b) Possible number of x-intercepts: 1, 2, or 3
y-intercept: 6
End behaviour: graph extends from quadrant III to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Possible number of turning points: 0 or 2
c) Possible number of x-intercepts: 0, 1, or 2
y-intercept: –1
End behaviour: graph extends from quadrant II to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \geq \text{minimum}, y \in \mathrm{R}\}$
Possible number of turning points: 1
d) Possible number of x-intercepts: 1, 2, or 3
y-intercept: 0
End behaviour: graph extends from quadrant III to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Possible number of turning points: 0 or 2

6. a) e.g., Since the function must have two turning points, it must be a cubic function. Since the graph must extend from quadrant III to quadrant I, the leading coefficient must be positive. Since the graph must pass through the origin, it must have a y-intercept of 0. This means that the constant term must be 0. Since the origin is the only x-intercept, the turning points must both lie above or below the x-axis. Therefore a function that satisfies these characteristics is $y = x^3 - 4x^2 + 5x$.
b) e.g., Since the function must extend from quadrant II to quadrant I, it must be a quadratic function with a positive leading coefficient. Since the graph must have a positive leading coefficient and two x-intercepts, the vertex of the parabola must lie below the origin. Therefore a function that satisfies these characteristics is $y = x^2 - 1$.
c) e.g., Since the function must have a degree of 1, the function must be linear. Since the function must extend from quadrant II to quadrant IV, the leading coefficient must be negative. Since the function must have a y-intercept of –3, the constant term must be –3. Therefore a function that satisfies these characteristics is $y = -x - 3$.
d) e.g., Since the function must have one turning point, it must be quadratic. Since the function must have only one x-intercept, the vertex of the parabola must lie on the x-axis. Since the function must have a y-intercept of 6, the constant term must be 6. Therefore a function that satisfies these characteristics is $y = 6(x - 1)^2$.
e) e.g., Since the range of the function is restricted, but not restricted to just one value, the function must be quadratic. Since the function must have a range of $y \geq -6$, the y-coordinate of the vertex must be –6 and the leading coefficient must be positive. Since the x-intercepts must be 2 and 6, the equation of the function in factored form is $y = a(x - 2)(x - 6)$, where $a > 0$.
Rearranging the equation into vertex form gives:
$y = a(x - 2)(x - 6)$
$y = a(x^2 - 8x + 12)$
$y = a(x^2 - 8x + 16 - 16) + 12a$
$y = a(x^2 - 8x + 16) - 16a + 12a$
$y = a(x - 4)^2 - 4a$
Since the y-coordinate of the vertex must be –6, we have:
$-6 = -4a$
$a = 1.5$
Therefore a function that satisfies these characteristics is $y = 1.5(x - 4)^2 - 6$.

Lesson 6.3: Modelling Data with a Line of Best Fit, page 407

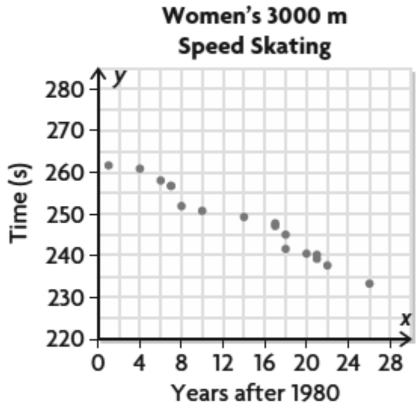
1. a) e.g., Slope: –1
e.g., y-intercept: 7.5
b) e.g., Slope: 0.1
e.g., y-intercept: 2

2. a) e.g., $y = -x + 7.5$
b) e.g., $y = 0.1x + 2$

3. a) The distance travelled in a car is dependent, because it depends on the average speed of the car. The average speed is independent, because it does not depend on the distance travelled by that car.
b) The size of the family is independent, because it does not depend on the number of cell phones. The number of cell phones is dependent, because it depends on the size of the family.
c) The number of people in a cafeteria is dependent, because it depends on the time of day. The time of day is independent, because it does not depend on the number of people in a cafeteria.
d) The number of hours of daylight is dependent, because it depends on the time of year. The time of year is independent, because it does not depend on the number of hours of daylight.

