2. The graph of $y = \sin x$ repeats itself after it passes through 360° or 2π.

3. e.g. The graph is symmetrical along the *x*-axis, with the axis of symmetry being at 90° and 270°, respectively. The graph is rotationally symmetrical around the point (180°, 0).

4. e.g., The graph is symmetrical along the *x*-axis, with the axis of symmetry being at 180° and 360°, respectively. The graph is rotationally symmetrical around the point (270°, 0).

5. a) The *y*-intercept of the graph of $y = \sin x$ is 0.
b) The *y*-intercept of the graph of $y = \cos x$ is 1.

6. Using the graph from chapter 1,
a) The *x*-intercepts of the graph of $y = \sin x$ for the interval from 0° to 720° are 0°, 180°, 360°, 540° and 720°.
b) The *x*-intercepts of the graph of $y = \cos x$ for the interval from 0° to 720° are 90°, 270°, 450°, and 630°.

7. e.g., The up-and-down pattern in the graph of $y = \sin x$ looks like the repeating pattern of waves in a lake.

8. Yes, because each point on the sine graph corresponds to a point on the cosine graph with the same *y*-coordinate but with an *x*-coordinate of 90° less. For example, consider the *x*-intercepts for
$y = \cos x$ and $y = \sin x$ on 0° to 720°:

$y = \cos x$	$y = \sin x$	y = cos x translated left by 90°
90°	0°	0°
270°	180°	180°
450°	360°	360°
630°	540°	540°
…	720°	720°

The same results occur for any value of $y = \cos x$.

9. e.g., It would look like the graph of cos *x*, as it would start at 1, decrease to 0 after spinning 90°, then decrease to 21 after spinning 180°, and so on.

History Connection, page 496

A. Looking for more precise values is a way to test the capabilities of newer computers. It is an interesting intellectual exercise. The digits in the decimal value of π occur in a random order, so it is a good random-number generator and it is useful for breaking codes.

B. i) The total surface area is 8.273… km^2.
ii) Using 3.2 as the value of π, the total surface area would have been 8.427… km^2. There would be 0.153… km^2 of wasted steel.

Lesson 8.3: The Graphs of Sinusoidal Functions, page 536

1. a)
$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{3.5 - (-3.5)}{2}$$
$$\text{Amplitude} = \frac{7}{2}$$
$$\text{Amplitude} = 3.5$$
The amplitude of the graph is 3.5 cm, and its range is $\{y | -3.5 \le y \le 3.5, y \in \text{R}\}$.

b)
$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{1 - (-3)}{2}$$
$$\text{Amplitude} = \frac{4}{2}$$
$$\text{Amplitude} = 2$$
The amplitude of the graph is 2 cm, and its range is $\{y | -3 \le y \le 1, y \in \text{R}\}$.

2. a) Equation of the midline:
$$y = \frac{\text{max} + \text{min}}{2} \qquad \text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$y = \frac{3.5 + 0.5}{2} \qquad \text{Amplitude} = \frac{3.5 - 0.5}{2}$$
$$y = \frac{4}{2} \qquad \text{Amplitude} = \frac{3}{2}$$
$$y = 2 \qquad \text{Amplitude} = 1.5$$
The equation of the midline of this graph is $y = 2$ cm, and its amplitude is 1.5 cm.
b) Equation of the midline:
$$y = \frac{\text{max} + \text{min}}{2} \qquad \text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$y = \frac{3 + (-5)}{2} \qquad \text{Amplitude} = \frac{3 - (-5)}{2}$$
$$y = \frac{-2}{2} \qquad \text{Amplitude} = \frac{8}{2}$$
$$y = -1 \qquad \text{Amplitude} = 4$$
The equation of the midline of this graph is $y = -1$ cm, and its amplitude is 4 cm.

3. a)
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 120° - 0°$$
$$\text{Period} = 120°$$
The period of this graph is 120°.
b)
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 3.75 - 0.75$$
$$\text{Period} = 3$$
The period of this graph is 3.

