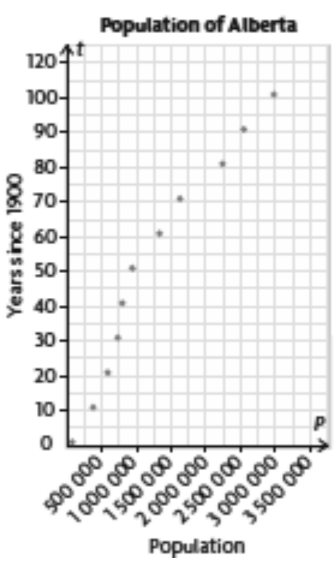
**c)** This earthquake was about 3000 times as intense than the Saskatchewan earthquake.

**15. a)** The *A*-intercept is 3000 and there is no *t*-intercept. In context, the domain of this graph is $\{A \mid A \geq 3000, A \in R\}$, and the range is $\{t \mid t \geq 0, t \in R\}$. This function is increasing.
**b)** $t = 58.7 \log A - 204.1$
$t = 58.7 \log(10\ 000) - 204.1$
$t = 30.7$
It will take about 31 years for Christie's investment to reach $10 000.
**c)** $t = 58.7 \log A - 204.1$
$t = 58.7 \log(6000) - 204.1$
$t = 17.677\ldots$
It will take about 18 years for Christie's investment to double.

## Lesson 7.5: Modelling Data Using Logarithmic Functions, page 494

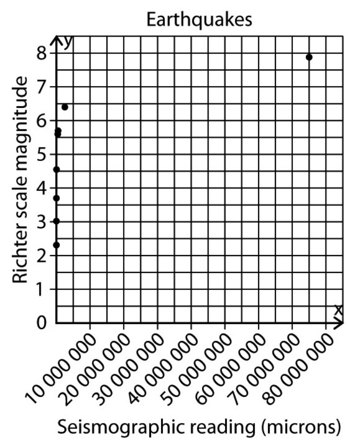
**1. a)** e.g., Graph is logarithmic because graph is increasing, *x*-intercept is 1, there is no *y*-intercept, end behaviour is QIV to QI, domain is $\{x \mid x > 0, x \in R\}$, and the range is $\{y \mid y \in R\}$.
**b)** When the decibel level increases by 10, the relative sound intensity increases by a factor of 10.
**c)** When the relative sound intensity doubles, the decibel level increases by a factor of 3.

**2.** The logarithmic regression equation that models this data is $y = -6.653\ldots + 108.491\ldots \ln x$.
*x*-intercept: 1.063…
*y*-intercept: none
End behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in R\}$
Range: $\{y \mid y \in R\}$
Function: increasing
**3. a)**




**b)** The regression equation that models the data is $t = -346.090\ldots + 28.957\ldots \ln P$
*P*-intercept: 155 076.923…
*t*-intercept: none
End behaviour: QIV to QI
Domain: $\{P \mid P > 0, P \in W\}$
Range: $\{t \mid t \geq 0, t \in W\}$
Function: increasing
**c)** $t = -346.090\ldots + 28.957\ldots \ln P$
$t = -346.090\ldots + 28.957\ldots \ln(2\ 000\ 000)$
$t = 74.043\ldots$
The population exceeded 2 000 000 in 1974.

**4. a)** The independent variable is seismographic reading and the dependent variable is Richter Scale magnitude.
**b)**




**c)** The regression equation for the data is $M = -0.006\ldots + 0.434\ldots \ln r$.
**d)** $M = -0.006\ldots + 0.434\ldots \ln r$

Let $M = 5.7$
$5.7 = -0.006\ldots + 0.434\ldots \ln r_1$
$5.706\ldots = 0.434\ldots \ln r_1$
$13.122\ldots = \ln r_1$
$r_1 = e^{13.122\ldots}$