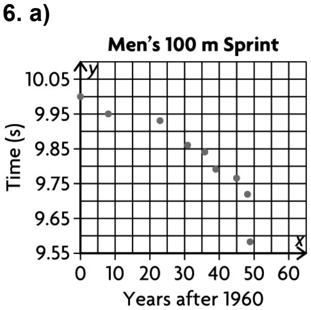
4. a) e.g., The line of best fit has a negative slope. Approximately the same number of points lie above and below the line. There seems to be a strong representation of the data because all the points are very close to the line of best fit.
b) e.g., I estimate that when $x = 47$, $y = 75$. I used interpolation, because the point is within the domain of the data.
c) e.g., I estimate that when $y = 70$, $x = 52$. I used interpolation, because the point is within the domain of the data.
d) e.g., I estimate that when $x = 15$, $y = 105$. I used extrapolation, because the point is outside the domain of the data.

5. a) Let x represent the number of years after 1980, and let y represent the record-breaking time, in seconds.



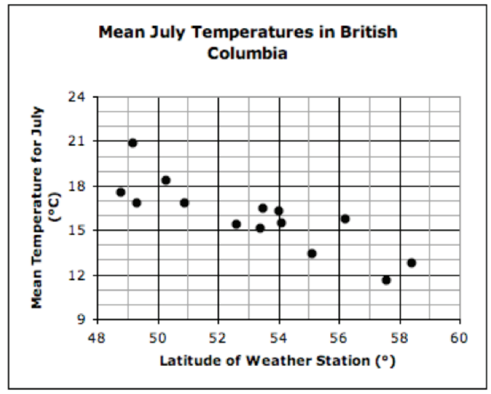
b) As the number of years after 1980 increases, the record time in the women's 3000 m speed skating decreases.
c) The linear regression function is
$y = -1.147\ldots x + 264.178\ldots$. The slope represents the number of seconds that the record time decreases each year and the y-intercept represents the record time in 1980.
d) e.g., It appears that in 2005, the record-breaking time is 235.489… s, or 3:55.49 min.
e) Cindy Klassen's actual time in 2005 was 3:55.75 min, which is only about 0.26 seconds higher than my estimate.

6. a)



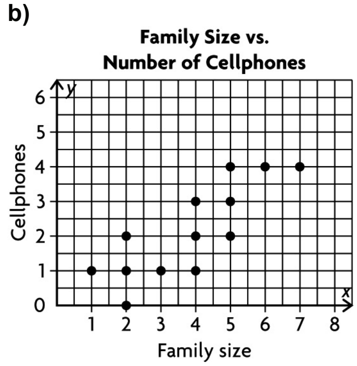
b) Most of the data follows a general pattern, which is that as the years pass, the world-record time will decrease slightly. The main outlier is the record at 49 years, set by Usain Bolt.
c) The linear regression function is
$y = -0.006\ldots x + 10.032\ldots$. The slope represents the amount of time in seconds that the world-record time will decrease every year and the y-intercept represents the record in 1960.
d) e.g., A possible world-record time for 2007 could be 9.72 s.
e) Asafa Powell's time in 2007 was 9.74 s, slightly higher than my estimate.

7. a) e.g., I would expect to see that when the latitude of a weather station increases, the mean temperature will decrease.



b) The linear regression equation is
$y = -0.637\ldots x + 49.730\ldots$.
c) $y = -0.637\ldots x + 49.730\ldots$
$y = -0.637\ldots(52.0) + 49.730\ldots$
$y = 16.600\ldots$
At a latitude of 52.0°N, the mean temperature for July will be 16.6°C.
d) $y = -0.637\ldots x + 49.730\ldots$
$18 = -0.637\ldots x + 49.730\ldots$
$-31.730\ldots = -0.637\ldots x$
$x = 49.803\ldots$
A mean July temperature of 18 °C can be expected when the latitude of the weather station is 49.8° N.

8. a) As the family size increases, the number of cellphones increases.
b)



The linear regression equation is
$y = 0.613\ldots x - 0.259\ldots$.
c) e.g., $y = 0.613\ldots x - 0.259\ldots$
$y = 0.613\ldots(3) - 0.259\ldots$
$y = 1.581\ldots$
According to the regression equation, a household of 3 people will have about 1.58 cell phones. This is not possible, so the number should be rounded up to 2.

