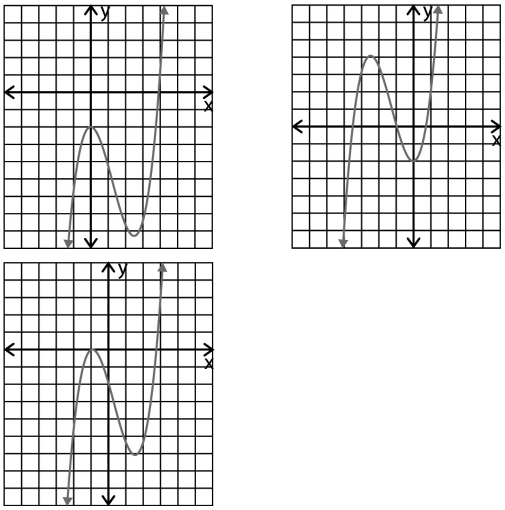
c)



Lesson 6.2: Characteristics of the Equations of Polynomial Functions, page 393

1. a) Degree: 2
Leading coefficient: 6
Constant term: –2
b) Degree: 1
Leading coefficient: $-\frac{2}{3}$
Constant term: 10
c) Degree: 3
Leading coefficient: –1
Constant term: 6
d) Degree: 3
Leading coefficient: 4
Constant term: –10

2. a) i) Minimum: 0
Maximum: 2
ii) End behaviour: graph extends from quadrant II to quadrant I
Domain: $\{x \mid x \in \text{R}\}$
Range: $\{y \mid y \geq \text{minimum}, y \in \text{R}\}$
iii) Minimum: 1
Maximum: 1
b) i) Minimum: 1
Maximum: 1
ii) End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in \text{R}\}$
Range: $\{y \mid y \in \text{R}\}$
iii) Minimum: 0
Maximum: 0
c) i) Minimum: 1
Maximum: 3
ii) End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in \text{R}\}$
Range: $\{y \mid y \in \text{R}\}$
iii) Minimum: 0
Maximum: 2
d) i) Minimum: 1
Maximum: 3
ii) End behaviour: graph extends from quadrant III to quadrant I
Domain: $\{x \mid x \in \text{R}\}$
Range: $\{y \mid y \in \text{R}\}$
iii) Minimum: 0
Maximum: 2

3. a) Degree: 2
Sign of leading coefficient: –
Constant term: 2
b) Degree: 1
Sign of leading coefficient: +
Constant term: 2
c) Degree: 3
Sign of leading coefficient: +
Constant term: –1
d) Degree: 3
Sign of leading coefficient: –
Constant term: 6

4. a) e.g., $y = 5$
b) e.g., $y = x + 5$
c) e.g., $y = x^2 + x + 5$
d) e.g., $y = x^3 + x^2 + x + 5$

5. a) The graph extends from quadrant III to quadrant I.
b) The graph extends from quadrant III to quadrant IV.
c) The graph extends from quadrant II to quadrant I.
d) The graph extends from quadrant II to quadrant IV.
e) The graph extends from quadrant II to quadrant IV.
f) The graph extends from quadrant III to quadrant I.

6. a) The correct function for this graph is v. Only v and vi are possible choices, because they are the only ones that are linear. The graph extends from quadrant II to quadrant IV, and the slope is –1. This means that the leading coefficient is –1. Therefore, v is the proper choice.
b) The correct function for this graph is i. Only i and iv are possible choices, because they are the only ones that are cubic. The graph extends from quadrant II to quadrant IV, which means that the leading coefficient is negative. Therefore, i is the proper choice.
c) The correct function for this graph is ii. Only ii and iii are possible choices, because they are the only ones that are quadratic. The graph has a *y*-intercept of 4, which means that the constant term is 4. Therefore, ii is the proper choice.

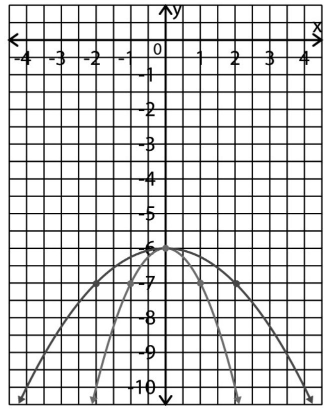
d) The correct function for this graph is vi. Only v and vi are possible choices, because they are the only ones that are linear. The graph extends from quadrant II to quadrant IV, and the slope is –2. This means that the leading coefficient is –2. Therefore, vi is the proper choice.
e) The correct function for this graph is iii. Only ii and iii are possible choices, because they are the only ones that are quadratic. The graph has a *y*-intercept of 2, which means that the constant term is 2. Therefore, iii is the proper choice.
f) The correct function for this graph is iv. Only i and iv are possible choices, because they are the only ones that are cubic. The graph extends from quadrant III to quadrant I, which means that the leading coefficient is positive. Therefore, iv is the proper choice.