4. a) Maximum value = 3 cm, Minimum value = –7 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{3 - (-7)}{2}$$
$$\text{Amplitude} = \frac{10}{2}$$
$$\text{Amplitude} = 5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{3 + (-7)}{2}$$
$$y = \frac{-4}{2}$$
$$y = -2$$
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 225^\circ - 45^\circ$$
$$\text{Period} = 180^\circ$$

The range of this graph is $\{y \mid -7 \le y \le 3, y \in \text{R}\}$, and its amplitude is 5 cm. The equation of the midline is $y = -2$ cm, and the period is 180°.

b) Maximum value = 6.5 cm
Minimum value = –0.5 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{6.5 - (-0.5)}{2}$$
$$\text{Amplitude} = \frac{7}{2}$$
$$\text{Amplitude} = 3.5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{6.5 + (-0.5)}{2}$$
$$y = \frac{6}{2}$$
$$y = 3$$
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 5 - 0$$
$$\text{Period} = 5$$

The range of this graph is $\{y \mid -0.5 \le y \le 6.5, y \in \text{R}\}$, and its amplitude is 3.5 cm. The equation of the midline is $y = 3$ cm, and the period is 5.

5. Maximum value = 3 cm
Minimum value = –2 cm

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$\text{Amplitude} = \frac{3 - (-2)}{2}$$
$$\text{Amplitude} = \frac{5}{2}$$
$$\text{Amplitude} = 2.5$$

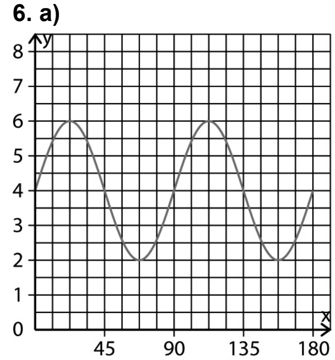
Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{3 + (-2)}{2}$$
$$y = \frac{1}{2}$$
$$y = 0.5$$

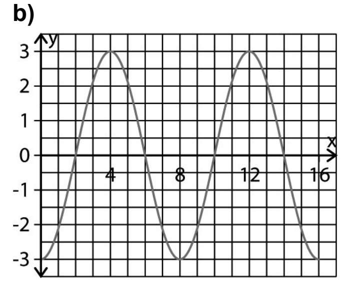
Period = second max – first max
Period = 6.45 – 2.2
Period = 4.25

The range of this graph is $\{y \mid -2 \le y \le 3, y \in \text{R}\}$, and its amplitude is 2.5 cm. The equation of the midline is $y = 0.5$ cm, and the period is 4.25.

6. a)



b)



7. The period would be the same but the amplitude (and the minima and maxima) would be smaller.

8. a) The depth of the water with no waves is represented by the equation of the midline of the graph.

$$y = \frac{\max + \min}{2}$$
$$y = \frac{2.4 + 0.8}{2}$$
$$y = \frac{3.2}{2}$$
$$y = 1.6$$

The depth of the water when no waves are being generated is 1.6 m.

b) The height of each wave is represented by the amplitude of the graph.

$$\text{Amplitude} = \frac{\max - \min}{2}$$
$$\text{Amplitude} = \frac{2.4 - 0.8}{2}$$
$$\text{Amplitude} = \frac{1.6}{2}$$
$$\text{Amplitude} = 0.8$$

The height of each wave is 0.8 m.

c) The amount of time it takes for one complete wave to pass is represented by the period of the graph.

$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 3.125 - 0.625$$
$$\text{Period} = 2.5$$

It takes 2.5 s for one complete wave to pass.

d) After 4 seconds, the water will be about 1.2 m deep, as can be seen from the graph. Since the depth of the water at 5 s is 1.6 m, and the period is 2.5 s, we can assume that at 7.5 s, the water will also be 1.6 m deep.

9. a) Equation of the midline:

$$y = \frac{\max + \min}{2}$$
$$y = \frac{0.8 + (-0.8)}{2}$$
$$y = \frac{0}{2}$$
$$y = 0$$

The equation of the midline is y = 0. In this situation, it represents the brief instance in time between inhaling and exhaling which is the velocity of the air between breathes.

b) $\text{Amplitude} = \frac{\max - \min}{2}$

$$\text{Amplitude} = \frac{0.8 - (-0.8)}{2}$$
$$\text{Amplitude} = \frac{1.6}{2}$$
$$\text{Amplitude} = 0.8$$

The amplitude of this function is 0.8 L/s.

c) $\text{Period} = \text{second max} - \text{first max}$

$$\text{Period} = 6.25 - 1.25$$
$$\text{Period} = 5$$

The period of this function is 5 s, which represents the time it takes to breathe in and out completely

10. a) Both: the increased amplitude means that the breathes are deeper and the decreased period means more breaths per minute.

b) The amplitude and period of this graph has changed compared to the graph in Question 9.

c) The maximum velocity of air entering the lungs has increased to 1.2 L/s.