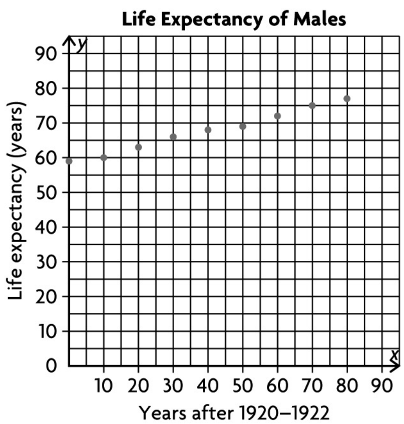
9. The linear regression equation is
$y = 404.891\ldots x - 300.362\ldots.$
Using the equation:

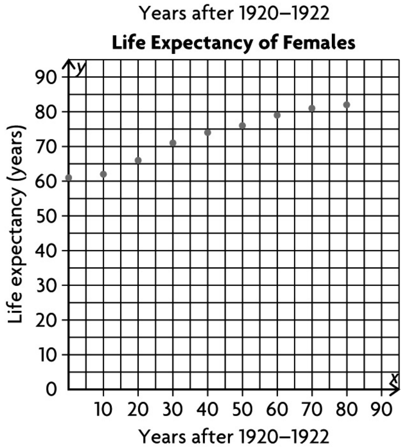
$$y = 404.891\ldots x - 300.362\ldots$$
$$750 = 404.891\ldots x - 300.362\ldots$$
$$1050.362\ldots = 404.891\ldots x$$
$$x = 2.594\ldots$$

According to the equation, it will cost $750 to attend 2.6 events. Since this is not possible, it would be wiser for Devin to plan to attend 2 events.

10. The linear regression equation is
$y = 0.078\ldots x - 48.742\ldots.$
Using the equation:
$y = 0.078\ldots x - 48.742\ldots$
$y = 0.078\ldots(2000) - 48.742\ldots$
$y = 108.646\ldots$
In order to fill the 2000 sq ft store, the retailer should plan on stocking 109 bikes at a time.

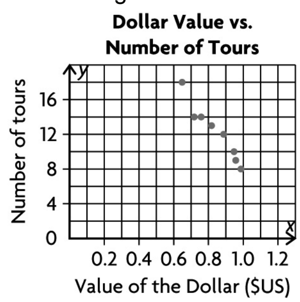
11. a) Let x represent the number of years after 1920–1922.





b) The linear regression equation for the male plot is $y = 0.23x + 58.466\ldots.$ The linear regression equation for the female plot is $y = 0.286\ldots x + 60.977\ldots.$
c) For males:
$y = 0.23x + 58.466\ldots$
$y = 0.23(90) + 58.466\ldots$
$y = 79.166\ldots$
For females:
$y = 0.286\ldots x + 60.977\ldots$
$y = 0.286\ldots(90) + 60.977\ldots$
$y = 86.777\ldots$
In 2010, the life expectancy in Canada will be 79 years for males and 87 years for females.

12. I created a scatter plot using the value of the Canadian dollar as the independent variable, and the number of tours as the dependent variable. I then used a linear regression.



The formula of the linear regression is
$y = -24.879\ldots x + 33.314\ldots.$
e.g., Using this formula:
$y = -24.879\ldots x + 33.314\ldots$
$y = -24.879\ldots(1.00) + 33.314\ldots$
$y = 8.434\ldots$
Extrapolating using the graph:
$y = 8.5$
Since a part of a tour is not really possible, it is likely that this number should be rounded down to 8. This makes sense, because when the value of the dollar was 0.99 US dollars, the number of tours made was 8. Since all the points are close to the line of best fit, there is a strong linear relationship between the dollar value and the number of tours.

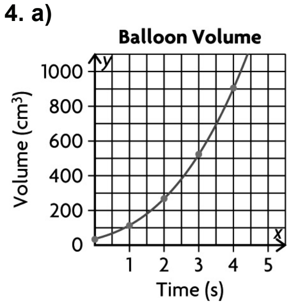
13. a) e.g., The easiest method would be to create a scatter plot using the data and then perform a linear regression. Insert the independent variable into the equation for the linear regression and solve for the dependent variable. For example, if a graph has a linear regression with the equation $y = 5x + 1$, then to estimate y when $x = 6$, simply insert $x = 6$ and you get $y = 31$.
b) Again, it is best to create a scatter plot using the data and then perform a linear regression. Then insert the dependent variable and rearrange the equation to solve for the independent variable.

