Lesson 5.3: Probabilities Using Counting Methods, page 321

1. Let *A* represent a passcode with 4 different even digits, and let *O* represent all 4-digit passcodes. There are 5 different possibilities for even digits: 0, 2,4,6, and 8. Since these numbers cannot be repeated, the total number of passcodes using 4 different even digits is equal to ${}_5P_4$.

$$n(A) = {}_5P_4$$
$$n(A) = \frac{5!}{(5-4)!}$$
$$n(A) = \frac{5!}{1!}$$
$$n(A) = \frac{5\cdot4\cdot3\cdot2\cdot1}{1}$$
$$n(A) = 5\cdot4\cdot3\cdot2$$
$$n(A) = 120$$

The number of 4 digit passcodes is 10^4, because there are 10 possible digits to use in 4 spaces, and repeating is allowed.

$$P(A) = \frac{n(A)}{n(O)}$$
$$P(A) = \frac{120}{10^4}$$
$$P(A) = \frac{120}{10\ 000}$$

The probability that Suri's passcode is made up of four different even digits is $\frac{120}{10\ 000}$, or 0.012 or 1.2%.

2. Let *A* represent an 8-card hand containing 8 hearts, and let *O* represent all 8-card hands.
In a standard deck of cards used for Crazy Eights, there are 13 hearts. Therefore, the total number of hands with 8 hearts is ${}_{13}C_8$.

$$n(A) = {}_{13}C_8$$
$$n(A) = \frac{13!}{(13-8)!\cdot 8!}$$
$$n(A) = \frac{13!}{5!\cdot 8!}$$
$$n(A) = \frac{13\cdot12\cdot11\cdot10\cdot9\cdot8!}{5\cdot4\cdot3\cdot2\cdot1\cdot8!}$$
$$n(A) = \frac{13\cdot12\cdot11\cdot10\cdot9}{5\cdot4\cdot3\cdot2}$$
$$n(A) = \left(\frac{12}{4\cdot3}\right)\left(\frac{10}{5\cdot2}\right)(13)(11)(9)$$
$$n(A) = 13\cdot11\cdot9$$

The total number of hands containing 8 cards is ${}_{52}C_8$, because there are 52 cards to choose from, and repeating is not possible.

$$n(O) = {}_{52}C_8$$
$$n(O) = \frac{52!}{(52-8)!\cdot 8!}$$
$$n(O) = \frac{52!}{44!\cdot 8!}$$
$$n(O) = \frac{52\cdot51\cdot50\cdot49\cdot48\cdot47\cdot46\cdot45\cdot44!}{44!\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$
$$n(O) = \frac{52\cdot51\cdot50\cdot49\cdot48\cdot47\cdot46\cdot45}{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2}$$
$$n(O) = \left(\frac{48}{8\cdot6}\right)\left(\frac{49}{7}\right)\left(\frac{45}{5}\right)\left(\frac{52}{4}\right)\left(\frac{51}{3}\right)\left(\frac{46}{2}\right)(47)(50)$$
$$n(O) = 1\cdot7\cdot9\cdot13\cdot17\cdot23\cdot47\cdot50$$
$$n(O) = 7\cdot9\cdot13\cdot17\cdot23\cdot47\cdot50$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$
$$P(A) = \frac{13\cdot11\cdot9}{7\cdot9\cdot13\cdot17\cdot23\cdot47\cdot50}$$
$$P(A) = \frac{11}{7\cdot17\cdot23\cdot47\cdot50}$$
$$P(A) = \frac{11}{6\ 431\ 950}$$

The probability that a hand will consist of 8 hearts is $\frac{11}{6\,431\,950}$ or about 0.000 001 71.

3. Let *A* represent Ben and Jen being chosen as president and secretary, and let *O* represent all possible committees.
Since order is important, the number of ways in which Ben and Jen can be chosen for president and secretary is ${}_2P_2$.

$$n(A) = {}_2P_2$$
$$n(A) = \frac{2!}{(2-2)!}$$
$$n(A) = \frac{2!}{0!}$$
$$n(A) = \frac{2\cdot1}{1}$$
$$n(A) = 2$$

The total number of possible committees is ${}_{12}P_2$, because there are 12 people to choose from, and order is important.

$$n(O) = {}_{12}P_2$$
$$n(O) = \frac{12!}{(12-2)!}$$
$$n(O) = \frac{12!}{10!}$$
$$n(O) = \frac{12\cdot11\cdot10!}{10!}$$
$$n(O) = 12\cdot11$$
$$n(O) = 132$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{2}{132}$$

$$P(A) = \frac{1}{66}$$

The probability Ben and Jen will be chosen is $\frac{1}{66}$, or about 0.0152 or 1.52%.