Let $M = 4.5$
$4.5 = -0.006\ldots + 0.434\ldots \ln r_2$
$4.506\ldots = 0.4348 \ln r_2$
$10.362\ldots = \ln r_2$
$r_2 = e^{10.362\ldots}$

$$\frac{r_1}{r_2} = \frac{e^{13.122\ldots}}{e^{10.362\ldots}}$$

$$\frac{r_1}{r_2} = e^{(13.122\ldots-10.362\ldots)}$$

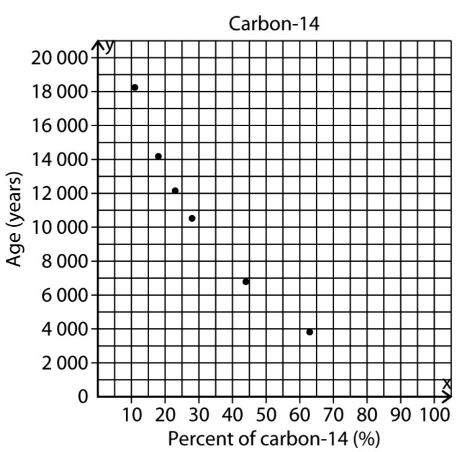
$$\frac{r_1}{r_2} = e^{2.759\ldots}$$

$$\frac{r_1}{r_2} = 15.794\ldots$$

A magnitude 5.7 earthquake is about 15.8 times more intense than a magnitude 4.5 earthquake.

**5. a)** The independent variable is pressure and the dependent variable is altitude.
**b)** The regression equation for this data is
$h = 30\ 665.960\ldots - 6640.436\ldots \ln P$.
**c)** $P$-intercept: 101.297…
$h$-intercept: none
End behaviour: QI to QIV
Domain: $\{P \mid P > 0, P \in \mathrm{R}\}$
Range: $\{h \mid h \in \mathrm{R}\}$
Function: decreasing
**d)**
$$y = 30\ 665.960\ldots - 6640.436\ldots \ln x$$
$$139 = 30\ 665.960\ldots - 6640.436\ldots \ln x$$
$$-30\ 526.960\ldots = -6640.436\ldots \ln x$$
$$4.597\ldots = \ln x$$
$$x = e^{4.597\ldots}$$
$$x = 99.199\ldots$$
The pressure setting that Michael will need to use is 99.2 kPa.
**e)**
$$y = 30\ 665.960\ldots - 6640.436\ldots \ln x$$
$$8848 = 30\ 665.960\ldots - 6640.436\ldots \ln x$$
$$-21817.960\ldots = -6640.436\ldots \ln x$$
$$3.285\ldots = \ln x$$
$$x = e^{3.285\ldots}$$
$$x = 26.725\ldots$$
The atmospheric pressure at the summit of Mt. Everest is 26.7 kPa.

**6. a)** Here is the graph.

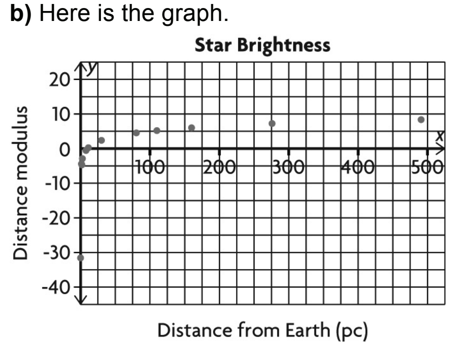



The regression equation that represents this data is $t = 38\ 070.332\ldots - 8266.961\ldots \ln P$
**b)** $t = 38\ 070.332\ldots - 8266.961\ldots \ln P$
$t = 38\ 070.332\ldots - 8266.961\ldots \ln (96.8)$
$t = 268.437\ldots$
The age of these fragments is about 268 years.
**c)** $t = 38\ 070.332\ldots - 8266.961\ldots \ln P$
$t = 38\ 070.332\ldots - 8266.961\ldots \ln (50)$
$t = 5729.790\ldots$
The half-life of carbon-14 is 5730 years.