7. a) Possible number of *x*-intercepts: 1
y-intercept: 5
End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Possible number of turning points: 0
b) Possible number of *x*-intercepts: 0, 1, or 2
y-intercept: –6
End behaviour: graph extends from quadrant II to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \geq \text{minimum}, y \in \mathrm{R}\}$
Possible number of turning points: 1
c) Possible number of *x*-intercepts: 1, 2, or 3
y-intercept: –1
End behaviour: graph extends from quadrant III to quadrant I
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Possible number of turning points: 0 or 2
d) Possible number of *x*-intercepts: 1, 2, or 3
y-intercept: 0
End behaviour: graph extends from quadrant II to quadrant IV
Domain: $\{x \mid x \in \mathrm{R}\}$
Range: $\{y \mid y \in \mathrm{R}\}$
Possible number of turning points: 0 or 2

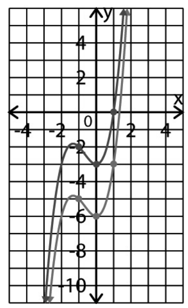
8. a) e.g., $y = -x^2 + 2$
b) e.g., $y = x^3 - 3x^2 - x + 3$
c) e.g., $y = x - 3$
d) e.g., $y = x^3 - 5x^2 - x + 5$
e) e.g., $y = -x^2 + 2$

9. e.g., The equation $f(x) = -3x^3 + 5x^2 + 11x$ is equivalent to $f(x) = -3x^3 + 5x^2 + 11x + 0$. This function does have a *y*-intercept, which is equal to 0.

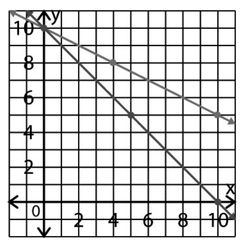
10. a) e.g.,



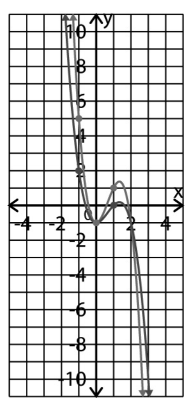
b) e.g.,



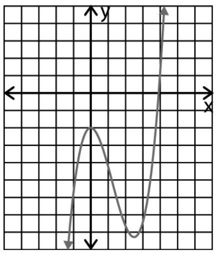
c) e.g.,



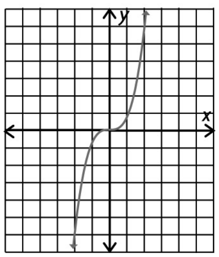
d) e.g.,



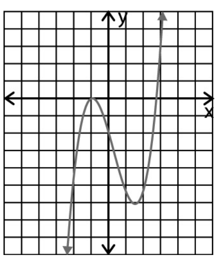
11. e.g., A cubic function could have one x-intercept if either both of the turning points are above the x-axis, or both of the turning points are below the x-axis, as shown here.



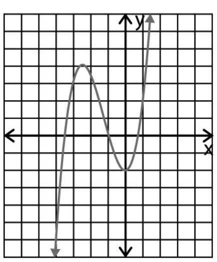
A cubic function could also have only one x-intercept if that function has no turning points.



A cubic function could have two x-intercepts if one of the turning points is on the x-axis.



A cubic function will have three x-intercepts if one of the turning points is above the x-axis, and the other one is below the x-axis.



12. e.g., Quadratic polynomial functions have equal end behaviour. This means that as x gets very large and very small, y either gets very large at both ends, or very small at both ends. Because of this, quadratic functions will always have either a maximum or minimum. Cubic polynomial functions have opposite end behaviour, which means that the range of the function is the set of real numbers. Therefore, a cubic function can never have a maximum or minimum, and will only have turning points.

13. a) This graph has a y-intercept at 0. Therefore, only i and iv are possibilities. The graph extends from quadrant II to quadrant IV, which means that the leading coefficient must be negative. Therefore, i must represent this function.
b) This graph has a y-intercept at 3. Therefore, only ii and iii are possibilities. The graph extends from quadrant II to quadrant IV, which means that the leading coefficient must be negative. Therefore, iii must represent this function.
c) This graph has a y-intercept at 0. Therefore, only i and iv are possibilities. The graph extends from quadrant III to quadrant I, which means that the leading coefficient must be positive. Therefore, iv must represent this function.
d) This graph has a y-intercept at 3. Therefore, only ii and iii are possibilities. The graph extends from quadrant III to quadrant I, which means that the leading coefficient must be positive. Therefore, ii must represent this function.

14. a) The degree of this function is 3, so it is a cubic function. The leading coefficient is positive so the function is increasing from left to right. The curve extends from quadrant III to quadrant I. It has a y-intercept of 25.720 and may have 1, 2, or 3 x-intercepts. Also, it may have 0 or 2 turning points.
b) In this case, the constant term equals the retail price of gas in 1979.

