Chapter 5: Probability

Lesson 5.1: Exploring Probability, page 303

1. e.g., Reverse the rules for Players 1 and 2 on each turn.
2. a) Outcome Table:

MATT

PAT	H	T
H	HH	HT
T	HT	TT

$P(\text{Matt wins}) = \frac{2}{4}$ $P(\text{Pat wins}) = \frac{2}{4}$

$P(\text{Matt wins}) = \frac{1}{2}$ $P(\text{Pat wins}) = \frac{1}{2}$

Each player has an equal chance of winning, so the game is fair.
b) Sample Space:

H	H	H
H	H	T
H	T	H
H	T	T

T	H	H
T	H	T
T	T	H
T	T	T

$P(\text{Treena wins}) = \frac{1}{8}$ $P(\text{Leena wins}) = \frac{1}{8}$ $P(\text{Gina wins}) = \frac{6}{8}$

This game is not fair. Gina has a 6 in 8 chance of winning.
c) Outcome Table:

Die 1

Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{Ann wins}) = \frac{15}{36}$ $P(\text{Dan wins}) = \frac{15}{36}$

$P(\text{Ann wins}) = \frac{5}{12}$ $P(\text{Dan wins}) = \frac{5}{12}$

Each player has an equal chance of winning, so the game is fair.
3. No. e.g., A certain chance is 100%. 120% > 100%.
4. Sample Space

Product	**1**	**2**	**3**	**4**
1	**1**	**2**	**3**	**4**
2	**2**	**4**	**6**	**8**
3	**3**	**6**	**9**	**12**
4	**4**	**8**	**12**	**16**

Sum	**1**	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Point Scoring

	1	2	3	4
1	Player 1	Player 1	Player 1	Player 1
2	Player 1	TIE	Player 2	Player 2
3	Player 1	Player 2	Player 2	Player 2
4	Player 1	Player 2	Player 2	Player 2

The probability of Player 1 winning is now $\frac{7}{16}$, or 43.75%. The probability of Player 2 winning is now $\frac{8}{16}$, or 50%. This is not a fair game. Player 2 has more opportunities to win.

Lesson 5.2: Probability and Odds, page 310

1. a) The odds against Marcia passing the driver's test on the first try are 3 : 5.
b) Let *A* represent Marcia passing her driver's test on the first try.

$P(A) = \frac{5}{5+3}$

$P(A) = \frac{5}{8}$

The probability Marcia will pass her driver's test on the first try is $\frac{5}{8}$, or 0.625.

2. a) The probability the coin is a loonie is $\frac{3}{10}$, or 0.3.

b) The probability the coin is not a loonie is $\frac{7}{10}$, or 0.7.

So, the odds against the coin being a loonie are 7 : 3.
3. a) Let *R* represent a red card being drawn.

$P(R) = \frac{26}{52}$

$P(R) = \frac{1}{2}$

$P(R) = 0.5$

The probability that Lily draws a red card is 0.5.
b) Let R' represent a card that is not red being drawn.

$P(B) = \frac{(52-26)}{52}$

$P(B) = \frac{26}{52}$

So, the odds in favour of the card being red are 26 : 26, or 1 : 1.
c) Let *S* represent a spade being drawn.

$P(S) = \frac{13}{52}$ $P(S') = \frac{4-1}{4}$

$P(S) = \frac{1}{4}$ $P(S') = \frac{3}{4}$

The odds against the card being a spade are 3 : 1.

d) Let F represent a face card being drawn.

$P(F)=\frac{12}{52}$

$P(F)=\frac{3}{13}$

The probability the card drawn is a face card is $\frac{3}{13}$, or about 0.231

4. a) Let A represent apple juice being available at the grocery store.

$P(A)=\frac{2}{5}$

$P(A')=\frac{(5-2)}{5}$

$P(A')=\frac{3}{5}$

The odds in favour of apple juice being available are 2 : 3.

b) The odds against apple juice being available are 3 : 2.

