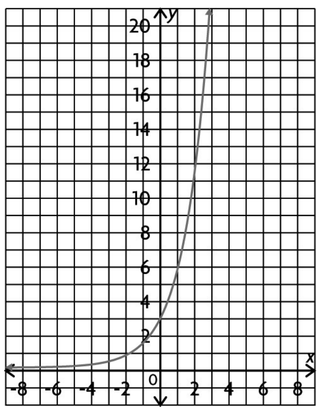
19. a)

Hour	Students who are Told
0	3
1	6
2	12
3	24
4	48
5	96

b) Yes. e.g., After each hour, the number of students who are told in that hour about the presentation doubles.
c) $y = 3(2)^x$; domain: $\{x \mid x \geq 0, x \in R\}$
range: $\{y \mid y \geq 3, y \in R\}$

d) After 8 h, 768 students would be told.

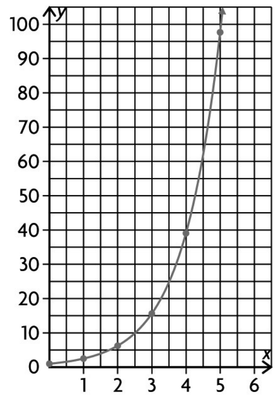


20. a) Yes. e.g., Any data with a constant doubling time can be expressed with an exponential function.
b) $y = 4(1.26)^x$; a represents the initial number of requests, b represents the rate of growth of the number of requests, x represents the amount of time in hours since the news broke, y represents the total number of interview requests.
c) domain: $\{x \mid x > 0, x \in N\}$
range: $\{y \mid y > 4, y \in N\}$
d)

Time	*x*	*y*
9:00 a.m.	0	4.0
10:00 a.m.	1	5.0
11:00 a.m.	2	6.4
12:00 p.m.	3	8.0
1:00 p.m.	4	10.1
2:00 p.m.	5	12.7
3:00 p.m.	6	16.0
4:00 p.m.	7	20.2

Lesson 7.3: Modelling Data Using Exponential Functions, page 461

1. a) This data set does not involve exponential growth or decay because the differences are constant. It is a linear function.
b) This data set does involve exponential growth because it has constant ratios between the y-values with each consequent unit value of x. The function is $y = (2.5)^x$.



c) This data set does not involve exponential growth or decay because the differences are not constant, or a constant ratio.
d) This data set does not involve exponential growth or decay the differences are not constant, or a constant ratio.

2. a) Using a graphing calculator, the exponential regression function for the data is $y = 10.097\ldots(0.200\ldots)^x$.
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
y-intercept: $y = 10.097\ldots$;
End Behaviour: QII to QI
This function is decreasing and shows exponential decay.
b) Using a graphing calculator, the exponential regression function for the data is $y = 2.780\ldots(1.054\ldots)^x$.
Domain: $\{x \mid x \in R\}$; Range: $\{y \mid y > 0, y \in R\}$
y-intercept: $y = 2.780\ldots$; End Behaviour: QII to QI
This function is increasing and shows exponential growth.

3. a)

Years since Retirement	Rent ($)	Ratios
0	9600	
1	9960	1.037…
2	10 344	1.038…
3	10 752	1.039…

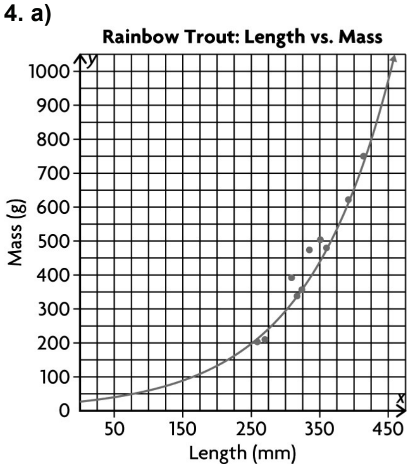
Yes, an exponential model can represent the data since the ratio of consecutive pairs of y-values are close.

b) The equation will be of the form $f(x) = a(b)^x$. Thus, We can create the formula by setting a to the first rent value and b to the average ratio of consecutive pairs of y-values. x will be the number of years since retirement. $f(x)$ will be the rent after x years.
c) Using graphing technology, the regression equation is $y = 9595.433\ldots(1.038\ldots)^x$.