**7. a)** The exponential regression equation that represents this data is $A = 14\ 999.826\ldots(1.045\ldots)^t$.
**b)** The logarithmic regression equation that represents this data is $t = -218.442\ldots + 22.717\ldots \ln A$.
**c)** Exponential:
$$A = 14\ 999.826\ldots(1.045\ldots)^t$$
$$25\ 000 = 14\ 999.826\ldots(1.045\ldots)^t$$
$$1.666\ldots = (1.045\ldots)^t$$
$$\log 1.666\ldots = \log (1.045\ldots)^t$$
$$\log 1.666\ldots = t \log 1.045\ldots$$
$$\frac{\log 1.666\ldots}{\log 1.045\ldots} = t$$
$$11.604\ldots = t$$

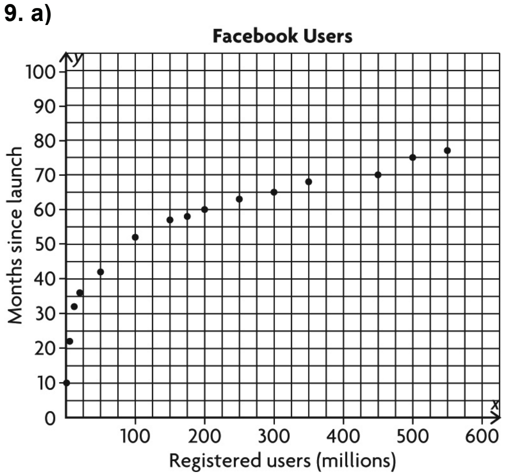
Logarithmic:
$t = -218.442\ldots + 22.717\ldots \ln A$
$t = -218.442\ldots + 22.717\ldots \ln (25\ 000)$
$t = 11.604\ldots$
It will take about 12 years for the balance to equal $25 000 according to both equations.
e.g., I prefer using the logarithmic because it makes the calculations simpler.

**8. a)** The independent variable is distance from Earth and the dependent variable is distance modulus.
**b)** Here is the graph.




**c)** The logarithmic regression equation for this data is $m = -5.086\ldots + 2.174\ldots \ln d$.
**d)** $m = -5.086\ldots + 2.174\ldots \ln d$
$m = -5.086\ldots + 2.174\ldots \ln (2.39)$
$m = -3.191\ldots$
The distance modulus of Wolf 359 is –3.19.
**e)**
$$m = -5.086\ldots + 2.174\ldots \ln d$$
$$-1.66 = -5.086\ldots + 2.174\ldots \ln d$$
$$3.426\ldots = 2.174\ldots \ln d$$
$$1.575\ldots = \ln d$$
$$e^{1.575\ldots} = d$$
$$d = 4.834\ldots$$
Gliese 876 is 4.8 pc from Earth.

**9. a)**




**b)** The logarithmic regression equation for this data is $t = 5.888\ldots + 10.833\ldots \ln n$.

**c)** $t = 5.888\ldots + 10.833\ldots \ln n$

$t = 5.888\ldots + 10.833\ldots \ln (275)$

$t = 66.736\ldots$

Facebook first surpassed 275 million registered users during the 66th month after February 2004, which is August 2009.

**10.** Enter the data in my calculator, perform a logarithmic regression, graph the data points in a scatter plot and the logarithmic function on the same axes, and identify the point with the known *x*-value and the unknown *y*-value.

**11. a) i)**

$$A = P(1+i)^n$$
$$A = 5000(1+0.05)^n$$
$$\frac{A}{5000} = 1.05^n$$
$$\ln\left(\frac{A}{5000}\right) = \ln 1.05^n$$
$$\ln A - \ln 5000 = n \ln 1.05$$
$$n = \frac{\ln A - \ln 5000}{\ln 1.05}$$

**ii)**

$$A = P(1+i)^n$$
$$A = 5000\left(1+\frac{0.05}{2}\right)^{2n}$$
$$\frac{A}{5000} = 1.025^{2n}$$
$$\ln\left(\frac{A}{5000}\right) = \ln 1.025^{2n}$$
$$\ln A - \ln 5000 = 2n \ln 1.025$$
$$2n = \frac{\ln A - \ln 5000}{\ln 1.025}$$
$$n = \frac{\ln A - \ln 5000}{2\ln 1.025}$$

**iii)**

$$A = P(1+i)^n$$
$$A = 5000\left(1+\frac{0.05}{4}\right)^{4n}$$
$$\frac{A}{5000} = 1.0125^{4n}$$
$$\ln\left(\frac{A}{5000}\right) = \ln 1.0125^{4n}$$
$$\ln A - \ln 5000 = 4n \ln 1.0125$$
$$4n = \frac{\ln A - \ln 5000}{\ln 1.0125}$$
$$n = \frac{\ln A - \ln 5000}{4\ln 1.0125}$$