11. e.g., The gymnast completes one jump every 3 seconds, which is the period of the graph. The jump has a total jump height of 27 feet, the amplitude of the graph. When the gymnast hits the trampoline, the height above the ground is 3 feet, the minimum value of the graph. The gymnast reaches a height of 30 feet, the maximum value of the graph.

Thus, the gymnast completes a jump every 3 seconds, going from 3 feet above the ground to 30 feet, with a total jump height of 27 feet.

12. Range: $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$

$$\text{Amplitude} = \frac{\max - \min}{2}$$
$$\text{Amplitude} = \frac{3 - (-7)}{2}$$
$$\text{Amplitude} = \frac{10}{2}$$
$$\text{Amplitude} = 5$$

Equation of the midline:

$$y = \frac{\max + \min}{2}$$
$$y = \frac{3 + (-7)}{2}$$
$$y = \frac{-4}{2}$$
$$y = -2$$
$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 0.875 - 0.375$$
$$\text{Period} = 0.5$$

The range of this graph is $\{y \mid -7 \le y \le 3, y \in \mathrm{R}\}$ and the amplitude is 5. The equation of the midline is y = –2, and the period is 0.5.

13. a) Pendulum A:

$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 1 - 0$$
$$\text{Period} = 1$$

Minimum value = 10 cm
Maximum value = 26 cm

$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{26 - 10}{2}$

$\text{Amplitude} = \frac{16}{2}$

$\text{Amplitude} = 8$

Pendulum B:

$\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 0.5 - 0$

$\text{Period} = 0.5$

Minimum value = 10 cm
Maximum value = 18 cm

$\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{18 - 10}{2}$

$\text{Amplitude} = \frac{8}{2}$

$\text{Amplitude} = 4$

The period for Pendulum A is 1 second, and the period for pendulum B is only 0.5 seconds. Pendulum B is swinging twice as fast as Pendulum A. Both pendulums have a minimum value of 10 cm above the table. Pendulum A reaches a maximum height of 26 cm, while Pendulum B reaches a height of 18 cm. The amplitude of graph A is 8 cm, and of graph B is 4 cm.

b) Pendulum A is longer because its amplitude is greater. The amplitude of graph A is twice as large as the amplitude of graph B.

14. a) The edge of the hoop will always be on the arm of the hoop dancer, and since half the total height of the hoop is the amplitude, the amplitude must represent the radius.

b) Hoop 1: $\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{7.5 - 2.5}{2}$

$\text{Amplitude} = \frac{5}{2}$

$\text{Amplitude} = 2.5$

Hoop 2: $\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{7.5 - 2.5}{2}$

$\text{Amplitude} = \frac{5}{2}$

$\text{Amplitude} = 2.5$

Hoop 3: $\text{Amplitude} = \frac{\text{max} - \text{min}}{2}$

$\text{Amplitude} = \frac{6.5 - 2.5}{2}$

$\text{Amplitude} = \frac{4}{2}$

$\text{Amplitude} = 2$

The amplitude of the first two graphs is 2.5 m, so the first two hoops have a radius of 2.5 m. The amplitude of the third graph is 2 m, so the third hoop has a radius of 2 m. The third hoop is the smallest.

c) In this situation, the period represents the amount of time it takes to spin the hoop one revolution.

Hoop 1: $\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 1.5 - 0.5$

$\text{Period} = 1$

Hoop 2: $\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 1 - 0.333...$

$\text{Period} = 0.666...$

Hoop 3: $\text{Period} = \text{second max} - \text{first max}$

$\text{Period} = 1 - 0.333...$

$\text{Period} = 0.666...$

The first graph has a period of one second, while the second and third graphs have a period of 0.67 s. This is the length of time for one spin. Therefore, the hoop in the first graph is being spun the slowest. Hoops 2 and 3 are being spun the fastest.

d) In this situation, the equation of the midline represents the arm height of the person spinning the hoop.