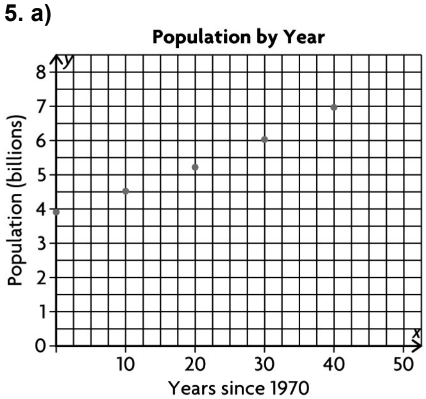
$$f(x) = 9595.433...(1.038...)^x$$
$$f(x) = 9595.433...(1.038...)^{10}$$
$$f(x) = 9595.433...(1.459...)$$
$$f(x) = 14\ 000.638...$$

I predict the Belands will pay about $14 000 in 10 years.

4. a)

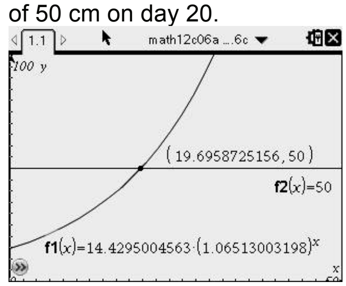


b) Using a graphing calculator, the exponential regression function for the data is $y = 26.934\ldots(1.008\ldots)^x$.
c) e.g., 690 g; I identified the point on the curve that had an x-value of 400.
d) I would expect this fish to be about 446 mm long. I identified the point on the curve that had a y-value of 1000.
5. a)



b) Using a graphing calculator, the exponential regression function for the data is $y = 3.911\ldots(1.014\ldots)^x$.
c) The population is estimated to be 8.05 billion. To get this number, 50 (for 2020) was substituted for x in the regression equation so that y could be determined.
d) The population is expected to reach 9.50 billion 61.5 years after 1970, that is, during the year 2031. I plotted $y = 9.50$ and the regression function on a graphing calculator. The x-coordinate of the point at which these two functions intersect is the point at which the population is expected to reach 9.50 billion.

6. a) Using a graphing calculator, the exponential regression equation for the data is $y = 14.429\ldots(1.065\ldots)^x$.
b) $y = 14.429\ldots(1.065\ldots)^{10}$
$y = 14.429\ldots(1.879\ldots)$
$y = 27.119\ldots$ cm
The height of the sunflower on day 10 was about 27.1 cm.
c) $y = 14.429\ldots(1.065\ldots)^{30}$
$y = 14.429\ldots(6.638\ldots)$
$y = 95.792\ldots$ cm
The height of the sunflower on day 30 was about 95.8 cm. The answer does not make sense since the height was 98.0 cm on day 28.
d) I would expect the sunflower to reach a height of 50 cm on day 20.

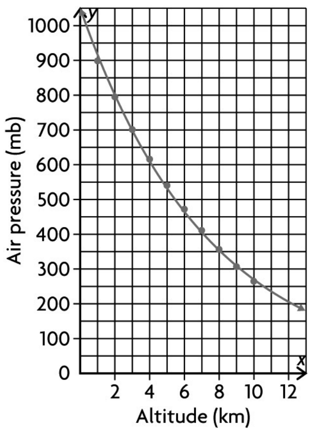


7. a) Using graphing technology, I input the x and y values into the spreadsheet. I then used the exponential regression function to determine the equation that models this growth. The equation is $y = 2^x$.
b) Under this context, the domain is $\{x \mid x \in \mathrm{N}\}$, and the range is $\{y \mid y > 0, y \in \mathrm{N}\}$.
c) If we extend the first and last column of the table, we get the following:

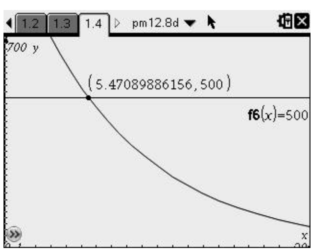
Number of Digits	Number of Possible Codes
1	2
2	4
3	8
4	16
5	32
6	64
7	128

Based on the table, 104 codes lies in between the codes for 6 digits and the codes for 7 digits. 6 digits would not provide enough codes. Therefore, 7 digits would be needed to have 104 codes.

