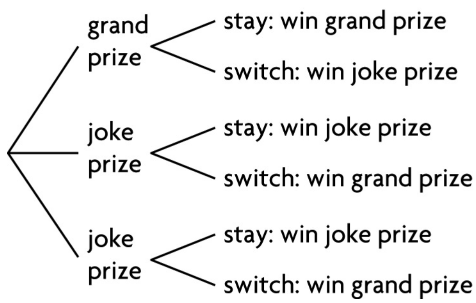


After you select, Monty opens a door with a joke prize. If you originally selected the grand prize, and you switch, you will lose. But if you originally selected one of the two joke prizes, and you switch, you will win. So, if you switch, you will win $\frac{2}{3}$ of the time. The better strategy is to switch.

Lesson 5.5: Conditional Probability, page 350

1. a) These two events are dependent.
b) Let *R* represent the red die showing 4, let *S* represent rolling a sum that is greater than 7.

$$P(R) = \frac{1}{6}$$
$$P(S|R) = \frac{1}{2}$$
$$P(S \cap R) = P(R) \cdot P(S|R)$$
$$P(S \cap R) = \frac{1}{6} \cdot \frac{1}{2}$$
$$P(S \cap R) = \frac{1}{12}$$

The probability that Austin will win a point is $\frac{1}{12}$, or about 0.0833 or 8.33%.

2. a) These two events are dependent.
b) Let *A* represent the first card being a diamond, and let *B* represent the second card being a diamond.

$$P(A) = \frac{13}{52} \qquad P(A \cap B) = P(A) \cdot P(B|A)$$
$$P(A) = \frac{1}{4} \qquad P(A \cap B) = \frac{1}{4} \cdot \frac{4}{17}$$
$$P(B|A) = \frac{12}{51} \qquad P(A \cap B) = \frac{1}{17}$$
$$P(B|A) = \frac{4}{17}$$

The probability that both cards are diamonds is $\frac{1}{17}$, or about 0.0588 or 5.88%.

3. a) These two events are independent.
b) Let *D* represent drawing a diamond.

$$P(D) = \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{4} \cdot \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{16}$$

The probability that both cards are diamonds is $\frac{1}{16}$, 0.0625 or 6.25%.

4. a) i) Let *B* represent Lexie pulling a black sock from her drawer.

$$P(B) = \frac{6}{14} \qquad P(B \cap B) = P(B) \cdot P(B \mid B)$$
$$P(B) = \frac{3}{7} \qquad P(B \cap B) = \frac{3}{7} \cdot \frac{5}{13}$$
$$P(B \mid B) = \frac{5}{13} \qquad P(B \cap B) = \frac{15}{91}$$

The probability of drawing two black socks is $\frac{15}{91}$, or about 0.165 or 16.5%.

ii) Let *W* represent Lexie pulling a white sock from her drawer.

$$P(W) = \frac{8}{14} \qquad P(W \cap W) = P(W) \cdot P(W|W)$$
$$P(W) = \frac{4}{7} \qquad P(W \cap W) = \frac{4}{7} \cdot \frac{7}{13}$$
$$P(W|W) = \frac{7}{13} \qquad P(W \cap W) = \frac{4}{13}$$

The probability of drawing two white socks is $\frac{4}{13}$, or about 0.308 or 30.8%.

iii) Let *B* represent pulling a black sock, and let *W* represent pulling a white sock. Let *A* represent drawing a pair of socks.

$$P(A) = P(B \cap B) + P(W \cap W)$$
$$P(A) = \frac{15}{91} + \frac{4}{13}$$
$$P(A) = \frac{43}{91}$$

The probability of drawing a pair of socks is $\frac{43}{91}$, or about 0.473 or 47.3%.

b) No, the answers would not change, because there is still no replacement.

5. a) Let *A* represent a student who plans to attend UBC, and let *O* represent all graduating students

$$P(A)=\frac{n(A)}{n(O)}$$
$$P(A)=\frac{30+50}{80+110}$$
$$P(A)=\frac{80}{190}$$
$$P(A)=\frac{8}{19}$$

The probability that a graduating student will attend UBC is $\frac{8}{19}$, or about 0.421 or 42.1%.

b) Let *A* represent a student who plans to attend UBC, and let *F* represent a female student.

$$P(F|A)=\frac{n(F)}{n(A)}$$
$$P(F|A)=\frac{50}{80}$$
$$P(F|A)=\frac{5}{8}$$

The probability this student is female is $\frac{5}{8}$, 0.625 or 62.5%.

