equation and b must be half the value of the original equation. Therefore, the function must be $y = 6 \sin 8x$.

b)



11. a) If the equation is in the form $y = a \sin b\,(x + c) + d$, the amplitude is a, the midline is $y = d$, it is translated to the right by c degrees or radians and the period is $\dfrac{360°}{b}$ or $\dfrac{2\pi}{b}$. Thus, for the amplitude and period to be a fourth of the original value, a must be one-fourth the value of the original equation and b must be four times the value of the original equation. Therefore, the function must be $y = 0.5 \cos 4x$.

b)



12. a) The diameter is 86 ft because the amplitude is 43 ft.

b) $\dfrac{2\pi}{b} = \dfrac{2\pi}{3.5}$

$= 1.795...$

It takes approximately 1.8 minutes or 1 minute and 48 seconds.

c) Minimum value = $d - a$

Minimum value = 47 – 43

Minimum value = 4

Ashley is approximately 4 ft above the ground at the lowest point.

13. a) The maximum value of this graph is 5, and the minimum value is –1. This eliminates equations iv) and v) because equation iv) has a minimum of –4 and equation v) has a minimum of 0.
The midline of this graph is y = 2.
The graph first intersects the midline and is increasing at x = 120°, and the first maximum is at 210°. Therefore, the horizontal translation is 120° for a sine graph and 210° for a cosine graph. Equation i) is eliminated, because its horizontal translation is –120° or 240°, and equation ii) is eliminated, because its horizontal translation is 90° or 450°.
Equation iii) corresponds to this graph.

b) The maximum value of this graph is 4, and the minimum value is –2. This eliminates equations iv) and v) because equation iv) has a minimum of –4, and equation v) has a minimum of 0.
The midline of this graph is y = 1.
The graph first intersects the midline and is decreasing at x = 60°, and the first minimum is at 150°. Therefore the translation is –120° for a sine function and –210° for a cosine function.
Equation i) corresponds to this graph.

14. a) $\text{Period} = \text{Second minimum} - \text{first minimum}$

$\text{Period} = 300° - 120°$

$\text{Period} = 180°$

$\text{Period} = \frac{360°}{b}$

$180° = \frac{360°}{b}$

$b = \frac{360°}{180°}$

$b = 2$

The equation that correctly represents this graph will have a value of 2 as *b*. Therefore, equations ii), iii), and iv) are eliminated, because ii) has *b* equal to 0.5, and iii) and iv) have *b* equal to 1. The first maximum is at 30°. So, the horizontal translation of the cosine function is 30°. The graph first intersects the midline and is increasing at 165° so the translation of the sine function is 165°. Therefore, equation i) cannot represent this graph, because it has *c* equal to –30°.
Equation v) corresponds to this graph.

b) The distance between the first maximum and first minimum is half the period of a sine or cosine function. Here, that distance is 360°, so the period of this graph is 720°.

$\text{Period} = \frac{360°}{b}$

$b = \frac{360°}{Period}$

$b = \frac{360°}{720°}$

$b = 0.5$

The only equation with a *b* value of 0.5 is equation ii). The values for *a*, *c* and *d* also fit the graph. Therefore, equation ii) corresponds to this graph.

15. a) Range: $\{y \mid -4 \le y \le 2, y \in \mathrm{R}\}$
Amplitude = 3
Maximum value = $d + a$
Maximum value = $-1 + 3$
Maximum value = 2
Minimum value = $d - a$
Minimum value = $-1 - 3$
Minimum value = -3
Equation of the midline: $y = -1$

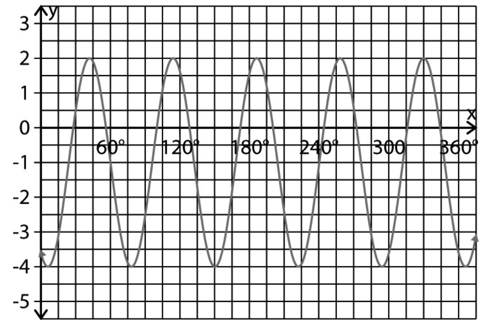
$\text{Period} = \frac{360°}{b}$

$\text{Period} = \frac{360°}{5}$

$\text{Period} = 72°$

Horizontal translation = –30°
The range of this function is $\{y \mid -4 \le y \le 2, y \in \mathrm{R}\}$, and the amplitude is 3. The equation of the midline is $y = -1$. The period is 72°, and the graph has been translated 30° to the left (or –30°).

b) e.g., I plotted the equation using graphing technology and the graph matched the characteristics.