4. a) Let *B* represent the possibility that only boys will be on the trip, and let *O* represent all possibilities.

$$n(B) = {}_5C_4$$

$$n(B) = \frac{5!}{(5-4)!4!}$$

$$n(B) = \frac{5!}{1! \cdot 4!}$$

$$n(B) = \frac{5 \cdot 4!}{1 \cdot 4!}$$

$$n(B) = 5$$

The number of possibilities of there being all boys on the trip is 5.

$$n(O) = {}_{11}C_4$$

$$n(O) = \frac{11!}{(11-4)!4!}$$

$$n(O) = \frac{11!}{7! \cdot 4!}$$

$$n(O) = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 4!}$$

$$n(O) = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2}$$

$$n(O) = 11 \cdot 10 \cdot 3$$

$$n(O) = 330$$

The total number of possibilities for selecting four students for the trip is 330.

$$P(B) = \frac{n(B)}{n(O)}$$

$$P(B) = \frac{5}{330}$$

$$P(B) = \frac{1}{66}$$

Therefore, the probability that only boys will be on the trip is $\frac{1}{66}$ or 0.0152 or 1.52%.

b) Let *C* represent that there is an equal number of boys and girls on the trip.

$$n(C) = {}_5C_2 \cdot {}_6C_2$$

$$n(C) = \frac{5!}{(5-2)!2!} \cdot \frac{6!}{(6-2)!2!}$$

$$n(C) = \frac{5!}{3! \cdot 2!} \cdot \frac{6!}{4! \cdot 2!}$$

$$n(C) = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} \cdot \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1}$$

$$n(C) = \frac{5 \cdot 4}{2} \cdot \frac{6 \cdot 5}{2}$$

$$n(C) = 10 \cdot 15$$

$$n(C) = 150$$

The total number of ways there can be an equal number of boys and girls on the trip is 150.

$$P(C) = \frac{n(C)}{n(O)}$$

$$P(C) = \frac{150}{330}$$

$$P(C) = \frac{5}{11}$$

The probability that an equal number of boys and girls will go on the trip is $\frac{5}{11}$, or about 0.455 or 45.5%.

c) Let *D* represent that there are more girls than boys on the trip. This is true if 4 girls and no boys are selected and if 3 girls and 1 boy are selected.

$$n(D) = {}_6C_4 + {}_6C_3 \cdot {}_5C_1$$

$$n(D) = \frac{6!}{(6-4)!4!} + \frac{6!}{(6-3)!3!} \cdot \frac{5!}{(5-1)!1!}$$

$$n(D) = \frac{6!}{2!4!} + \frac{6!}{3!3!} \cdot \frac{5!}{4!1!}$$

$$n(D) = \frac{6 \cdot 5 \cdot 4!}{2!4!} + \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!3!} \cdot \frac{5 \cdot 4!}{4!}$$

$$n(D) = 15 + 20 \cdot 5$$

$$n(D) = 115$$

There are 115 ways in which there can be more girls than boys on the trip.

$$P(D) = \frac{n(D)}{n(O)}$$

$$P(D) = \frac{115}{330}$$

$$P(D) = \frac{23}{66}$$

The probability that more girls than boys will go is $\frac{23}{66}$ or about 0.348 or 34.8%.

5. Let *S* represent a password that contains S and Q, and let *W* represent all possible passwords.
a) The number of possible passwords containing S and Q is ${}_2P_2 \cdot {}_{10}P_3$, or $2! \cdot {}_{10}P_3$. Order in passwords is important, so permutations are used.

$$n(S) = 2! \cdot {}_{10}P_3$$
$$n(S) = 2! \cdot \frac{10!}{(10-3)!}$$
$$n(S) = 2! \cdot \frac{10!}{7!}$$
$$n(S) = (2 \cdot 1)\left(\frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}\right)$$
$$n(S) = 2 \cdot 10 \cdot 9 \cdot 8$$
$$n(S) = 1440$$

The total number of passwords of this form is ${}_{26}P_2 \cdot {}_{10}P_3$.

$$n(W) = {}_{26}P_2 \cdot {}_{10}P_3$$
$$n(W) = \frac{26!}{(26-2)!} \cdot \frac{10!}{(10-3)!}$$
$$n(W) = \frac{26!}{24!} \cdot \frac{10!}{7!}$$
$$n(W) = \frac{26 \cdot 25 \cdot 24!}{24!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$$
$$n(W) = 26 \cdot 25 \cdot 10 \cdot 9 \cdot 8$$
$$n(W) = 468\ 000$$

Now determine the probability.