6. Let *T* represent a true or false question, and let *M* represent a multiple-choice question. Let *C* represent a correct question.

$$P(T\cap C)=P(T)\cdot P(C|T)$$
$$P(T\cap C)=0.30\cdot 0.60$$
$$P(T\cap C)=0.18$$
$$P(M\cap C)=P(M)\cdot P(C|M)$$
$$P(M\cap C)=0.70\cdot 0.80$$
$$P(M\cap C)=0.56$$
$$P(C)=0.18+0.56$$
$$P(C)=0.74$$
$$P(M|C)=\frac{P(M\cap C)}{P(C)}$$
$$P(M|C)=\frac{0.56}{0.74}$$
$$P(M|C)=\frac{28}{37}$$

The probability the question was multiple-choice is $\frac{28}{37}$, or about 0.757 or 75.7%.

7. Let *F* represent drawing a loonie the first time, and let *S* represent drawing a loonie the second time.

$$P(F)=\frac{4}{12}$$
$$P(F)=\frac{1}{3}$$
$$P(S|F)=\frac{3}{11}$$
$$P(F\cap S)=P(F)\cdot P(S|F)$$
$$P(F\cap S)=\frac{1}{3}\cdot\frac{3}{11}$$
$$P(F\cap S)=\frac{1}{11}$$

The probability both coins are loonies is $\frac{1}{11}$, or about 0.091 or 9.1%.

8. Let *S* represent Anita remembering to set her alarm, and let *N* represent Anita not remembering to set her alarm. Let *L* represent Anita being late for school.

$$P(S\cap L)=P(S)\cdot P(L|S)$$
$$P(S\cap L)=0.62\cdot 0.20$$
$$P(S\cap L)=0.124$$
$$P(N\cap L)=P(N)\cdot P(L|N)$$
$$P(N\cap L)=0.38\cdot 0.70$$
$$P(N\cap L)=0.266$$
$$P(C)=0.124+0.266$$
$$P(C)=0.39$$
$$P(S|L)=\frac{P(S\cap L)}{P(L)}$$
$$P(M|C)=\frac{0.124}{0.39}$$
$$P(M|C)=\frac{62}{195}$$

The probability Anita's alarm clock was set is $\frac{62}{195}$, or about 0.317 or 31.7%.

9. Let *N* represent a nice day, and let *R* represent a rainy day. Let *J* represent Ian jogging 8 km.

$P(N \cap J) = P(N) \cdot P(J \mid N)$

$P(N \cap J) = 0.70 \cdot 0.85$

$P(N \cap J) = 0.595$

$P(R \cap J) = P(\text{R}) \cdot P(J \mid R)$

$P(R \cap J) = 0.30 \cdot 0.40$

$P(R \cap J) = 0.12$

$P(J) = 0.595 + 0.12$

$P(J) = 0.715$

The probability Ian will jog for 8 km tomorrow is 0.715, or 71.5%.

10. Let *C* represent a user having call display, and let *D* represent a user having a data plan.

$$P(C\cap D)=P(D)\cdot P(C|D)$$
$$P(C\cap D)=0.40\cdot 0.75$$
$$P(C\cap D)=0.30$$
$$P(D|C)=\frac{P(D\cap C)}{P(C)}$$
$$P(D|C)=\frac{0.30}{0.70}$$
$$P(D|C)=\frac{3}{7}$$

The probability that a cell phone user with call display also has a data plan is $\frac{3}{7}$, or about 0.429 or 42.9%.

11. e.g., A student selected at random goes to a fast-food outlet that particular day. What is the probability that the student had more than 1 h for lunch?

12. e.g.,

a) Survey questions for a group of classmates over one month (weekdays only):

How often do you cycle to school?	140
How often do you get to school other than by cycling?	60
How often do you cycle when the weather is fine?	80
How often do you cycle when it is raining or snowing?	60

b) A randomly selected student cycled to school on a particular day. What is the probability that the weather was fine that day?

