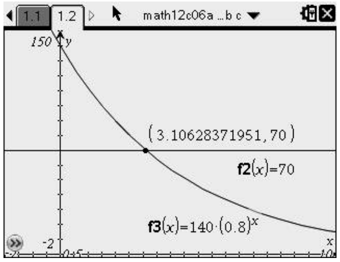
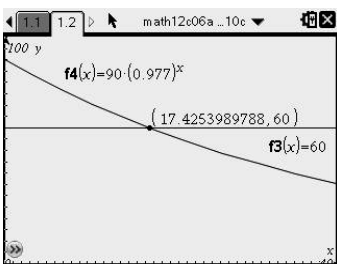
c) $x = 3.106\ldots$ when $y = 70$ cm

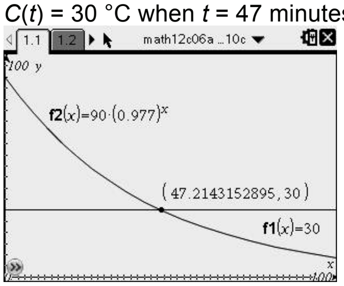


The height was less than half the initial drop height on the 4th bounce.

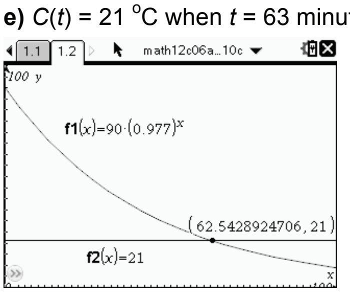
11. a) The function is decreasing. It has one y-intercept and no x-intercepts.
b) The domain is time, so it must be positive: $\{t \mid t > 0, x \in \mathrm{R}\}$
The range is all the possible temperatures from 90 °C to 21 °C: $\{C(t) \mid 21 < C(t) < 90, C(t) \in \mathrm{R}\}$
c) $C(t) = 90(0.977)^t$
$C(10) = 90(0.977)^{10}$
$C(10) = 71.316\ldots$
The temperature of the coffee after 10 min is 71 °C.
d) $C(t) = 60$ °C when $t = 17$ minutes.



$C(t) = 30$ °C when $t = 47$ minutes.



e) $C(t) = 21$ °C when $t = 63$ minutes.



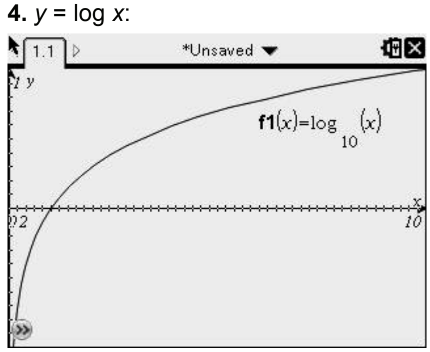
Lesson 7.4: Characteristics of Logarithmic Functions with Base 10 and Base *e*, page 482

1. x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$

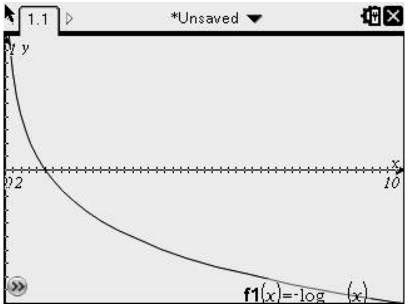
2. a) No. e.g., no x-intercept and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept. It is an exponential function.
b) No. e.g., two x-intercepts and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept. It is a quadratic function.
c) Yes. e.g., one x-intercept and no y-intercept.
d) No. e.g., one x-intercept and one y-intercept whereas logarithmic functions have x-intercepts of 1.
e) Yes. e.g., one x-intercept and no y-intercept.
f) No. e.g., no x-intercept and one y-intercept whereas logarithmic functions have no y-intercepts and one x-intercept.