$$P(S) = \frac{n(S)}{n(W)}$$
$$P(S) = \frac{1440}{468\,000}$$
$$P(S) = \frac{1}{325}$$

The probability that a password chosen at random will include S and Q is $\frac{1}{325}$, or about 0.003 08 or 0.308%.

b) Since repetition is allowed, the number of passwords containing S and Q is ${}_2P_2 \cdot 10^3$, or $2! \cdot 10^3$. The total number of passwords available is $26^2 \cdot 10^3$.

$$P(S) = \frac{n(S)}{n(W)}$$
$$P(S) = \frac{2! \cdot 10^3}{26^2 \cdot 10^3}$$
$$P(S) = \frac{2!}{26^2}$$
$$P(S) = \frac{2 \cdot 1}{26^2}$$
$$P(S) = \frac{2}{676}$$
$$P(S) = \frac{1}{338}$$

The probability that a password chosen at random will include S and Q is $\frac{1}{338}$, or about 0.002 96 or 0.296%.

6. Let *F* represent the four friends being on the team, and let *T* represent all the possible teams.
a) The number of ways 4 people can be placed in 4 spots, when order is important, is ${}_4P_4$.

$$n(F) = {}_4P_4$$
$$n(F) = \frac{4!}{(4-4)!}$$
$$n(F) = \frac{4!}{0!}$$
$$n(F) = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1}$$
$$n(F) = 4 \cdot 3 \cdot 2$$
$$n(F) = 24$$

The number of ways 9 people can be placed in 4 spots, when order is important, is ${}_9P_4$.

$$n(T) = {}_9P_4$$
$$n(T) = \frac{9!}{(9-4)!}$$
$$n(T) = \frac{9!}{5!}$$
$$n(T) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!}$$
$$n(T) = 9 \cdot 8 \cdot 7 \cdot 6$$
$$n(T) = 3024$$

The probability can now be determined.

$$P(F) = \frac{n(F)}{n(T)}$$
$$P(F) = \frac{24}{3024}$$
$$P(F) = \frac{1}{126}$$

The probability the 4 friends will all be on the team is $\frac{1}{126}$, or about 0.0079 or 0.79%.

b) If only 8 students were applying for the team, then the total number of teams would change from ${}_9P_4$ to ${}_8P_4$.

$$n(T) = {}_8P_4$$
$$n(T) = \frac{8!}{(8-4)!}$$
$$n(T) = \frac{8!}{4!}$$
$$n(T) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$$
$$n(T) = 8 \cdot 7 \cdot 6 \cdot 5$$
$$n(T) = 1680$$

Now determine the probability.

$$P(F) = \frac{n(F)}{n(T)}$$

$$P(F) = \frac{24}{1680}$$

$$P(F) = \frac{1}{70}$$

The probability increases from $\frac{1}{126}$ to $\frac{1}{70}$, or about 0.0143 or 1.43%.

7. Let T represent Tara and Laura being chosen to play in the infield. Let O represent all possible infield lineups. The number of ways to arrange Tara and Laura in the infield positions is $2 \cdot 3!$. The number of ways to arrange the other 7 players in the remaining 2 infield positions is $_7P_2$. Therefore, the total number of infield lineups that include Tara and Laura is $(2 \cdot 3!) \cdot {}_7P_2$.

$$n(T) = (2 \cdot 3!) \cdot {}_7P_2$$

$$n(T) = (2 \cdot 3!) \cdot \frac{7!}{(7-2)!}$$

$$n(T) = (2 \cdot 3!) \cdot \frac{7!}{5!}$$

$$n(T) = (2 \cdot 3 \cdot 2 \cdot 1) \cdot \frac{7 \cdot 6 \cdot 5!}{5!}$$

$$n(T) = 2 \cdot 3 \cdot 2 \cdot 7 \cdot 6$$

$$n(T) = 504$$

The total number of infield lineups possible is $_9P_4$.

$$n(O) = {}_9P_4$$

$$n(O) = \frac{9!}{(9-4)!}$$

$$n(O) = \frac{9!}{5!}$$

$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!}$$

$$n(O) = 9 \cdot 8 \cdot 7 \cdot 6$$

$$n(O) = 3024$$

Now determine the probability.

$$P(T) = \frac{n(T)}{n(O)}$$

$$P(T) = \frac{504}{3024}$$

$$P(T) = \frac{1}{6}$$

The probability that Tara and Laura will both play in the infield is $\frac{1}{6}$. Therefore the odds in favour of this event are 1 : (6 – 1) or 1 : 5.