13. Let F represent tires lasting 5 years, and let S represent tires lasting 6 years. Since all tires that have lasted 6 years have also lasted 5 years, it is true that $P(F \cap S) = P(S)$.

$$P(S|F) = \frac{P(F \cap S)}{P(F)}$$

$$P(S|F) = \frac{P(S)}{P(F)}$$

$$P(S|F) = \frac{0.5}{0.8}$$

$$P(S|F) = 0.625$$

The probability that tires that have lasted 5 years will last 6 years is 0.625, or 62.5%.

14. Let T represent windshield wipers lasting 3 years, and let F represent windshield wipers lasting 4 years. Since all windshield wipers that have lasted 3 years have also lasted 4 years, $P(T \cap F) = P(F)$.

$$P(F|T) = \frac{P(T \cap F)}{P(T)}$$

$$P(F|T) = \frac{P(F)}{P(T)}$$

$$P(F|T) = \frac{0.6}{0.7}$$

$$P(F|T) = \frac{6}{7}$$

The probability that windshield wipers that have lasted 3 years will last 4 years is $\frac{6}{7}$, or about 0.857 or 85.7%.

15. Let S represent badminton shoes lasting six months, and let Y represent badminton shoes lasting one year. Since all badminton shoes that have lasted one year have also lasted 6 months, $P(S \cap Y) = P(Y)$

$$P(Y|S) = \frac{P(S \cap Y)}{P(S)}$$

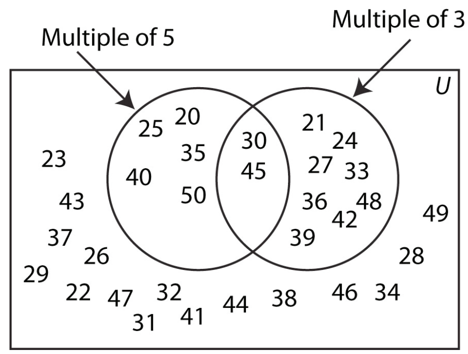
$$P(Y|S) = \frac{P(Y)}{P(S)}$$

$$P(Y|S) = \frac{0.2}{0.9}$$

$$P(Y|S) = \frac{2}{9}$$

The probability that badminton shoes that have lasted six months will last one year is $\frac{2}{9}$, or about 0.222 or 22.2%.

16. a) The probability that a multiple of 5 will also be a multiple of 3 is $\frac{2}{7}$, or about 0.286 or 28.6%.



b) Let F represent a number being a multiple of 5, let T represent a number being a multiple of 3.

F: {20, 25, 30, 35, 40, 45, 50}

T: {21. 24. 27. 30, 33, 36, 39, 42, 45, 48}

30 and 45 are in both sets.

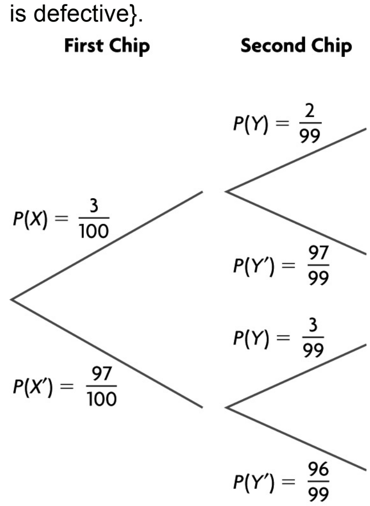
$$P(T|F) = \frac{P(T \cap F)}{P(F)}$$

$$P(T|F) = \frac{\left(\frac{2}{31}\right)}{\left(\frac{7}{31}\right)}$$

$$P(T|F) = \frac{2}{7}$$

The probability that a multiple of 5 will also be a multiple of 3 is $\frac{2}{7}$, or about 0.286 or 28.6%.

17. a) Let X = {1st chip is defective} and Y = {2nd chip is defective}.



b) Let D represent a chip being defective, and let N represent a chip not being defective. Let O represent exactly one of the two chips being defective.
There are two ways to draw two chips, with one being defective. Either the first is defective, or the second is defective. Therefore, the probability of both these events must be determined and added together, to get the total probability.