3. c: x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
a: $a > 0$ e.g., since the graph is increasing
e: x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QI to QIV
Domain: $\{x \mid x > 0, x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
a: $a < 0$ e.g., since the graph is decreasing

4. $y = \log x$:

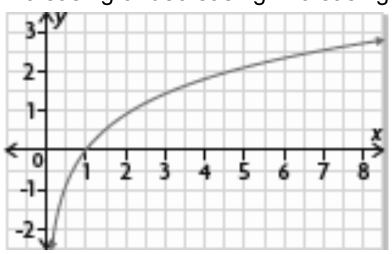


x-intercept: 1
Number of y-intercepts: 0
End Behaviour: QIV to QI
Domain: $\{x \mid x > 0, x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Increasing or decreasing: increasing
$y = -\log x$:

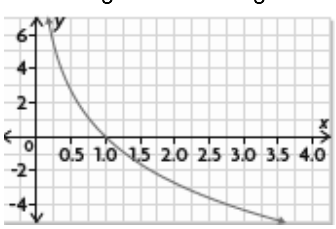


x-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: decreasing

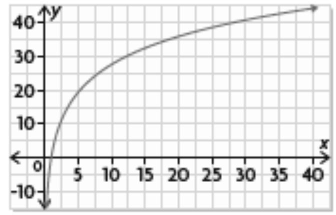
5. a) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: increasing



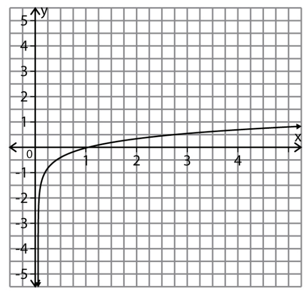
b) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: decreasing



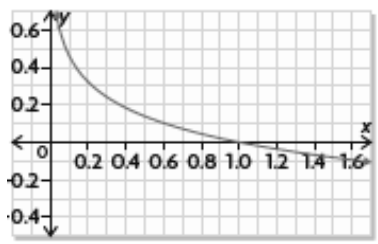
c) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: increasing



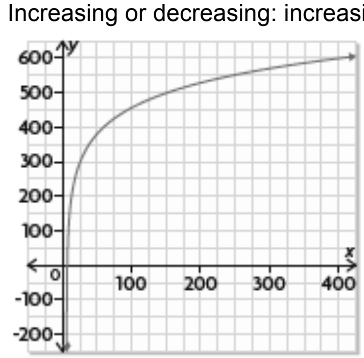
d) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: increasing



e) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QI to QIV
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: decreasing

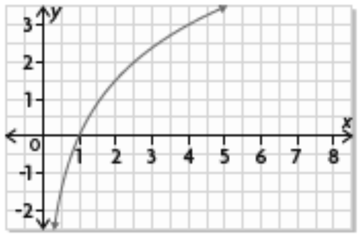


f) *x*-intercept: 1
Number of *y*-intercepts: 0
End Behaviour: QIV to QI
Domain: {*x* | *x* > 0, *x* ∈ R}
Range: {*y* | *y* ∈ R}
Increasing or decreasing: increasing

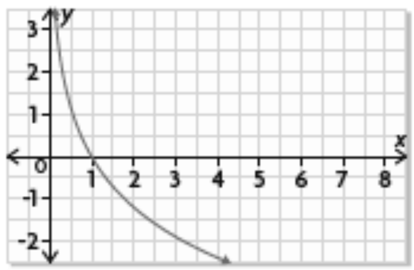


6. e.g., one x-intercept of 1, no y-intercepts, domain: $\{x \mid x > 0, x \in \mathrm{R}\}$.