16. a) G: Period:

$\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{1\,231.5}$

Period = 0.0051...

Frequency:

$\text{Frequency} = \frac{1}{\text{Period}}$

$\text{Frequency} = \frac{1}{0.0051...}$

Frequency = 195.999...

The period and frequency is 0.0051 s and 196 Hz respectively.

D: Period:

$\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{1845.4}$

Period = 0.0034...

Frequency:

$\text{Frequency} = \frac{1}{\text{Period}}$

$\text{Frequency} = \frac{1}{0.0034...}$

Frequency = 293.704...

The period and frequency is 0.0034 s and 293.7 Hz respectively.

A: Period:

$\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{2764.6}$

Period = 0.0022...

Frequency:

$\text{Frequency} = \frac{1}{\text{Period}}$

$\text{Frequency} = \frac{1}{0.0022...}$

Frequency = 439.999...

The period and frequency is 0.022 s and 440 Hz respectively.
E:
Period:

$$\text{Period} = \frac{2\pi}{b}$$

$$\text{Period} = \frac{2\pi}{4\,142.5}$$

$$\text{Period} = 0.0015...$$

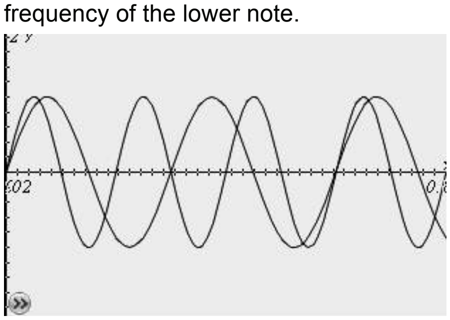
Frequency:

$$\text{Frequency} = \frac{1}{\text{Period}}$$

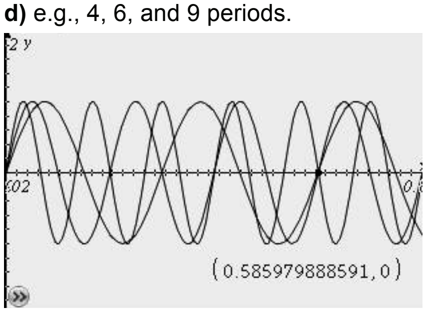
$$\text{Frequency} = \frac{1}{0.0015...}$$

$$\text{Frequency} = 659.299...$$

The period and frequency is 0.0015 s and 659.3 Hz respectively.
b) e.g., Each string is tuned to almost 1.5 times the frequency of the previous string.
c) e.g., About 2 periods of the lower note and 3 periods of the higher note. The frequency of the higher note is approximately 1.5 times the frequency of the lower note.



d) e.g., 4, 6, and 9 periods.



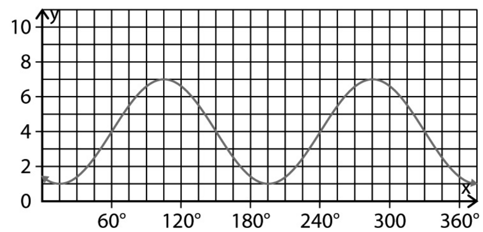
17. a) i) Amplitude = 3
Range: $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$
Equation of the midline: $y = 4$

$$\text{Period} = \frac{360°}{b}$$

$$\text{Period} = \frac{360°}{2}$$

$$\text{Period} = 180°$$

Horizontal translation = 60°
This graph will have an amplitude of 3, the range is $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$ and the equation of the midline will be $y = 4$. The period will be 180°, and the graph will be translated 60° to the right.



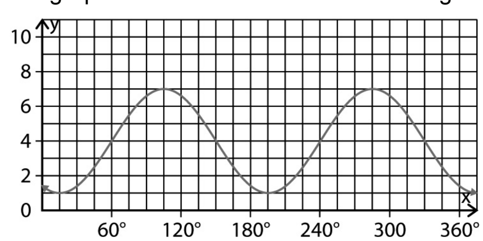
ii) Amplitude = 3
Range: $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$
Equation of the midline: $y = 4$

$$\text{Period} = \frac{360°}{b}$$

$$\text{Period} = \frac{360°}{2}$$

$$\text{Period} = 180°$$

Horizontal translation = 240°.
This graph will have an amplitude of 3, the range is $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$ and the equation of the midline will be $y = 4$. The period will be 180°, and the graph will be translated 240° to the right.