5. Let S represent two students in Mario's class sharing a birthday.

$P(S)=\frac{7}{7+3}$

$P(S)=\frac{7}{10}$

$P(S)=0.7$

The probability two students share a birthday is 0.7.

6. Let C represent Jamia climbing to the top.

a) $P(C)=\frac{12}{24}$

$P(C)=\frac{1}{2}$

The probability Jamia will make it to the top is $\frac{1}{2}$, or 0.5.

b) $P(C')=\frac{24-12}{24}$

$P(C')=\frac{12}{24}$

$P(C')=\frac{1}{2}$

The odds against Jamia making it to the top are 12 : 12, or 1 : 1.

c) e.g., The odds against and the odds in favour are both 1 : 1.

7. If the probability of snow is 60%, then the probability that of no snow is 40%. Therefore, the odds against snow are 40 : 60, or 2 : 3.

8. If Allan has an 8% chance of having red-green colourblindness, then he has a 92% chance of not having red-green colourblindness. Therefore, the odds in favour of Allan having red-green colourblindness are 8 : 92, or 2 : 23

9. If Katherine had scored 4 times out of 20, then she had not scored 16 times out of 20. Therefore, the odds in favour are 4:16, or 1:4. This can be written as 1 to 4. The probability of her scoring is 4 in 20, or 1 : 5. Therefore, Katherine is not correct.

10. a) If Jason has scored 5 times in his last 10 penalty shots, then he didn't score 5 times. Therefore, the odds in favour of scoring are 5 : 5, or 1 : 1.

b) Gilles blocked 8 of the last 10 penalty shots, and let 2 shots in. Therefore, the odds in favour of Jason scoring on Gilles are 2 : 8, or 1 : 4.

c) e.g., Jason's data reflects his record against all goalies, not just Gilles. Gilles' data suggests that he is better than average at blocking penalty shots.

11. Let S represent a person between the ages of 18 and 35 who uses social networking.

$P(S)=\frac{31}{(31+19)}$

$P(S)=\frac{31}{50}$

$P(S)=0.62$

The probability that a person between the ages of 18 and 35 uses a social networking site is 0.62.

12. Let W represent a win, let L represent a loss, and let T represent a tie.

$P(W)=\frac{3}{3+2}$ $\quad P(L)=\frac{1}{1+4}$ $\quad P(T)=\frac{1}{1+4}$

$P(W)=\frac{3}{5}$ $\quad P(L)=\frac{1}{5}$ $\quad P(T)=\frac{1}{5}$

The probability of a win is 3 in 5 (60%), the probability of a loss is 1 in 5 (20%), and the probability of a tie is 1 in 5 (20%). The probabilities add up to 100%.

13. a) 65% of the audience are male, and 35% of the audience are not male. Therefore, the odds in favour of someone watching *Show Trial* being male are 65 : 35, or 13 : 7.

b) 40% of the audience are over 45, and 60% of the audience are under 45, Therefore, the odds in favour of someone watching *Show Trial* being over 45 are 40 : 60, or 2 : 3.

14. a) 70% of the people vaccinated did not get sick, so 30% of the people vaccinated did get sick. So, the odds against getting sick if you are vaccinated are 70 : 30, or 7 : 3.

b) 42% of the people who were not vaccinated got sick, so 58% of the people who were not vaccinated did not get sick. Therefore, the odds against getting sick if you are not vaccinated are 58 : 42, or 29 : 21.

c) The first ratio, 7 : 3, can be written as 49 : 21 by multiplying both numbers by 7. So the odds of getting sick are 49 : 21 and the odds against getting sick are 29 : 21.

d) e.g., Yes, because your chances of not getting sick are much better.