14. a) Using technology, the regression equation is
$y = 2.445\ldots x - 23.118\ldots$.
b) $y = 2.445\ldots x - 23.118\ldots$
$y = 2.445\ldots(5) - 23.118\ldots$
$y = -10.893\ldots$
e.g., This answer doesn't make sense because area cannot be negative. Extrapolating doesn't always make sense because the trend does not always continue outside the domain of the data.
c) e.g., Based on the graph, the data does not appear to be linear.
d) e.g., Quadratic regression may be better because the data fits a quadratic function.

Lesson 6.4: Modelling Data with a Curve of Best Fit, page 419

1. a) e.g., The number of births slowly increased until about 1960, where it then started to decrease slowly for a few years before beginning to decline rapidly.
b) Based on the graph, the greatest number of births occurred about 1960.
c) About 425 000 births occurred in 1965.
d) More than 400 000 births per year occurred from 1953 until 1965.
2. a) If the athlete's power output is 310 W, his heart rate will be 149 beats/min.
b) If the athlete's heart rate is 130 beats/min, his power output will be 244 W.
3. a) The ball rises, peaks at about 3 s, and then falls.
b) The regression equation is
$h(t) = -10.071\ldots t^2 + 51.408\ldots t + 11$
c) i) $h(0) = -10.071\ldots(0)^2 + 51.408\ldots(0) + 11$
$h(0) = 11$
The height of the ball at 0 s is 11 m.
ii) $h(2.5) = -10.071\ldots(2.5)^2 + 51.408\ldots(2.5) + 11$
$h(2.5) = 76.575\ldots$
The height of the ball at 2.5 s is 76.6 m.
iii) $h(4.5) = -10.071\ldots(4.5)^2 + 51.408\ldots(4.5) + 11$
$h(4.5) = 38.392\ldots$
The height of the ball at 4.5 s is 38.4 m.
d) The ball hit the ground when its height was zero. Determine the solutions to the equation
$0 = -10.071\ldots t^2 + 51.408\ldots t + 11$
Graph each side of the equation as a separate function. The t-coordinates of the intersection points are solutions to the equation.
$t = 5.310\ldots$ or $t = -0.205\ldots$
Since time cannot be negative, the ball hit the ground at about 5.3 s.

4. a)



e.g., As the time increases, the volume is increasing at an increasing rate, but it does not appear to be quadratic.
b) The cubic regression equation is
$y = 4.189\ldots x^3 + 25.130\ldots x^2 + 50.267\ldots x + 33.510\ldots$
c) $y = 4.189\ldots x^3 + 25.130\ldots x^2 + 50.267\ldots x + 33.510\ldots$
$y = 4.189\ldots(10.5)^3 + 25.130\ldots(10.5)^2 + 50.267\ldots(10.5) + 33.510\ldots$
$y = 8181.469\ldots$
At 10.5 s, the volume of the balloon will be about 8181.5 cm^3.

5. a) The cubic regression equation is
$y = -0.00006\ldots x^3 + 0.008\ldots x^2 - 0.064\ldots x + 999.869\ldots$.
Using technology, the water will have the minimum volume at 3.9777°C or about 4.0°C.
b)
$y = -0.000\,06\ldots x^3 + 0.008\ldots x^2 - 0.064\ldots x + 999.869\ldots$
$y = -0.000\,06\ldots(40)^3 + 0.008\ldots(40)^2 - 0.064\ldots(40) + 999.869\ldots$
$y = 1006.547\ldots$
At 40 °C, the water will have a volume of 1006.55 cm^3.

6. The regression equation is
$y = -0.746\ldots x^2 + 2.303\ldots x - 0.364\ldots$.
Using technology, the maximum height of the dolphin was about 1.41 m.

7. a) e.g., The quadratic regression equation is
$y = 0.007\ldots x^2 - 0.065\ldots x + 3.521\ldots$.
Interpolations:
When $x = 1$, $y = 3.5$
When $x = 2$, $y = 3.4$
When $x = 7$, $y = 3.4$
When $x = 8$, $y = 3.5$
When $x = 12$, $y = 3.8$
When $x = 13$, $y = 3.9$
When $x = 18$, $y = 4.8$
When $x = 19$, $y = 5.0$
When $x = 20$, $y = 5.2$
When $x = 24$, $y = 6.3$
When $x = 25$, $y = 6.6$
b) e.g., The values found for when x is equal to 12, 13, and 24 are exactly correct, while the estimate for when x is equal to 2 is very close. The interpolated values found when x is equal to 1, 2, 18,19, and 20 are too small, while the values found when x is equal to 7, 8, and 25 are too large.