8. a) Let Y represent Yuko, Luigi and Justin being chosen, and let T represent all of the ways that treasurer, secretary and liaison can be chosen. In this case, there is only 1 way to achieve the favourable outcome. The number of ways to choose treasurer, secretary and liaison is $_{15}P_3$.

$$n(T) = {}_{15}P_3$$

$$n(T) = \frac{15!}{(15-3)!}$$

$$n(T) = \frac{15!}{12!}$$

$$n(T) = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12!}$$

$$n(T) = 15 \cdot 14 \cdot 13$$

$$n(T) = 2730$$

Now determine the probability.

$$P(Y) = \frac{n(Y)}{n(T)}$$

$$P(Y) = \frac{1}{2730}$$

The probability is $\frac{1}{2730}$, or about 0.000 366 or 0.0366%.

b) Let Y represent Yuko, Luigi and Justin being chosen, and let L represent all of that ways that three students can be chosen. Again, there is only 1 way to achieve the favourable outcome. The number of ways to choose 3 people from 15 to clean up is $_{15}C_3$.

$$n(L) = {}_{15}C_3$$

$$n(L) = \frac{15!}{(15-3)! \cdot 3!}$$

$$n(L) = \frac{15!}{12! \cdot 3!}$$

$$n(L) = \frac{15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 3 \cdot 2 \cdot 1}$$

$$n(L) = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2}$$

$$n(L) = \frac{2730}{6}$$

$$n(L) = 455$$

Now determine the probability.

$$P(Y) = \frac{n(Y)}{n(L)}$$

$$P(Y) = \frac{1}{455}$$

The probability that Yuko, Luigi and Justin will be picked is $\frac{1}{455}$, or about 0.0022 or 0.220%.

9. a) Let *F* represent a password that is greater than 5000, and let *O* represent all possible passwords. There are 4999 4-digit numbers greater than 5000. However, in this range, there are 5 numbers that repeat all 4 digits (5555, 6666, 7777, 8888 and 9999). Therefore, the total number of usable passwords, greater than 5000, is 4994. The total number of four digits possible is 10 000. However, there are 10 numbers that repeat all 4 digits. So, there are 9990 usable passwords.

$$P(F) = \frac{n(F)}{n(O)}$$

$$P(F) = \frac{4994}{9990}$$

$$P(F) = \frac{2497}{4995}$$

The probability Lesley's password is greater than 5000 is $\frac{2497}{4995}$, or about 0.4999 or 49.99%.

b) Let *B* represent a password that begins and ends with the number 4, and let *O* represent all possible passwords. Because it is assumed that the first and last digits are 4s, only the two middle digits need to be dealt with. Since repeating two digits is allowed, the number of ways to arrange the two middle digits is 10^2, or 100. However, all the digits could be 4, which is not allowed. Therefore, the total number of usable passwords, where the first and last digits are 4s, is 99. Again, the total number of usable passwords is 9990.

$$P(B) = \frac{n(B)}{n(O)}$$

$$P(B) = \frac{99}{9990}$$

$$P(B) = \frac{11}{1110}$$

The probability Lesley's password begins and ends with 4 is $\frac{11}{1110}$, or about 0.0099 or 0.99%.

c) Let *E* represent a password that begins with an odd number and ends with an even number, and let *O* represent all possible passwords. If the first digit is odd and the last digit is even, then there is no possible way that all four digits can be the same. The first digit can be one of five odd numbers (1, 3, 5, 7 and 9), and the last digit can be one of 5 even numbers (0, 2, 4, 6 and 8). The two middle digits can be one of ten digits. Therefore the total number of usable passwords that begin with an odd digit and end with an even digit is 5 · 10 · 10 · 5, or 2500. The total number of usable passwords is 9990.

$$P(E) = \frac{n(E)}{n(O)}$$

$$P(E) = \frac{2500}{9990}$$

$$P(E) = \frac{250}{999}$$

The probability that Lesley's password begins with an odd digit and ends with an even digit is $\frac{250}{999}$, or about 0.2503 or 25.03%.

10. Let *T* represent three girls and two boys being chosen to form a subcommittee, and let *S* represent all possible subcommittees.
In this example, order is not important. The number of ways to arrange three girls and two boys from 16 girls and 7 boys is ${}_{16}C_3 \cdot {}_7C_2$.

$$n(T) = {}_{16}C_3 \cdot {}_7C_2$$

$$n(T) = \frac{16!}{(16-3)! \cdot 3!} \cdot \frac{7!}{(7-2)! \cdot 2!}$$

$$n(T) = \frac{16!}{13! \cdot 3!} \cdot \frac{7!}{5! \cdot 2!}$$

$$n(T) = \frac{16 \cdot 15 \cdot 14 \cdot 13!}{13! \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5!}{5! \cdot 2 \cdot 1}$$

$$n(T) = \frac{16 \cdot 15 \cdot 14}{3 \cdot 2} \cdot \frac{7 \cdot 6}{2}$$

$$n(T) = \frac{3360}{6} \cdot \frac{42}{2}$$

$$n(T) = 560 \cdot 21$$

$$n(T) = 11\,760$$

The number of ways to arrange 23 people in a five-person committee is ${}_{23}C_5$.