**iv)**

$$A = P(1+i)^n$$
$$A = 5000\left(1+\frac{0.05}{365}\right)^{365n}$$
$$\frac{A}{5000} = 1.000\,136\ldots^{365n}$$
$$\ln\left(\frac{A}{5000}\right) = \ln 1.000\,136\ldots^{365n}$$
$$\ln A - \ln 5000 = 365n \ln 1.000\,136\ldots$$
$$365n = \frac{\ln A - \ln 5000}{\ln 1.000\,136\ldots}$$
$$n = \frac{\ln A - \ln 5000}{365\ln 1.000\,136\ldots}$$

**b)** e.g., The denominator approaches ln 1 = 0 as the number of compounding periods per year increases.

## Math in Action, page 500

• It looks as if you need to ask no more than 10 questions to guess a number from 1 to 1000.

• Let *n* represent the numbers remaining and let *q* represent the number of questions asked.
Since the secret number is from 1 to 1000, it means that there are 1000 possible numbers. Each time you do a binary search, you are dividing the set of possible numbers in half.

I wrote a table of values for the first few guesses.

| ***n*** | ***q*** |
|---|---|
| 1000 | 0 |
| $1000\left(\frac{1}{2}\right)$ | 1 |
| $\left[1000\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right)=1000\left(\frac{1}{2}\right)^2$ | 2 |
| $\left[1000\left(\frac{1}{2}\right)^2\right]\left(\frac{1}{2}\right)=1000\left(\frac{1}{2}\right)^3$ | 3 |

I noticed that the exponent on the base $\left(\frac{1}{2}\right)$ is equal to *q*, the number of questions asked. So, I set up an equation expressing *n* in terms of *q* to model the situation.

$$n=1000\left(\frac{1}{2}\right)^q$$

• I substituted $n = 1$ into my equation.

$$1=1000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1000}=\left(\frac{1}{2}\right)^q$$

$$q=\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1000}\right)$$

$$q=9.965...$$

You would need to ask 10 questions to find a number from 1 to 1000. This answer matches my experimental answer.

• To determine the number of questions needed to guess a number from 1 to one billion, I changed 1000 to 1 000 000 000 in my equation.

$$n = 1\ 000\ 000\ 000\left(\frac{1}{2}\right)^q$$

Then I substituted $n = 1$ and solved for *q*:

$$1=1\ 000\ 000\ 000\left(\frac{1}{2}\right)^q$$

$$\frac{1}{1\ 000\ 000\ 000}=\left(\frac{1}{2}\right)^q$$

$$q=\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{1\ 000\ 000\ 000}\right)$$

$$q=29.897...$$

You would need to ask 30 questions to guess a number from 1 to 1 000 000 000.

• Answers will vary. e.g., One possibility is to allow the guesser to divide the list of remaining numbers into three groups, instead of two. Questions would now be like this: "Is your number between *a* and *b*, *b* and *c*, or *c* and *d*?" This would make the game go much more quickly, since each turn reduces the number of remaining numbers by $\frac{2}{3}$.

To guess a number from 1 to 1000, the equation that would model the situation would now be

$$n=1000\left(\frac{1}{3}\right)^q$$

$$1=1000\left(\frac{1}{3}\right)^q$$

$$\frac{1}{1000}=\left(\frac{1}{3}\right)^q$$

$$q=\log_{\left(\frac{1}{3}\right)}\left(\frac{1}{1000}\right)$$

$$q=6.287...$$

## Chapter Self-Test, page 501

**1.** To match the functions with the graphs, look at the *x*- or *y*-intercepts and directions of the functions and of the graphs. This is because each function and each graph has a unique *y*-intercept.

**a)** It is a decreasing exponential function with *y*-intercept of 0.2 so it must match with ii.

**b)** It is a decreasing logarithmic function, so it must match with iv.

**c)** It is an increasing exponential function with *y*-intercept of 2, so it must match with iii.

**d)** It is an increasing logarithmic function, so it must match with i.