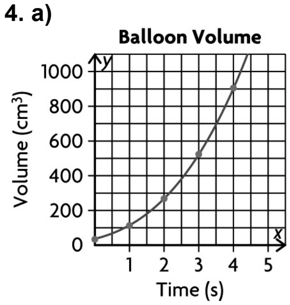
14. a) Using technology, the regression equation is $y = 2.445\ldots x - 23.118\ldots$.
b) $y = 2.445\ldots x - 23.118\ldots$
$y = 2.445\ldots(5) - 23.118\ldots$
$y = -10.893\ldots$
e.g., This answer doesn't make sense because area cannot be negative. Extrapolating doesn't always make sense because the trend does not always continue outside the domain of the data.
c) e.g., Based on the graph, the data does not appear to be linear.
d) e.g., Quadratic regression may be better because the data fits a quadratic function.

Lesson 6.4: Modelling Data with a Curve of Best Fit, page 419

1. a) e.g., The number of births slowly increased until about 1960, where it then started to decrease slowly for a few years before beginning to decline rapidly.
b) Based on the graph, the greatest number of births occurred about 1960.
c) About 425 000 births occurred in 1965.
d) More than 400 000 births per year occurred from 1953 until 1965.
2. a) If the athlete's power output is 310 W, his heart rate will be 149 beats/min.
b) If the athlete's heart rate is 130 beats/min, his power output will be 244 W.
3. a) The ball rises, peaks at about 3 s, and then falls.
b) The regression equation is
$h(t) = -10.071\ldots t^2 + 51.408\ldots t + 11$
c) i) $h(0) = -10.071\ldots(0)^2 + 51.408\ldots(0) + 11$
$h(0) = 11$
The height of the ball at 0 s is 11 m.
ii) $h(2.5) = -10.071\ldots(2.5)^2 + 51.408\ldots(2.5) + 11$
$h(2.5) = 76.575\ldots$
The height of the ball at 2.5 s is 76.6 m.
iii) $h(4.5) = -10.071\ldots(4.5)^2 + 51.408\ldots(4.5) + 11$
$h(4.5) = 38.392\ldots$
The height of the ball at 4.5 s is 38.4 m.
d) The ball hit the ground when its height was zero. Determine the solutions to the equation
$0 = -10.071\ldots t^2 + 51.408\ldots t + 11$
Graph each side of the equation as a separate function. The t-coordinates of the intersection points are solutions to the equation.
$t = 5.310\ldots$ or $t = -0.205\ldots$
Since time cannot be negative, the ball hit the ground at about 5.3 s.

4. a)



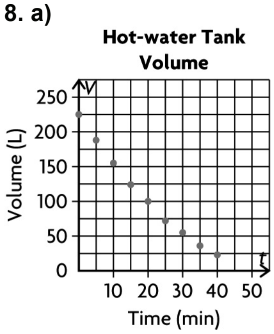
e.g., As the time increases, the volume is increasing at an increasing rate, but it does not appear to be quadratic.
b) The cubic regression equation is
$y = 4.189\ldots x^3 + 25.130\ldots x^2 + 50.267\ldots x + 33.510\ldots$
c) $y = 4.189\ldots x^3 + 25.130\ldots x^2 + 50.267\ldots x + 33.510\ldots$
$y = 4.189\ldots(10.5)^3 + 25.130\ldots(10.5)^2 + 50.267\ldots(10.5) + 33.510\ldots$
$y = 8181.469\ldots$
At 10.5 s, the volume of the balloon will be about 8181.5 cm^3.

5. a) The cubic regression equation is
$y = -0.00006\ldots x^3 + 0.008\ldots x^2 - 0.064\ldots x + 999.869\ldots$.
Using technology, the water will have the minimum volume at 3.9777°C or about 4.0°C.
b)
$y = -0.000\,06\ldots x^3 + 0.008\ldots x^2 - 0.064\ldots x + 999.869\ldots$
$y = -0.000\,06\ldots(40)^3 + 0.008\ldots(40)^2 - 0.064\ldots(40) + 999.869\ldots$
$y = 1006.547\ldots$
At 40 °C, the water will have a volume of 1006.55 cm^3.

6. The regression equation is
$y = -0.746\ldots x^2 + 2.303\ldots x - 0.364\ldots$.
Using technology, the maximum height of the dolphin was about 1.41 m.

7. a) e.g., The quadratic regression equation is
$y = 0.007\ldots x^2 - 0.065\ldots x + 3.521\ldots$.
Interpolations:
When $x = 1$, $y = 3.5$
When $x = 2$, $y = 3.4$
When $x = 7$, $y = 3.4$
When $x = 8$, $y = 3.5$
When $x = 12$, $y = 3.8$
When $x = 13$, $y = 3.9$
When $x = 18$, $y = 4.8$
When $x = 19$, $y = 5.0$
When $x = 20$, $y = 5.2$
When $x = 24$, $y = 6.3$
When $x = 25$, $y = 6.6$
b) e.g., The values found for when x is equal to 12, 13, and 24 are exactly correct, while the estimate for when x is equal to 2 is very close. The interpolated values found when x is equal to 1, 2, 18,19, and 20 are too small, while the values found when x is equal to 7, 8, and 25 are too large.