$P(D)=\frac{3}{100}$ $\quad$ $P(N)=\frac{97}{100}$

$P(N|D)=\frac{97}{99}$ $\quad$ $P(D|N)=\frac{3}{99}$

$P(D\cap N)=P(D)\cdot P(N|D)$ $\quad$ $P(D|N)=\frac{1}{33}$

$P(D\cap N)=\frac{3}{100}\cdot\frac{97}{99}$ $\quad$ $P(N\cap D)=P(N)\cdot P(D|N)$

$P(D\cap N)=\frac{291}{9900}$ $\quad$ $P(N\cap D)=\frac{97}{100}\cdot\frac{1}{33}$

$P(D\cap N)=\frac{97}{3300}$ $\quad$ $P(N\cap D)=\frac{97}{3300}$

Now determine the total probability of drawing exactly one defective chip.

$P(O)=P(D\cap N)+P(N\cap D)$

$P(O)=\frac{97}{3300}+\frac{97}{3300}$

$P(O)=\frac{97}{1650}$

The probability of drawing exactly one defective chip is $\frac{97}{1650}$, or about 0.0588 or 5.88%.
e.g., Multiply the probability of the first chip being defective by the second chip being non-defective, then multiply the probability of the first chip being non-defective by the second chip being defective, and then add the products.

18. Let D represent a chip being defective, and let N represent a chip not being defective. Let O represent exactly one of the chips being defective.

a) There is only one way to draw two defective chips.

$P(D)=\frac{3}{150}$ $\quad$ $P(D|D)=\frac{2}{149}$

$P(D\cap D)=P(D)\cdot P(D|D)$

$P(D\cap D)=\frac{3}{150}\cdot\frac{2}{149}$

$P(D\cap D)=\frac{6}{22\ 350}$

$P(D\cap D)=\frac{1}{3725}$

The probability of drawing two defective chips is $\frac{1}{3725}$ or about 0.000 268.

b) There is only one way to draw two non-defective chips.

$P(N)=\frac{147}{150}$ $\quad$ $P(N|N)=\frac{146}{149}$

$P(N\cap N)=P(N)\cdot P(N|N)$

$P(N\cap N)=\frac{147}{150}\cdot\frac{146}{149}$

$P(N\cap N)=\frac{21\ 462}{22\ 350}$

$P(N\cap N)=\frac{3577}{3725}$

The probability of drawing two non-defective chips is $\frac{3577}{3725}$ or about 0.960.

c) In this case, either one defective chip is drawn first and then one non-defective chip is drawn, or vice versa. The probability of drawing exactly one defective chip is the sum of the probabilities of these events.

$P(D)=\frac{3}{150}$ $\quad$ $P(N)=\frac{147}{150}$

$P(N|D)=\frac{147}{149}$ $\quad$ $P(D|N)=\frac{3}{149}$

$P(D\cap N)=P(D)\cdot P(N|D)$ $\quad$ $P(D\cap N)=P(N)\cdot P(D|N)$

$P(D\cap N)=\frac{3}{150}\cdot\frac{147}{149}$ $\quad$ $P(D\cap N)=\frac{147}{150}\cdot\frac{3}{149}$

$P(D\cap N)=\frac{441}{22\ 350}$ $\quad$ $P(D\cap N)=\frac{441}{22\ 350}$

$P(D\cap N)+P(D\cap N)=\frac{441}{22\ 350}+\frac{441}{22\ 350}$

$P(D\cap N)+P(D\cap N)=\frac{882}{22\ 350}$

$P(D\cap N)+P(D\cap N)=\frac{147}{3725}$

The probability of drawing exactly one defective chip is $\frac{147}{3725}$, or about 0.0395.

19. Let S represent a sunny day, and R represent a rainy day. Let W represent a win, and L represent a loss.

a) $P(R\cap W)=P(R)\cdot P(W\mid R)$
$P(R\cap W)=0.30\cdot 0.50$
$P(R\cap W)=0.15$
$P(S\cap W)=P(S)\cdot P(W\mid S)$
$P(S\cap W)=0.70\cdot 0.60$
$P(S\cap W)=0.42$
$P(W)=P(R\cap W)+P(S\cap W)$
$P(W)=0.15+0.42$
$P(W)=0.57$

The probability Savannah's team will win is 0.57, or 57%.

b) $P(R \cap L) = P(R) \cdot P(L \mid R)$
$P(R \cap L) = 0.30 \cdot 0.50$
$P(R \cap L) = 0.15$
$P(S \cap L) = P(S) \cdot P(L \mid S)$
$P(S \cap L) = 0.70 \cdot 0.40$
$P(S \cap L) = 0.28$
$P(L) = P(R \cap L) + P(S \cap L)$
$P(L) = 0.15 + 0.28$
$P(L) = 0.43$
The probability that Savannah's team will lose is 0.43, or 43%.