$$n(S) = {}_{23}C_5$$

$$n(S) = \frac{23!}{(23-5)! \cdot 5!}$$

$$n(S) = \frac{23!}{18! \cdot 5!}$$

$$n(S) = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!}{18! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(S) = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{5 \cdot 4 \cdot 3 \cdot 2}$$

$$n(S) = \frac{4\,037\,880}{120}$$

$$n(S) = 33\,649$$

Now determine the probability.

$$P(T) = \frac{n(T)}{n(S)}$$

$$P(T) = \frac{11\,760}{33\,649}$$

$$P(T) = \frac{1680}{4807}$$

The odds in favour that the committee will contain 3 girls and 2 boys is 1680 : 4807 – 1680 or 1680 : 3127.

11. The total number of outcomes is 2^4, or 16. There is only one option where no coins land on tails (all land on head). Therefore, the probability that at least one coin lands tails is $\frac{15}{16}$, or 0.9375 or 93.75%.

12. Let B represent Bilyana and Bojana sitting together, and let S represent all possible seating arrangements.

a) The number of ways to seat Bilyana and Bojana together is $4 \cdot {}_2P_2$, or $4 \cdot 2!$. The number of ways to seat the 3 other people is ${}_3P_3$, or 3!. The number of ways to seat the 5 friends in the row is $4 \cdot 2! \cdot 3!$, or 48.
There are 5! or 120 ways to arrange five people.

$$P(B) = \frac{n(B)}{n(S)}$$
$$P(B) = \frac{48}{120}$$
$$P(B) = \frac{2}{5}$$

The probability that Bilyana and Bojana are sitting together is $\frac{2}{5}$, 0.4 or 40%.

b) The probability that Bilyana and Bojana are not sitting together is the complement of the probability that they are sitting together

$$P(B') = 1 - P(B)$$
$$P(B') = 1 - \frac{2}{5}$$
$$P(B') = \frac{3}{5}$$

The probability Bilyana and Bojana are not sitting together is $\frac{3}{5}$, 0.6 or 60%.

13. a) Let L represent Tanya taking psychology, linear algebra and English in her first term, and let S represent all possible schedules.
There are $4 \cdot 3!$, or 24 ways to arrange psychology, linear algebra and English. The number of ways to fill the fourth slot is 5. Therefore, the number of schedules that contain psychology, linear algebra and English is $24 \cdot 5$, or 120.
The total number of ways to organize 8 courses into 4 slots is ${}_8P_4$.

$$n(S) = {}_8P_4$$
$$n(S) = \frac{8!}{(8-4)!}$$
$$n(S) = \frac{8!}{4!}$$
$$n(S) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$$
$$n(S) = 8 \cdot 7 \cdot 6 \cdot 5$$
$$n(S) = 1680$$

Now determine the probability.

$$P(L) = \frac{n(L)}{n(S)}$$
$$P(L) = \frac{120}{1680}$$
$$P(L) = \frac{1}{14}$$

The probability that three of the four courses will be psychology, linear algebra, and English is $\frac{1}{14}$, or about 0.0714 or 7.14%.

b) Let R represent Tanya taking religion, political studies and biology in her first term, and let S represent all possible schedules.
There are $10 \cdot 3!$, or 60, ways to arrange religion, political studies and biology. The number of ways to fill the remaining two slots is ${}_5P_2$.

$$n(R) = 60 \cdot {}_5P_2$$
$$n(R) = 60 \cdot \frac{5!}{(5-2)!}$$
$$n(R) = 60 \cdot \frac{5!}{3!}$$
$$n(R) = 60 \cdot \frac{5 \cdot 4 \cdot 3!}{3!}$$
$$n(R) = 60 \cdot 5 \cdot 4$$
$$n(R) = 1200$$

The total number of ways to organize 8 courses into 5 slots is ${}_8P_5$.

$$n(S) = {}_8P_5$$
$$n(S) = \frac{8!}{(8-5)!}$$
$$n(S) = \frac{8!}{3!}$$
$$n(S) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!}$$
$$n(S) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$
$$n(S) = 6720$$

Now determine the probability.

$$P(R) = \frac{n(R)}{n(S)}$$
$$P(R) = \frac{1200}{6720}$$
$$P(R) = \frac{5}{28}$$

The probability that three of the five courses will be religion, political studies and biology is $\frac{5}{28}$, or about 0.179 or 17.9%.