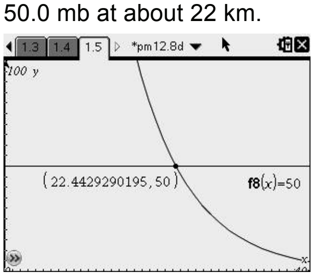
8. a) Here is the graph.



b) Using a graphing calculator, the exponential regression function for the data is
$y = 1050.311\ldots(0.873\ldots)^x$.
c) $y = 1050.311\ldots(0.873\ldots)^{15}$
$y = 1050.311\ldots(0.130\ldots)$
$y = 137.249\ldots$ mb
The pressure at 15 km is about 137.3 mb.
d) Using graphing technology, the pressure reaches 500.0 mb at about 5 km.



Using graphing technology, the pressure reaches 50.0 mb at about 22 km.



9. a)

Time (years)	Frog Population	Decrease in Population (%)
0	575	
1	460	20%
2	368	20%
3	294	20.1%
4	235	20.1%
5	188	20%

Average Decrease in Population = 20.04%
Therefore, the frog population is decreasing by 20% every year.
b) Using a graphing calculator, the exponential regression function for the data is
$y = 575.228\ldots(0.799\ldots)^x$.

c) e.g., Half the population is $\frac{575}{2}$ or about 288 frogs. One possible estimate could be that the population is halved in about 3 years, since the size of the population at 3 years is 294, almost equal to half the initial population. Using the regression equation, we determine that the time to halve the population is about 3.1 years.

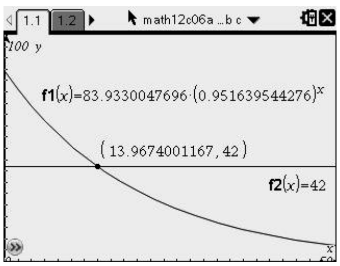
10. a)

Year	Atmospheric CO_2 (ppm)	First Differences
1960	319	
1965	322	3
1970	328	6
1975	333	5
1980	341	8
1985	348	7
1990	356	8
1995	364	8
2000	372	8
2005	381	9

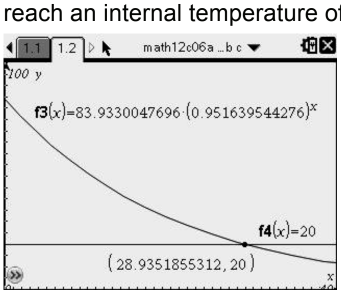
e.g., No, the rate of change of atmospheric CO_2 is increasing.
b) x: The number of years since 1960.
y: The atmospheric CO_2; $a = 319$
Solve for b using the a-value and one of the points in the table. Using (45, 381),
$381 = 319b^{45}$
$1.194\ldots = b^{2005}$
$b = 1.003\ldots$
Therefore, the exponential function is
$y = 319(1.003\ldots)^x$.
c) Using a graphing calculator, the exponential regression function for the data is
$y = 315.609\ldots(1.004\ldots)^x$.

d) Predicted Proportion in 2010
y = 315.609…(1.004…)^{50}
y = 315.609…(1.225…)
y = 386.609… cm
The predicted proportion in 2010 is 387 ppm.
Predicted Proportion in 2020
y = 315.609…(1.004…)^{60}
y = 315.609…(1.275…)
y = 402.687… cm
The predicted proportion in 2020 is 403 ppm.

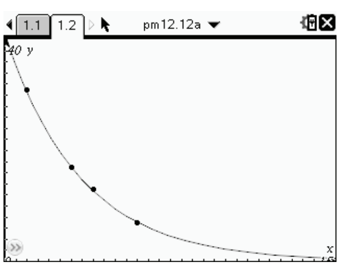
11. a) Using a graphing calculator, the exponential regression function for the data is y = 83.933…(0.951…)^x.
b) It will take the sandwich 14 minutes to have an internal temperature of half its initial temperature (42 °C).



c) The sandwich will need to sit for 29 minutes to reach an internal temperature of 20 °C.

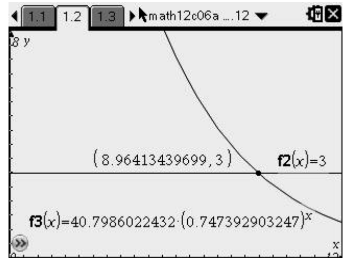


12. a) Using graphing technology to determine the regression function, a graphical exponential model is shown below.