8. a)



e.g., As the time increases, the volume of the tank decreases.

b) The quadratic regression equation is
$V = 0.064\ldots t^2 - 7.650\ldots t + 224.885\ldots$

c) The tank is half full when its volume is $\frac{225}{2}$ or 112.5 L. Use technology to determine where $V = 112.5$ intersects with the quadratic regression function. The t-coordinates of the intersection points are solutions to the equation
$0.064\ldots t^2 - 7.650\ldots t + 224.885\ldots = 112.5$

The t-coordinates of the intersection points are 17.2 min and 100.6 min.
$t = 100.6$ is not a possible value in this scenario since the tank is draining and the volume is already lower than half at 20 min.
After 17.2 min, the tank will be half full.

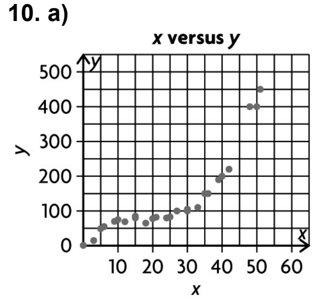
d) The tank is empty when its volume is 0 L. Use technology to determine where $V = 0$ intersects with the quadratic regression function. The t-coordinates of the intersection points are solutions to the equation
$0.064\ldots t^2 - 7.650\ldots t + 224.85\ldots = 0$

The t-coordinates of the intersection points are 56.2 and 61.6.
Since the tank is draining, once it is the empty the first time, it will always be empty. Therefore, the tank was empty at about 56.2 min.

9. a) The cubic regression equation is
$y = 0.004\ldots x^3 - 0.294\ldots x^2 + 4.548\ldots x + 74.659\ldots$
In 2010:
$y = 0.004\ldots(34)^3 - 0.294\ldots(34)^2 + 4.548\ldots(34) + 74.659\ldots$
$y = 65.342\ldots$
In 2010, the cancer incidence rate will be 65.3.

b) Canada should never expect to see the incidence of lung cancer drop below 65 per 100 000 males. Using technology, the minimum of this cubic regression equation for all years after 1976 is 65.3 and since this is the minimum, it is impossible to get a value of 65. Therefore the incidence of lung cancer will never drop below 65 per 100 000 males.

10. a)



b) The cubic regression equation is
$y = 0.010\ldots x^3 - 0.557\ldots x^2 + 11.078\ldots x + 1.409\ldots$

c) When $x = 25$:
$y = 0.010\ldots(25)^3 - 0.557\ldots(25)^2 + 11.078\ldots(25) + 1.409\ldots$
$y = 87.688\ldots$
Using the cubic regression equation, when $x = 25$, $y = 87.7$. This value is very close to the y-value in the chart. It is off by approximately 4.7.

11. a) e.g., From 0 s to 9 s, the data can be modelled with a linear regression. The linear regression equation for this portion of the data is
$y = 115.830\ldots x + 6999.963\ldots.$
From 10 s to 22 s, the data can be with a quadratic regression. The quadratic regression equation for this portion of the data is
$y = -9.783\ldots x^2 + 311.403\ldots x + 6023.164\ldots.$
From 23 s to 30 s, the data can be modelled with a linear regression. The linear regression equation for this portion of the data is
$y = -115.916\ldots x + 10\,688.916\ldots.$

b) e.g., The length of time that the feeling of weightlessness occurred was 13 s. This is because the section of the data from 10 s to 22 s is the only section that is truly parabolic, which is what is required for weightlessness.

History Connection, page 423

A.

x	$f(x)$
0	41
1	43
2	47
3	53
4	61
5	71
6	83
7	97
8	113
9	131
10	151
11	173
12	197
13	223
14	251
15	281
16	313
17	347
18	383
19	421
20	461
21	503
22	547
23	593
24	641
25	691
26	743
27	797
28	853
29	911
30	971
31	1033
32	1097
33	1163
34	1231
35	1301
36	1373
37	1447
38	1523
39	1601

B. $f(x) = x^2 + x + 41$
$f(40) = 40^2 + 40 + 41$
$f(40) = 1681$
I can tell that $f(40)$ is not prime because I can factor the expression.
$f(40) = 40^2 + 40 + 41$
$f(40) = 40(40 + 1) + 41$
$f(40) = 40(41) + 41$
$f(40) = 41(40 + 1)$
$f(40) = (41)(41)$

C. The quadratic function has a positive leading coefficient and a constant term of 41. This means that the graph is a parabola that opens up and has a y-intercept of 41. These characteristics are important because 41 is prime. If 41 were not prime, then the very first number generated by the function would not be prime. Opening up is also important. If the parabola opened down, then the function would reach zero very quickly and it would not likely generate many prime numbers.

D. $f(x) = x^2 + x + 5$ generates 4 consecutive prime numbers.
$f(x) = x^2 + x + 11$ generates 9 consecutive prime numbers.
$f(x) = x^2 - x + 11$ generates 10 consecutive prime numbers.

Chapter Self-Test, page 424

1. a) i) e.g., $y = -x + 3$
ii) e.g., $y = x - 6$
b) i) e.g., $y = -2x^3 - 7x^2 + 24x + 45$
ii) e.g., $y = 4x^3 + 12x^2 - 5x - 6$

2. a) Possible number of x-intercepts: 0, 1, or 2
y-intercept: –1
End behaviour: graph extends from quadrant III to quadrant IV.
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \le 3, y \in R\}$
Possible number of turning points: 1
b) Possible number of x-intercepts: 1
y-intercept: 1.5
End behaviour: graph extends from quadrant III to quadrant I.
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0
c) Possible number of x-intercepts: 1, 2, or 3
y-intercept: –42
End behaviour: graph extends from quadrant III to quadrant I.
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0 or 2
d) Possible number of x-intercepts: 1, 2, or 3
y-intercept: 2
End behaviour: graph extends from quadrant II to quadrant IV.
Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \in R\}$
Possible number of turning points: 0 or 2