14. a) Let *A* represent a hand that contains the ace to 8 of the same suit, and let *O* represent all 8-card hands. In this example, order is not important. There are 4 ways to have the ace to 8 of the same suit, because there are 4 suits. There are ${}_{52}C_8$ ways to arrange 52 cards in 8 positions.

$$n(O) = {}_{52}C_8$$

$$n(O) = \frac{52!}{(52-8)! \cdot 8!}$$

$$n(O) = \frac{52!}{44! \cdot 8!}$$

$$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44!}{44! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{48}{8 \cdot 6} \cdot \frac{49}{7} \cdot \frac{45}{5} \cdot \frac{52}{4} \cdot \frac{51}{3} \cdot \frac{46}{2} \cdot 47 \cdot 50$$

$$n(O) = 7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{4}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(A) = \frac{2 \cdot 2}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 2 \cdot 25}$$

$$P(A) = \frac{2}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 25}$$

$$P(A) = \frac{2}{376\ 269\ 075}$$

The probability that an eight-card hand will contain the A, 2, 3, 4, 5, 6, 7 and 8 of the same suit is $\frac{2}{376269075}$ or about 0.000 000 005 32.

b) Let *S* represent a hand containing 8 cards of the same colour, and let *O* represent all 8-card hands. The number of ways to arrange 8 cards from 26 cards of the same colour suit is ${}_{26}C_8$. But, both colours are considered. So, the number of ways to have 8 cards of the same colour is $2 \cdot {}_{26}C_8$.

$$n(S) = 2 \cdot {}_{26}C_8$$

$$n(S) = 2 \cdot \frac{26!}{(26-8)! \cdot 8!}$$

$$n(S) = 2 \cdot \frac{26!}{18! \cdot 8!}$$

$$n(S) = \frac{2 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!}{18! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(S) = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}$$

$$n(S) = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{8 \cdot 7 \cdot 3 \cdot 2 \cdot 5 \cdot 4 \cdot 3}$$

$$n(S) = \frac{24}{8 \cdot 3} \cdot \frac{21}{7 \cdot 3} \cdot \frac{25}{5} \cdot \frac{20}{4} \cdot \frac{26}{2} \cdot 19 \cdot 22 \cdot 23$$

$$n(S) = 5 \cdot 5 \cdot 13 \cdot 19 \cdot 22 \cdot 23$$

Now determine the probability.

$$P(S) = \frac{n(S)}{n(O)}$$

$$P(S) = \frac{5 \cdot 5 \cdot 13 \cdot 19 \cdot 22 \cdot 23}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(S) = \frac{5 \cdot 5 \cdot 19 \cdot 22}{7 \cdot 9 \cdot 17 \cdot 47 \cdot 50}$$

$$P(S) = \frac{5 \cdot 5 \cdot 19 \cdot 2 \cdot 11}{7 \cdot 9 \cdot 17 \cdot 47 \cdot 2 \cdot 5 \cdot 5}$$

$$P(S) = \frac{19 \cdot 11}{7 \cdot 9 \cdot 17 \cdot 47}$$

$$P(S) = \frac{209}{50\ 337}$$

The probability that all cards in an 8-card hand are the same colour is $\frac{209}{50337}$, or about 0.00415 or 0.415%.

c) Let *F* represent a hand containing 4 face cards and 4 other cards, and let *O* represent all 8-card hands. A standard deck of cards contains 12 face cards and 40 other cards. There are ${}_{12}C_4$ ways to choose 4 face cards, and ${}_{40}C_4$ ways to choose 4 other cards. Therefore, the number of ways to have 4 face cards and 4 other cards in a hand is ${}_{12}C_4 \cdot {}_{40}C_4$.

$$n(F) = {}_{12}C_4 \cdot {}_{40}C_4$$

$$n(F) = \frac{12!}{(12-4)! \cdot 4!} \cdot \frac{40!}{(40-4)! \cdot 4!}$$

$$n(F) = \frac{12!}{8! \cdot 4!} \cdot \frac{40!}{36! \cdot 4!}$$

$$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36!}{36! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} \cdot \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2}$$

$$n(F) = \left(\frac{12}{4 \cdot 3} \cdot \frac{10}{2} \cdot 9 \cdot 11\right) \cdot \left(\frac{40}{4} \cdot \frac{39}{3} \cdot \frac{38}{2} \cdot 37\right)$$

$$n(F) = 5 \cdot 9 \cdot 10 \cdot 11 \cdot 13 \cdot 19 \cdot 37$$

Now determine the probability.