20. e.g., Problem 1: On weekdays I have cereal for breakfast 70% of the time. On the weekends I have cereal for breakfast 40% of the time. On a random day, what is the probability that I do not have cereal?
Solution:
Let *WD* represent a weekday, and *WE* represent a weekend. Let *C* represent having cereal for breakfast, and let *N* represent not having cereal for breakfast.

$P(C \mid WD) = 0.70$ means $P(N \mid WD) = 0.30$
$P(C \mid WE) = 0.40$ means $P(N \mid WE) = 0.60$

$$P(N) = \frac{5}{7} \cdot P(N \mid WD) + \frac{2}{7} \cdot P(N \mid WE)$$
$$P(N) = \frac{5}{7} \cdot 0.30 + \frac{2}{7} \cdot 0.60$$
$$P(N) = \frac{50}{70} \cdot \frac{21}{70} + \frac{20}{70} \cdot \frac{42}{70}$$
$$P(N) = \frac{1050}{4900} + \frac{840}{4900}$$
$$P(N) = \frac{1890}{4900}$$
$$P(N) = \frac{27}{70}$$

Answer: $\frac{27}{70}$ or about 0.386

Problem 2: I draw two cards from a well shuffled standard deck, drawing the second card without replacing the first one. If my second card is a red card, what is the probability that my first card is black?
Solution: Let *B* represent the first card being black and *R* represent the second card being red. We are looking for $P(B \mid R)$.

$$P(B \mid R) = \frac{P(B \cap R)}{P(R)}$$
$$P(B \mid R) = \frac{\frac{26}{52} \cdot \frac{26}{51}}{\frac{26}{52}}$$
$$P(B \mid R) = \frac{26}{51}$$

Answer: $\frac{26}{51}$ or about 0.510

21. e.g., The probability of event *A* and *B* both occurring is the probability *A* occurs, multiplied by the probability *B* occurs given that *A* occurs. Example: If a 6-sided die is rolled twice,
P(rolling a 4 the first time and a total > 7)
= *P*(rolling a 4 the first time)
· *P*(a total > 7 | rolling a 4 the first time)
or $\frac{1}{12} = \frac{1}{6} \cdot \frac{1}{2}$.

22. Let *D* represent a chip being defective, and let *N* represent a chip not being defective.
a) Let *S* represent the second chip being defective, given that the first chip drawn was also defective. Let *T* represent the third chip being defective, given that the first two were also defective. Let *O* represent all three chips drawn being defective. In this case, there is only one way to draw three defective chips.

$$P(D) = \frac{4}{100} \qquad P(T) = \frac{2}{98}$$
$$P(D) = \frac{1}{25} \qquad P(T) = \frac{1}{49}$$
$$P(S) = \frac{3}{99} \qquad P(O) = P(D) \cdot P(S) \cdot P(T)$$
$$P(S) = \frac{1}{33} \qquad P(O) = \frac{1}{25} \cdot \frac{1}{33} \cdot \frac{1}{49}$$
$$P(O) = \frac{1}{40425}$$

The probability of drawing 3 defective chips is $\frac{1}{40\,425}$, or about 0.000 024 7 or 0.002 47%.

b) Let *B* represent the second chip not being defective, given that the first chip drawn was also not defective. Let *C* represent the third chip not being defective, given that the first two were also not defective. Let *F* represent all three chips drawn not being defective. In this case, there is only one way to draw three non-defective chips.

$$P(N) = \frac{96}{100} \qquad P(B) = \frac{95}{99} \qquad P(C) = \frac{94}{98}$$
$$P(N) = \frac{24}{25} \qquad P(C) = \frac{47}{49}$$

$$P(F) = P(N) \cdot P(B) \cdot P(C)$$
$$P(F) = \frac{24}{25} \cdot \frac{95}{99} \cdot \frac{47}{49}$$
$$P(F) = \frac{107160}{121275}$$
$$P(F) = \frac{7144}{8085}$$

The probability of drawing 3 defective chips is $\frac{7144}{8085}$, or about 0.884 or 88.4%.

c) In this case, more defective chips will be pulled when 2 or 3 defective chips are pulled. There are 3 different ways to pull 2 defective chips.
Let *S* represent the second chip being defective, given that the first chip drawn was also defective. Let *U* represent the third chip not being defective, given that the first two were defective. Let *G* represent two of the three chips drawn being defective. Let *A* represent at least two of the three chips drawn being defective.