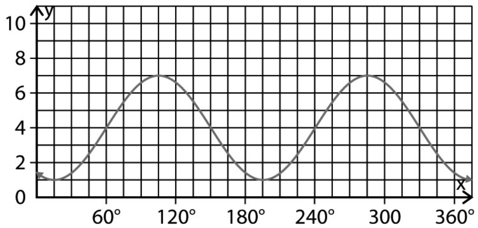
iii) Amplitude = 3
Range: $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$
Equation of the midline: $y = 4$

$$\text{Period} = \frac{360°}{b}$$

$$\text{Period} = \frac{360°}{2}$$

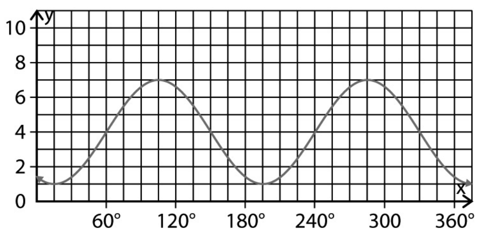
$$\text{Period} = 180°$$

Horizontal translation = –120°.
This graph will have an amplitude of 3, the range is $\{y \mid 1 \le y \le 7,\ y \in \mathrm{R}\}$ and the equation of the midline will be $y = 4$. The period will be 180°, and the graph will be translated 120° to the left.



b) The three graphs are exactly the same. The amplitude, equation of the midline and period are the same. But, they have all been translated by different amounts. Equation i) has been translated 60°, equation ii) has been translated 240° (or 60° + 1 period length), and equation iii) has been translated –120° (or 60° – 1 period length). Since all of these translations differ by 1 or 2 period lengths, the effect of the translation does the same thing for every graph.

c)



e.g., It is also the same because it is a cosine graph with the same attributes as the sine graphs, except that it is translated to the left by 45° (90° divided by b).

18. a) The highest and lowest points the apple reaches correspond to the maximum and minimum values of the function.
Maximum = $d + a$
Maximum = 6.5 + 4
Maximum = 10.5 cm
Minimum = $d – a$
Minimum = 6.5 – 4
Minimum = 2.4 cm
The highest point the apple reaches is 10.5 cm, and the lowest point the apple reaches is 2.5 cm.

b) $\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{8\pi}$

$\text{Period} = \frac{1}{4}$

$\text{Period} = 0.25$

The period of the equation is 0.25 s; the apple completes 4 bounces per second. In this context, the period corresponds to one full oscillation up and down of the apple attached to the spring.

19. a) The maximum and minimum heights you can reach riding the Ferris wheel correspond to the maximum and minimum values of the equation.
Maximum = $d + a$
Maximum = 18 + 15
Maximum = 33
Minimum = $d – a$
Minimum = 18 – 15
Minimum = 3
The maximum height you can reach is 33 m, and the minimum height is 3 m.

b) $\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{1}$

$\text{Period} = 2\pi$

$\text{Period} = 0.628\text{...min}$

The period of the equation is about 6.3 minutes. In this context, the period represents the time it takes for the Ferris wheel to make one full rotation so the wheel completes a rotation every 6.3 minutes.

20. a) $\text{Period} = \frac{2\pi}{b}$

$\text{Period} = \frac{2\pi}{8.4}$

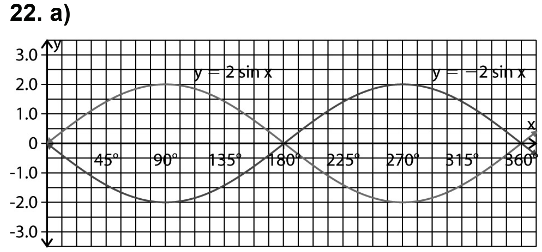
$\text{Period} = 0.747\ldots \text{ s}$

The period of the equation is about 0.75 s.

b) In this situation, the period of the equation corresponds to a person's blood pressure. Thus, A person's blood pressure makes a complete low-to-high cycle about every 0.75 seconds.

21. e.g. If the equation is in the form $y = a \sin b (x – c) + d$, the amplitude is a, the midline is $y = d$, it is translated to the right by c degrees or radians and the period is $\frac{360°}{b}$ or $\frac{2\pi}{b}$. For example, the equation $y = 2 \sin 2 (x – 45°) + 3$ has an amplitude of 1, a midline of $y = 3$, it is translated to the right 45°, and a period of 180°.