$$P(F) = \frac{n(F)}{n(O)}$$

$$P(F) = \frac{5 \cdot 9 \cdot 10 \cdot 11 \cdot 13 \cdot 19 \cdot 37}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(F) = \frac{5 \cdot 10 \cdot 11 \cdot 19 \cdot 37}{7 \cdot 17 \cdot 23 \cdot 47 \cdot 5 \cdot 10}$$

$$P(F) = \frac{11 \cdot 19 \cdot 37}{7 \cdot 17 \cdot 23 \cdot 47}$$

$$P(F) = \frac{7733}{128\ 639}$$

The probability that an 8-card hand contains 4 face cards and 4 other cards is $\frac{7733}{128639}$, or about 0.0601 or 6.01%.

15. Let *A* represent a 3-letter word, using the letters in the word "CABINET", that contains two vowels and one consonant. Let *O* represent all 3-letter words that use the letters in the word "CABINET". The word "CABINET" contains 3 vowels and 4 consonants. Therefore, there are ${}_3C_2 \cdot {}_4C_1$ ways to arrange 2 vowels and 1 consonant from 3 vowels and 4 consonants.

$$n(A) = {}_3C_2 \cdot {}_4C_1$$
$$n(A) = \frac{3!}{(3-2)! \cdot 2!} \cdot \frac{4!}{(4-1)! \cdot 1!}$$
$$n(A) = \frac{3!}{1! \cdot 2!} \cdot \frac{4!}{3! \cdot 1!}$$
$$n(A) = \frac{3 \cdot 2!}{2!} \cdot \frac{4 \cdot 3!}{3!}$$
$$n(A) = 3 \cdot 4$$
$$n(A) = 12$$

The number of possible arrangements of 3 letters from 7 letters is ${}_7C_3$.

$$n(O) = {}_7C_3$$
$$n(O) = \frac{7!}{(7-3)! \cdot 3!}$$
$$n(O) = \frac{7!}{4! \cdot 3!}$$
$$n(O) = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1}$$
$$n(O) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2}$$
$$n(O) = \frac{210}{6}$$
$$n(O) = 35$$

Now determine the probability.

$$P(A) = \frac{n(\text{A})}{n(O)}$$
$$P(A) = \frac{12}{35}$$

The probability the tiles will consist of 2 vowels and 1 consonant is $\frac{12}{35}$, or about 0.343 or 34.3%.

16. Let *D* represent an arrangement of the eight dogs such that the bearded collie and the sheltie are together. Let *O* represent all arrangements.
The number of ways to place the bearded collie and the sheltie together is 7 · 2!. The number of ways to place the other 6 dogs is 6!. Therefore, the total number of ways to seat all the dogs, with the collie and sheltie together, is 7 · 2! · 6!, or 2 · 7!.
There are 8! ways to arrange eight dogs.

$$P(D) = \frac{n(D)}{n(O)}$$
$$P(D) = \frac{2 \cdot 7!}{8!}$$
$$P(D) = \frac{2 \cdot 7!}{8 \cdot 7!}$$
$$P(D) = \frac{2}{8}$$
$$P(D) = \frac{1}{4}$$

The probability that the bearded collie and the sheltie will be next to each other is $\frac{1}{4}$, 0.25 or 25%.

17. Let *A* represent Maggie and Tanya in the starting lineup.

$$n(A) = {}_6C_2 \cdot {}_4C_1$$
$$n(A) = \frac{6!}{(6-2)! \cdot 2!} \cdot \frac{4!}{(4-1)! \cdot 1!}$$
$$n(A) = \frac{6!}{4! \cdot 2!} \cdot \frac{4!}{3! \cdot 1!}$$
$$n(A) = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2} \cdot \frac{4 \cdot 3!}{3!}$$
$$n(A) = \frac{6 \cdot 5}{2} \cdot 4$$
$$n(A) = 60$$

The number of ways in which Maggie and Tanya can be in the starting lineup is 60.

$$n(O) = {}_7C_3 \cdot {}_5C_2$$
$$n(O) = \frac{7!}{(7-3)! \cdot 3!} \cdot \frac{5!}{(5-2)! \cdot 2!}$$
$$n(O) = \frac{7!}{4! \cdot 3!} \cdot \frac{5!}{3! \cdot 2!}$$
$$n(O) = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2} \cdot \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2}$$
$$n(O) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} \cdot \frac{5 \cdot 4}{2}$$
$$n(O) = 350$$

The total number of possible starting lineups is 350.

$$P(A) = \frac{n(\text{A})}{n(O)}$$
$$P(A) = \frac{60}{350}$$
$$P(A) = \frac{6}{35}$$

The probability both Maggie and Tanya will be in the starting lineup is $\frac{6}{35}$, or about 0.171 or 17.1%.