Hoop 1: Equation of the midline:

$y = \frac{\text{max} + \text{min}}{2}$

$y = \frac{7.5 + 2.5}{2}$

$y = \frac{10}{2}$

$y = 5$

Hoop 2: Equation of the midline:

$y = \frac{\text{max} + \text{min}}{2}$

$y = \frac{7.5 + 2.5}{2}$

$y = \frac{10}{2}$

$y = 5$

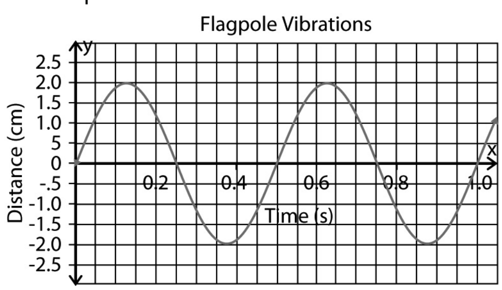
Hoop 3: Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2}$$
$$y = \frac{6.5 + 2.5}{2}$$
$$y = \frac{9}{2}$$
$$y = 4.5$$

e.g., I think the third hoop is being spun by the shorter person, because the midline of this graph is lower compared to the other two graphs. In addition, hoop 3 is smaller but is still spun at the same rate as hoop 2. Therefore, hoop 3 is spun by the shorter person.

15. e.g., The period and midline stay the same, while the amplitude decreases.
e.g., The range is the difference of the maximum and minimum *y*-values. The amplitude is half the difference f the maximum and minimum values. The equation of the midline is *y* = average of the minimum and maximum values.

16. Graph 1:

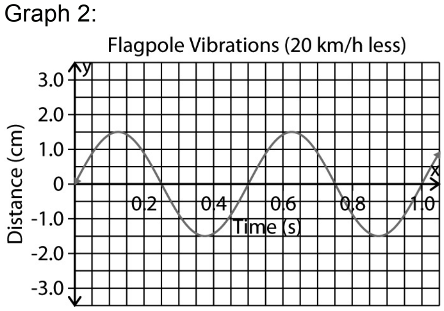


$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 0.625 - 0.125$$
$$\text{Period} = 0.5$$

Equation of the midline:

$$y = \frac{\text{max} + \text{min}}{2} \qquad \text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$y = \frac{2 + (-2)}{2} \qquad \text{Amplitude} = \frac{2 - (-2)}{2}$$
$$y = \frac{0}{2} \qquad \text{Amplitude} = \frac{4}{2}$$
$$y = 0 \qquad \text{Amplitude} = 2$$

Graph 2:



$$\text{Period} = \text{second max} - \text{first max}$$
$$\text{Period} = 0.625 - 0.125$$
$$\text{Period} = 0.5$$

Axis:

$$y = \frac{\text{max} + \text{min}}{2} \qquad \text{Amplitude} = \frac{\text{max} - \text{min}}{2}$$
$$y = \frac{1.5 + (-1.5)}{2} \qquad \text{Amplitude} = \frac{1.5 - (-1.5)}{2}$$
$$y = \frac{0}{2} \qquad \text{Amplitude} = \frac{3}{2}$$
$$y = 0 \qquad \text{Amplitude} = 1.5$$

The period and equation of midline are not affected when the wind decreases, because they are the same in both graphs. However, when the wind decreased, the amplitude decreased from 2 to 1.5.

17. a) A faster frequency will result in a smaller period, because more cycles will occur in a smaller amount of time. Since graph 2 has the largest period, it must have the slowest frequency, and therefore, represents the lowest pitch. By the same logic, Graph 1 represents the middle note, and Graph 3 represents the highest pitch. Thus, the order must be first graph 2, then graph 1 and lastly, graph 3.
b) Each A pitch has double the frequency of the previous one.
c) Graph 2 appears to have half the frequency of Graph 1, so its frequency would be 110 Hz. Graph 3 appears to have double the frequency of Graph 1, so its frequency would be 440 Hz.
d) e.g., Sound waves can be approximated by a sinusoidal function. The amplitude dictates the volume of the sound. The period, or frequency, varies with the size of the instrument (e.g., a tuba creates low-frequency/long-period waves while a piccolo creates high-frequency waves.)

18. a) If the frequency is 60 times per second, then one complete cycle takes $\frac{1}{60}$ = 0.0166… s which is about 17 ms.
b)