$P(D) = \frac{4}{100}$ $P(S) = \frac{3}{99}$ $P(U) = \frac{96}{98}$

$P(D) = \frac{1}{25}$ $P(S) = \frac{1}{33}$ $P(U) = \frac{48}{49}$

$P(G) = P(D) \cdot P(S) \cdot P(U) \cdot 3$

$P(G) = \frac{1}{25} \cdot \frac{1}{33} \cdot \frac{48}{49} \cdot 3$

$P(G) = \frac{144}{40425}$

$P(G) = \frac{48}{13475}$

The total probability can now be determined.

$P(A) = P(G) + P(O)$

$P(A) = \frac{48}{13475} + \frac{1}{40425}$

$P(A) = \frac{144}{40425} + \frac{1}{40425}$

$P(A) = \frac{145}{40425}$

$P(A) = \frac{29}{8085}$

The probability of drawing more defective chips than non-defective chips is $\frac{29}{8085}$, or about 0.0036 or 0.36%.

Lesson 5.6: Independent Events, page 360

1. a) These events are independent, because the result of the spinner does not affect the result of the die, and vice versa.

b) These events are independent, because the result of the red die does not affect the result of the green die, and vice versa.

c) These events are dependent. Because there is no replacement, the deck that the second card is drawn from is technically different than the original deck.

d) These events are independent, because replacement is occurring, which 'resets' the probability for each draw.

2. a) These events are likely independent. The cardio workouts focus on the heart, and use the legs the most often. Therefore, there would be no specific reason why one workout would be favoured over another.

b) Let *B* represent Celeste using a stationary bike, and let *F* represent Celeste using free weights.

$P(B) = \frac{1}{3}$ $P(F) = \frac{1}{2}$

$P(B \cap F) = P(B) \cdot P(F)$

$P(B \cap F) = \frac{1}{3} \cdot \frac{1}{2}$

$P(B \cap F) = \frac{1}{6}$

The probability Celeste will use a stationary bike and free weights for her next workout is $\frac{1}{6}$, or about 0.167 or 16.7%.

3. a) The two workouts are dependent. After Ian chooses a workout, he won't choose the same one again.

b) Let *T* represent Ian running the track, and let *E* represent Ian using an elliptical walker.

$P(T) = \frac{1}{4}$ $P(E|T) = \frac{1}{3}$

$P(T \cap E) = P(T) \cdot P(E|T)$

$P(T \cap E) = \frac{1}{4} \cdot \frac{1}{3}$

$P(T \cap E) = \frac{1}{12}$

The probability that Ian will run the track and use the elliptical walker is $\frac{1}{12}$, or about 0.0833 or 8.33%.

4. a) Let *R* represent spinning a red, and let *T* represent rolling a two.

$P(R) = \frac{1}{4}$ $P(T) = \frac{1}{6}$

$P(R \cap T) = P(R) \cdot P(T)$

$P(R \cap T) = \frac{1}{4} \cdot \frac{1}{6}$

$P(R \cap T) = \frac{1}{24}$

The probability the spinner will land on red and the die will land on 2 is $\frac{1}{24}$, or about 0.0417 or 4.17%.

b) Let *O* represent rolling a one on the red die, and *F* represent rolling a five on the green die.

$P(O) = \frac{1}{6}$ $P(F) = \frac{1}{6}$

$P(O \cap F) = P(O) \cdot P(F)$

$P(O \cap F) = \frac{1}{6} \cdot \frac{1}{6}$

$P(O \cap F) = \frac{1}{36}$

The probability of rolling a 1 on the red die and a 5 on the green die is $\frac{1}{36}$, or about 0.0278 or 2.78%.