7. a) e.g., $y = 5 \log x$

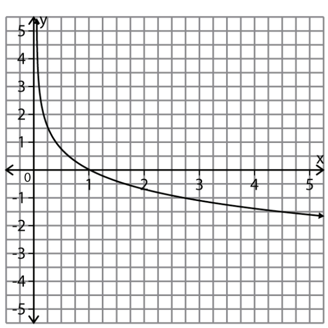


b) e.g., $y = -4 \log x$

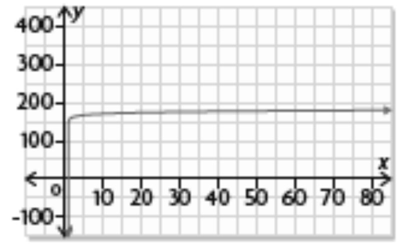


8. i) b, e.g., x-intercept is 1, no y-intercept, graph extends from QIV to QI. Thus, the function is logarithmic so b and c are the only options. The function is increasing so the correct graph is b.
ii) c, e.g., x-intercept is 1, no y-intercept, graph extends from QI to QIV. Thus, the function is logarithmic so b and c are the only options. The function is decreasing so the correct graph is c.
iii) d, e.g., no x-intercept, y-intercept is 1, graph extends from QII to QI. Thus, the function is exponential so a and d are the only options. The function is increasing so the correct graph is d.
iv) a, e.g., no x-intercept, y-intercept is 2, graph extends from QII to QI. Thus, the function is exponential a and d are the only options. The function is decreasing so the correct graph is a.
9. Yes, e.g., An exponential function has no x-intercepts, and a logarithmic function has one x-intercept.

10. As hydrogen ion concentration increases, pH decreases.



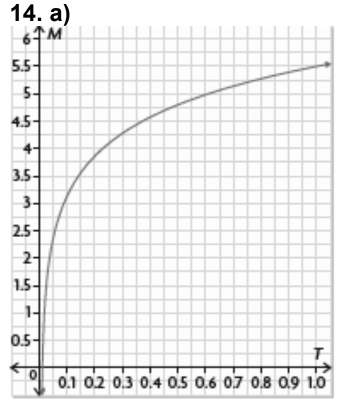
11. As the energy of the sound increases, the number of decibels increases.



12. a) P-intercept: 1, t-intercept: none, domain: $\{P \mid 0 < P \le 1, P \in \mathrm{R}\}$, range: $\{t \mid t \ge 0, t \in \mathrm{R}\}$, function: decreasing, $P = 1$: $t = 0$.
b) $t = -100.422 \log P$
$t = -100.422 \log(0.05)$
$t = 130.652\ldots$
It would take about 131 years for only 5% of the original amount to remain.
c) $t = -100.422 \log P$
$t = -100.422 \log(0.5)$
$t = 30.230\ldots$
The half-life of cesium-137 is about 30 years.

13. a) When $a < 0$, each function will be decreasing.
b) The domain of each function is restricted as for each function x must be greater than 0.
c) The range is unrestricted as all values of y are possible.

14. a)



The T-intercept is $10^{-5.5}$ and there is no M-intercept. In context, the domain of this graph is $\{T \mid T \ge 10^{-5.5}, T \in \mathrm{R}\}$, and the range is $\{M \mid M \ge 0, M \in \mathrm{R}\}$. The graph is increasing, and when $T = 1$, $M = 5.5$.
b) $M = \log T + 5.5$
$M = \log(50) + 5.5$
$M = 7.198\ldots$
An earthquake that is 50 times more intense than the Saskatchewan earthquake would have a magnitude of about 7.2.

c) This earthquake was about 3000 times as intense than the Saskatchewan earthquake.

15. a) The A-intercept is 3000 and there is no t-intercept. In context, the domain of this graph is $\{A \mid A \geq 3000, A \in \mathrm{R}\}$, and the range is $\{t \mid t \geq 0, t \in \mathrm{R}\}$. This function is increasing.

b) $t = 58.7 \log A - 204.1$

$t = 58.7 \log(10\,000) - 204.1$

$t = 30.7$

It will take about 31 years for Christie's investment to reach $10 000.

c) $t = 58.7 \log A - 204.1$

$t = 58.7 \log(6000) - 204.1$

$t = 17.677\ldots$

It will take about 18 years for Christie's investment to double.