22. a)



The period, amplitude and equation of midline are the same for both graphs in addition to minimum and maximum values. What is different is they are reflected across the x-axis or they can be considered reflections of each other.

b) The graph of $y = 2 \sin x$ first crosses the midline while increasing at $x = 0°$. Therefore, it has not shifted. The graph of $y = –2 \sin x$ first crosses the midline while increasing at $x = 180°$. So, either graph could have a translation of π or 180° to the left or right in order to match the other graph.
c) Yes, e.g., graphs of functions of the form $y = a \sin x$ and $y = –a \sin x$ are horizontal translations of each other by 180° or π.

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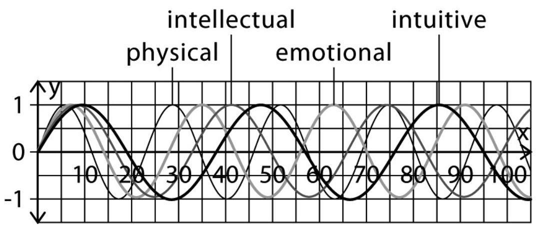
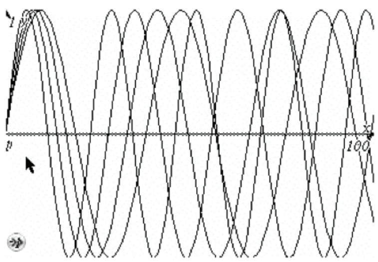


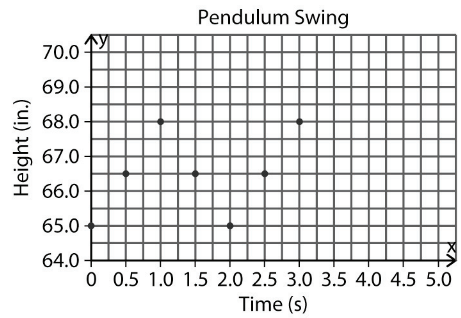


• All four cycles will again be at zero at 201 894 days. The person will be about 553 years old. A point at which all the cycles are at zero together is the least common multiple of the periods:
$23 \cdot (2^2 \cdot 7) \cdot (3 \cdot 11) \cdot 19 = 403\ 788$
However, since sine functions intersect the x-axis halfway through their period, the first point at which all the cycles will be zero is half the number above, or 201 894.

• The first maximum for the physical cycle will occur at 5.75 days and every 23 days after (28.75, 51.75, and so on). The first maximum for the emotional cycle will be at 7 days and then every 28 days after (35, 63, 91, and so on). These two maximum values will never coincide, because one is never a whole number and the other is always a whole number. The same is true for the minimum values.
• Today, I am 6239 days old. According to my equations, my physical and intuitive states should be strong, my intellectual state is increasing, and my emotional state is in decline. No, these charts do not match how I feel today. Today, I feel my intellectual and emotional states are strong and my physical state is average. My intuitive state is in decline.

Lesson 8.5: Modelling Data with Sinusoidal Functions, page 571

1. a)



b) The equation of the sinusoidal regression function is
$y = 1.5 \sin (3.141\ldots x – 1.570\ldots) + 66.5$.

2. a)

Revolution	Time (s)	Height (ft)
0	0	11
$\frac{1}{4}$	0.5	6
$\frac{1}{2}$	1	1
$\frac{3}{4}$	1.5	6
1	2	11
$1\frac{1}{4}$	2.5	6
$1\frac{1}{2}$	3	1
$1\frac{3}{4}$	3.5	6
2	4	11

b) The equation of the sinusoidal regression function is
$y = 5 \sin (3.141\ldots x + 1.570\ldots) + 6$.
c) $y = 5\sin(3.141...x + 1.570...) + 6$
$y = 5\sin(3.141...(1.15) + 1.570) + 6$
$y = 5\sin(3.612... + 1.570...) + 6$
$y = 5\sin(5.183...) + 6$
$y = 5(–0.891...) + 6$
$y = –4.455... + 6$
$y = 1.544...\text{ft}$
Convert the decimal feet to feet and inches.
$y = 1.544...\text{ft}$
$y = 1\text{ ft } (0.544... \cdot 12)\text{ in.}$
$y = 1\text{ ft } 6.539...\text{ in.}$
The height at 1.15 s is 1 ft 7 in. high.