15. a) When $t = 10$,
$f(10) = 0.001(10)^3 - 0.055(10)^2 + 0.845(10) + 0.293$
$f(10) = 4.243$ m
Actual tide depth = 4.2 m
The estimate using the formula is only 0.043 m off from the actual tide depth at that time.
b) The curve extends from quadrant III to quadrant I.
c) No. e.g., This function cannot provide accurate information outside the given time frame because the tide depth cannot be negative or infinitely large, which is what the function does outside the given time frame. Also, the data in the table shows that the tides don't keep the same schedule every day; for example, the maximum tide depth is at 8 a.m. on Jan. 6 and at 9 a.m. on Jan. 10.

16. e.g., I would ask for the degree of the function, its leading coefficient, and the x-intercepts. Knowing the degree of the function is essential to describing it, and knowing the leading coefficient as well will allow you to know the end behaviour of the function. With the x-intercepts, as well as the other 2 aspects of the function, you can create the formula, which will tell you the y-intercept as well. You may also be able to deduce how many turning points a function has from the number of x-intercepts.

17. a) i) The turning points occur when

$x = \dfrac{6+2\sqrt{3}}{3}$, or $x = \dfrac{6-2\sqrt{3}}{3}$

$x = 3.154\ldots$, or $x = 0.845\ldots$

So the turning points are (3.15, –3.08) and (0.85, 3.08).

ii) The turning points occur when

$x = \dfrac{7+\sqrt{37}}{3}$, or $x = \dfrac{7-\sqrt{37}}{3}$

$x = 4.360\ldots$, or $x = 0.305\ldots$

So the turning points are (4.36, –20.75) and (0.31, 12.60).

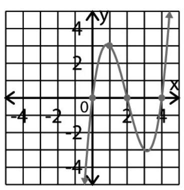
iii) The turning points occur when

$x = -3+\sqrt{3}$, or $x = -3-\sqrt{3}$

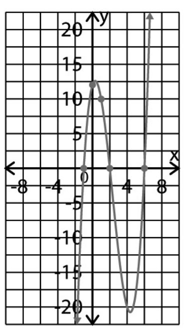
$x = -1.267\ldots$, or $x = -4.732\ldots$

So the turning points are (–1.27, 10.39) and (–4.73, –10.39).

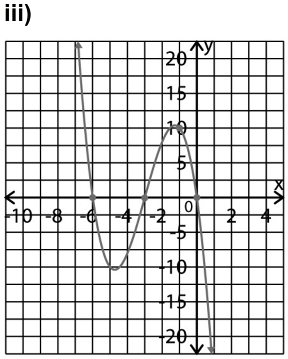
b) i)



ii)



iii)



c) e.g., All of these equations have 3 *x*-intercepts and 2 turning points, which can both be found between 2 adjacent *x*-intercepts. You could find points that are close to the middle of two *x*-intercepts, using the formula, and see where the turning point could be.

18. a) e.g., Because we are dealing with probability, *x* and *P*(*x*) must both be between 0 and 1.

When $x = 0$, $P(x) = 0$
When $x = 0.2$, $P(x) = 0.096$
When $x = 0.4$, $P(x) = 0.288$
When $x = 0.6$, $P(x) = 0.432$
When $x = 0.8$, $P(x) = 0.384$
When $x = 1$, $P(x) = 0$

The function seems reasonable. As the probability of sinking one free throw increases, you would expect the probability of sinking two out of three free throws to increase until you reach a point where you are more likely to make all three free throws instead of two out of the three.

b) Domain: $\{x \mid 0 \le x \le 1, x \in \mathrm{R}\}$

Range: $\{y \mid 0 \le y \le \frac{4}{9}, y \in \mathrm{R}\}$

c) The coordinates of the turning point are $\left(\frac{2}{3}, \frac{4}{9}\right)$. e.g., The probability of sinking two out of three free throws is highest when the probability of sinking one free throw is $\frac{2}{3}$, and as the probability of making one free throw increases beyond this point, the probability of making two out of three free throws begins to decrease.

d) The *x*-intercepts are 0 and 1. e.g., The probability of sinking two out of three free throws is zero if the probability of sinking one throw is 0. The probability of sinking exactly two out of three free throws is 0 if the probability of sinking one throw is 1 (that is, if the probability of sinking a free throw is 1, you are certain to make all shots, so you would sink 3 out of 3).

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- We could release a drop of water from a window and determine how long it takes to reach the ground. We could substitute the time we measured and the height, 0, into the equation and determine h_0. The vertex is on the *y*-axis at h_0, the height of the window.
- This polynomial function of degree 2 would be a parabola opening down, because *a* is negative. The vertex would be on the *y*-axis at h_0.
- Time for drop of water to hit the ground on three separate trials: 2.0 s, 2.2 s, 2.1 s

  Mean time: 2.1 s

  Substitute $t = 2.1$ and $h(t) = 0$ into the equation:

  $h(t) = -4.9t^2 + h_0$

  $0 = -4.9(2.1)^2 + h_0$

  $21.609 = h_0$

  The window is about 22 m high.