Lesson 7.5: Modelling Data Using Logarithmic Functions, page 494

1. a) e.g., Graph is logarithmic because graph is increasing, x-intercept is 1, there is no y-intercept, end behaviour is QIV to QI, domain is $\{x \mid x > 0, x \in \mathrm{R}\}$, and the range is $\{y \mid y \in \mathrm{R}\}$.

b) When the decibel level increases by 10, the relative sound intensity increases by a factor of 10.

c) When the relative sound intensity doubles, the decibel level increases by a factor of 3.

2. The logarithmic regression equation that models this data is $y = -6.653\ldots + 108.491\ldots\ln x$.

x-intercept: 1.063…

y-intercept: none

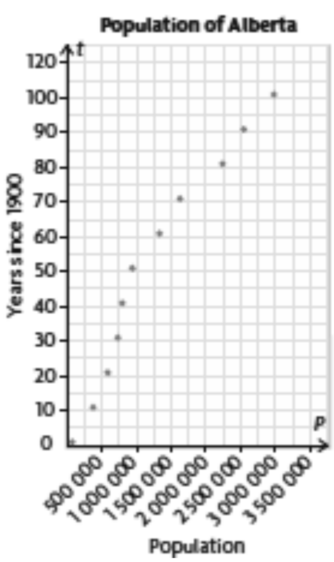
End behaviour: QIV to QI

Domain: $\{x \mid x > 0, x \in \mathrm{R}\}$

Range: $\{y \mid y \in \mathrm{R}\}$

Function: increasing

3. a)



b) The regression equation that models the data is $t = -346.090\ldots + 28.957\ldots \ln P$

P-intercept: 155 076.923…

t-intercept: none

End behaviour: QIV to QI

Domain: $\{P \mid P > 0, P \in \mathrm{W}\}$

Range: $\{t \mid t \geq 0, t \in \mathrm{W}\}$

Function: increasing

c) $t = -346.090\ldots + 28.957\ldots \ln P$

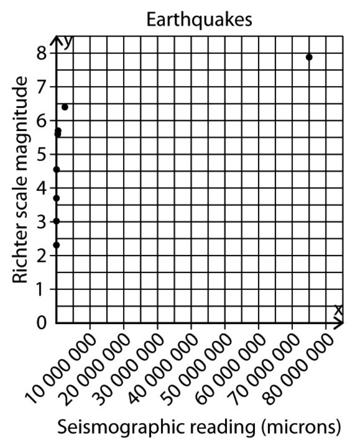
$t = -346.090\ldots + 28.957\ldots \ln(2\,000\,000)$

$t = 74.043\ldots$

The population exceeded 2 000 000 in 1974.

4. a) The independent variable is seismographic reading and the dependent variable is Richter Scale magnitude.

b)



c) The regression equation for the data is $M = -0.006\ldots + 0.434\ldots \ln r$.

d) $M = -0.006\ldots + 0.434\ldots \ln r$

Let $M = 5.7$

$5.7 = -0.006\ldots + 0.434\ldots \ln r_1$

$5.706\ldots = 0.434\ldots \ln r_1$

$13.122\ldots = \ln r_1$

$r_1 = e^{13.122\ldots}$

Let $M = 4.5$

$4.5 = -0.006\ldots + 0.434\ldots \ln r_2$

$4.506\ldots = 0.4348 \ln r_2$

$10.362\ldots = \ln r_2$

$r_2 = e^{10.362\ldots}$

$$\frac{r_1}{r_2} = \frac{e^{13.122\ldots}}{e^{10.362\ldots}}$$

$$\frac{r_1}{r_2} = e^{(13.122\ldots - 10.362\ldots)}$$

$$\frac{r_1}{r_2} = e^{2.759\ldots}$$

$$\frac{r_1}{r_2} = 15.794\ldots$$