15. a) The odds in favour of getting 7 points, by scoring a touchdown, and the convert, are 5 : 7. The odds in favour of getting 3 points by kicking a field goal, are 5 : 1 or 35 : 7.
b) e.g., The odds in favour of getting 3 points are much better, so the coach should choose the field goal.
16. a) The odds in favour of Eduard winning are 45 : (100 – 45).This is equal to 45 : 55, or 9 : 11. The odds in favour of Julie winning are 35 : (100 – 35). This is equal to 35 : 65, or 7 : 13. The odds in favour of Bill winning are 20 : (100 – 20). This is equal to 20 : 80, or 1 : 4.
b) If Bill's 20% support goes to Julie, then her support will now be 55%, and the odds in favour of Julie winning will be the same as the odds against Edie winning. So, the odds in favour of Julie winning are 11 : 9.
17. a) e.g., The odds in favour of passing are 11 : 9, so the probability of passing is slightly more than 50%.
For every 20 people who take the exam, 11 pass.

$$P(\text{pass}) = \frac{11}{20}$$
$$P(\text{pass}) = 0.55$$
$$P(\text{pass}) = 55\%$$

The practice exams cost \$65. The cost to rewrite the exam is \$235.
Yes. If he pays the \$65, he can reduce the 45% chance of having to pay the additional \$235.
b) No. e.g., With odds of 17 : 4, Grant has about a 19% chance of failing even without the practice exams, so he should probably not buy them.
Yes. e.g., With odds of 3 : 7, his chance of failing is 70% without the practice exams, so he should buy them.
18. a) e.g., If the odds for an event are *m* : *n*, then

$P(A) = \frac{m}{m+n}$ and $P(A') = \frac{n}{m+n}$, so

$P(A') : P(A) = \frac{n}{m+n} : \frac{m}{m+n}$. This ratio is equal to *n* : *m*.

b) e.g., The probability of the event happening is $\frac{a}{a+b}$. If the odds in favour of rain tomorrow are 2 : 3, then the probability is $\frac{2}{2+3} = \frac{2}{5}$, or 40%.
c) e.g., The odds against the event happening are *c* – *a* : *a*. If the probability of winning the lottery is $\frac{1}{1\,000\,000}$, the odds against are 999 999 : 1.

19. e.g., I prefer using probability because if I express the probability as a percent, it tells me how many times out of a hundred I could expect the event to occur. e.g., I prefer using odds because it compares the chances for and against the event occurring.
20. a) Let *A* represent a child between the ages of 6 and 18 who will need corrective lenses. Let *R* represent a girl between the ages of 6 and 18 who will need corrective lenses.

$$P(A) = 25.4\% \qquad P(R) = \frac{141}{141+100}$$
$$P(A) = \frac{25.4}{100} \qquad P(R) = \frac{141}{241}$$
$$P(A) = \frac{127}{500}$$
$$P(A \cap R) = \frac{127}{500} \cdot \frac{141}{241}$$
$$P(A \cap R) = \frac{17\,907}{120\,500}$$

The probability that a randomly selected girl between the ages of 6 and 18 will need corrective lenses is $\frac{17\,907}{120\,500}$, or about 0.149.
b) Let *A* represent a child between the ages of 6 and 18 who will need corrective lenses. Let *B* represent a boy between the ages of 6 and 18 who will need corrective lenses.

$$P(A) = 25.4\% \qquad P(B) = \frac{100}{141+100}$$
$$P(A) = \frac{25.4}{100} \qquad P(B) = \frac{100}{241}$$
$$P(A) = \frac{127}{500}$$
$$P(A \cap B) = \left(\frac{127}{500}\right)\left(\frac{100}{241}\right)$$
$$P(A \cap B) = \frac{12\,700}{120\,500}$$
$$P(A \cap B) = \frac{127}{1205}$$

The probability that a randomly selected 18-year-old boy will need corrective lenses is $\frac{127}{1205}$. So, the odds in favour of a randomly selected 18-year-old boy will need corrective lenses are 127 : (1205 – 127). This is equal to 127 : 1078.