18. e.g., I would use permutations in a problem where the order of the items was important, and use combinations in a problem where order was not important. Permutations: Determine the probability that two items are next to each other in a lineup of seven different items that has been placed in a random order. Combinations: Determine the probability that, given eight books, four of which are about math, if I choose five of the eight books, I choose three math books.

19. Let *A* represent a route that passes the pool, and let *O* represent all routes. The route Tuyet takes consists of 5 'rights' and 4 'downs'. Therefore, the total number of possible routes is the total number of moves (9!), dividing out repetition (5! · 4!)

$$n(O) = \frac{9!}{5! \cdot 4!}$$
$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2}$$
$$n(O) = \frac{3024}{24}$$
$$n(O) = 126$$

To pass by the pool, Tuyet must first take 3 'rights' and 2 'downs', then must take 2 'rights' and 2 'downs' in any order. Again, repetition must be divided out.

$$n(A) = \frac{5!}{3! \cdot 2!} \cdot \frac{4!}{2! \cdot 2!}$$
$$n(A) = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2 \cdot 1}$$
$$n(A) = \frac{5 \cdot 4}{2} \cdot \frac{4 \cdot 3}{2}$$
$$n(A) = 5 \cdot 2 \cdot 2 \cdot 3$$
$$n(A) = 60$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$
$$P(A) = \frac{60}{126}$$
$$P(A) = \frac{10}{21}$$

The probability that Tuyet passes the pool is $\frac{10}{21}$, or 47.6%.

20. I know from chapter 4 that the formula to use is $1 - \frac{{}_{365}P_n}{365^n}$, where n is the number of people. I tried a large number, because I thought that there would need to be a lot of people for the percentage to be as high as 80%. I tried *n* = 100.

$1 - \frac{{}_{365}P_{100}}{365^{100}}$ The calculator gave an error message. This number was too high. I tried *n* = 30.

$1 - \frac{{}_{365}P_{30}}{365^{30}} = 0.706...$. This answer was about 71%, so I thought n would be a little more. I tried n = 35.

$1 - \frac{{}_{365}P_{35}}{365^{35}} = 0.814...$ This is a little over 80%. There would need to be 35 people in a room for the probability that two of them have the same birthday is 80%.

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Let us now follow in Pascal's footsteps and analyze correctly the chances of winning in these two games.
Single die Rolling a single die once leads to precisely one of 6 possible outcomes: Exactly one of the numbers 1, 2, 3, 4, 5, or 6 will be on top. The die is called fair, if each of these outcomes is equally likely. Players of dice games usually assume they are using fair dice, so I will assume this too. If I roll a die 4 times, then the total number of all possible outcomes is 6 · 6 · 6 · 6 = 1296
Out of these there are 5 · 5 · 5 · 5 = 625 outcomes with **no** 6 in them. Thus, if I bet on getting at least one 6 when rolling a die 4 times, there are 625 possibilities of losing, and 1296 – 625 = 671 possibilities of winning. So, my chances of winning are higher than my chances of losing.
Two dice Let us now turn to the two-dice game. Rolling two dice once leads to one of 36 possible outcomes, namely all possible outcomes of rolling die number 1 combined with all possible outcomes of rolling die number two. Thus, if we roll two dice 24 times, then the total number of possible outcomes is 36 · 36 · … · 36 (36 multiplied with itself 24 times), which is approximately $2.245\ldots \cdot 10^{37}$. Out of these there are 35 · 35 · … · 35 (35 multiplied with itself 24 times), which is approximately $1.141\ldots \cdot 10^{37}$ outcomes with **no** double 6. Thus, if I gamble on getting at least one double 6 when rolling two dice 24 times, there are approximately $1.141\ldots \cdot 10^{37}$ possibilities of losing, and $1.103\ldots \cdot 10^{37}$ possibilities of winning. This means that the chances of winning with this game are lower than the chances of losing—as the Chevalier De Mere learned the hard way.

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1. a) Outcome table

Dice roll	Product	Sum	Who wins?
(1,1,1)	1	3	Erik
(1,1,2)	2	4	Erik
(1,1,3)	3	5	Erik
(1,1,4)	4	6	Erik
(1,2,1)	2	4	Erik
(1,2,2)	4	5	Erik
(1,2,3)	6	6	Ethan
(1,2,4)	8	7	Ethan
(1,3,1)	3	5	Erik
(1,3,2)	6	6	Ethan
(1,3,3)	9	7	Ethan