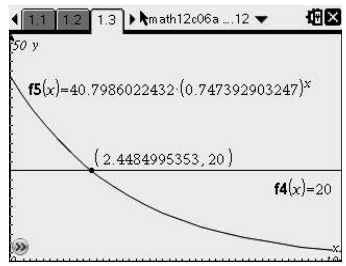


A function that models this situation algebraically is y = 40.799…(0.747…)^x.

b) Tristan will have less than 3 mg of caffeine in his body after 9 hours.



c) Half the initial amount of caffeine in Tristan's body is 20 mg. Half the initial amount of caffeine in Tristan's body remains after about 2 h.



13. a) Using graphing technology, the equation is y = 4120.075…(1.499…)^x.
b) y = 4120.075…(1.499…)^{10}
y = 4120.075…(57.659…)
y = 237 559.646…
There will be 237 560 bacteria.
c) Assuming that the observations are made every hour, the function that models the growth of these bacteria is y = 4120.075…(1.2)^x. The base has changed to 1.2 because the bacteria are growing at a rate of 20% per hour. Since the population is growing, the value must be greater than 1.

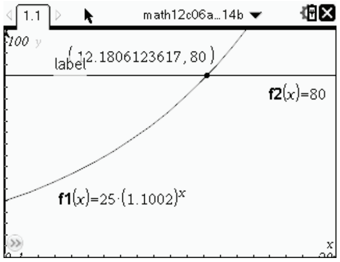
14. a)

Week	Pond Surface Covered by Algae (%)	Growth (%)
0	25.0	
1	27.5	10.0
2	30.3	10.2
3	33.3	9.9
4	36.6	9.9
5	40.3	10.1

$$\frac{(10.0+10.2+9.9+9.9+10.1)}{5}=10.02\%$$

The algae are growing at a rate of about 10% every week.

b) Use the equation $y = 25(1.100\ldots)^x$ and a graphing calculator to solve this.



It will take 12.2 weeks for the algae to cover 80% of the pond.

15. a) i) $P(26) = 200\left(\frac{1}{2}\right)^{\frac{26}{48}}$

$P(26) = 200\left(\frac{1}{2}\right)^{0.541\ldots}$

$P(26) = 200(0.686\ldots)$

$P(26) = 137.395\ldots$ barrels/week

The expected production of this well, 26 weeks after production begins to drop is 137 barrels/week.

ii) $P(26) = 200\left(\frac{1}{2}\right)^{\frac{100}{48}}$

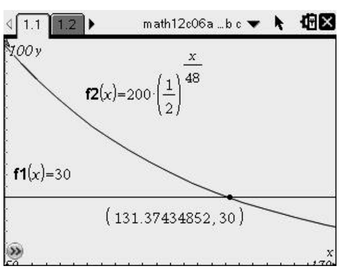
$P(26) = 200\left(\frac{1}{2}\right)^{2.083\ldots}$

$P(26) = 200(0.235\ldots)$

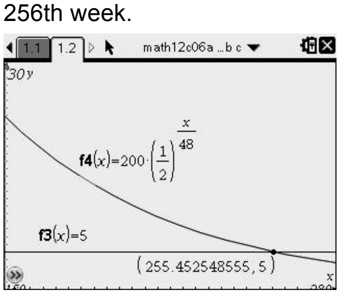
$P(26) = 47.193\ldots$ barrels/week

The expected production of this well, 100 weeks after production begins to drop is 47 barrels/week.

b) Production will reach this level during the 132nd week.



c) The company should cap the well during the 256th week.



16. a) $y = 3000(0.9)^x$, where x is the number of weeks that the well has operated for and y is the weekly amount of oil produced, in barrels.

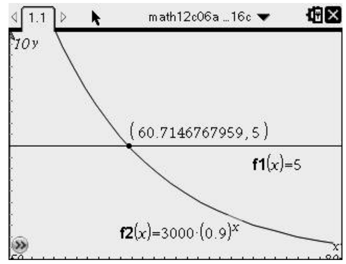
b) $y = 3000(0.9)^{20}$

$y = 3000(0.121\ldots)$

$y = 364.729\ldots$

At the end of 20 weeks of operation, 365 barrels of oil will be extracted from the well.

c) The well will be closed during the 61st week.



17. a) e.g., Calculate the rate of change in the decrease (b) and use 100 for a. Use software or a calculator to perform exponential regression to get those values.

b) Using the method from part a) to create an algebraic model, we get our equation as so:

$97 = 100b^1$

$b = 0.97$

$y = 100(0.97)^x$

Now substitute 7 for x:

$y = 100(0.97)^7$

$y = 100(0.807\ldots)$

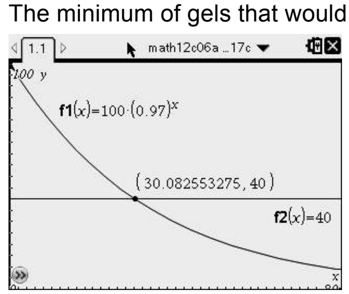
$y = 80.798\ldots\%$

The light intensity would be 80.80%.

c) 100 – 60 = 40

The light intensity has to go below 40%.

The minimum of gels that would be needed is 31.



18. a) Perimeter: 4(4) = 16; Inner Rows: 4(3) = 12

Inner Columns: 4(3) = 12; 16 + 12 + 12 = 40

40 toothpicks will be needed to create the next figure in the pattern.

b)

Figure	Number of Toothpicks	1st Diff.	2nd Diff.
1	4		
2	12	8	
3	24	12	4
4	40	16	4

The second differences are constant, so the data is best modelled by a quadratic function.

c) Continuing the table:

Figure	Number of Toothpicks	1st Diff.	2nd Diff.
1	4		
2	12	8	
3	24	12	4
4	40	16	4
5	60	20	4
6	84	24	4
7	112	28	4

112 toothpicks will be needed to make the 7th figure.

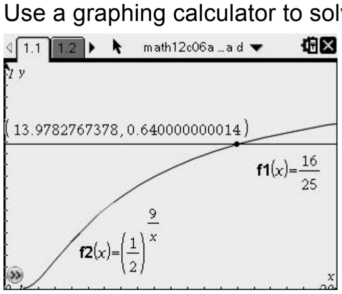
19. a) Using point (9, 64.0), calculate *h* as shown:

$$64.0 = 100.0\left(\frac{1}{2}\right)^{\frac{9}{h}}$$

$$\frac{64}{100} = \left(\frac{1}{2}\right)^{\frac{9}{h}}$$

$$\frac{16}{25} = \left(\frac{1}{2}\right)^{\frac{9}{h}}$$

Use a graphing calculator to solve this equation.



The value of *h* is 13.978… or 14 days.

b) $A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{13.978...}}$

c) $a = 100$

Use the same point used to calculate *h* in a) to calculate *b* for this part. Using (9, 64.0), *b* is calculated as shown:

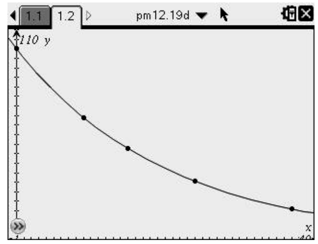
$$64.0 = 100b^9$$

$$\frac{16}{25} = b^9$$

$$b = 0.951...$$

The exponential regression equation is $y = 100(0.951\ldots)^x$.

d) Using graphing technology, the exponential regression equation is $y = 100.001\ldots(0.951\ldots)^x$. It is a good model for the data because the function lies very close to the scatter plot.



The equation determined by hand and the one determined using graphing technology are very close because the difference in numbers start showing up in the thousandth decimal place.

History Connection, page 468

A. For example, uranium, thorium, radon, and radium.

B. The radioactive decay of an element can be modeled by a half-life equation, which is an exponential decay function.

The half-lives of the four elements listed in prompt A is given below.

- U-235: 7.1×10^8 years
- U-238: 4.51×10^9 years
- Rn-220: 55 s
- Rn-222: 3.823 days
- Ra-226: 1600 years
- Th-227: 18.5 days
- Th-230: 8.0×10^4 years
- Th-231: 25.5 h
- Th-234: 24.1 days

To determine the half-life equations for the elements shown above, I substituted each half-life into the half-life equation. $A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{h}}$,

where $A(x)$ is the amount of the element at time *x* and A_0 is the original amount. The half-life equations for the elements are (where *x* represents time in years, except as noted):

U-235:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{7.1\times10^8}}$$

U-238:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{4.51\times10^9}}$$

Rn-220:

$$A(x) = A_0\left(\frac{1}{2}\right)^{\frac{x}{55}}$$

where *x* represents time in seconds