c) There is only one time where your favourite song from each of the 5 CDs will be played.
Probability (P):

$$P = \frac{1}{13\,019\,909} \times 100\%$$

$$P = 0.000008\%$$

There is about a 0.000008% or 1 in 13 019 909 chance that your favourite song from each of the 5 CDs will be played.

21. ${}_nC_3 + {}_nC_2 + {}_nC_1$

$$= \frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!} + \frac{n!}{1!(n-1)!}$$

$$= \frac{n!}{6(n-3)!} + \frac{n!}{2(n-2)!} + \frac{n!}{(n-1)!}$$

$$= \frac{n!}{6(n-3)!} + \frac{3n(n-1)(n-3)!}{6(n-3)!} + \frac{6n(n-3)!}{6(n-3)!}$$

$$= \frac{n! + 3n(n-1)(n-3)! + 6n(n-3)!}{6(n-3)!}$$

$$= \frac{(n-3)!\left[n(n-1)(n-2) + 3n(n-1) + 6n\right]}{6(n-3)!}$$

$$= \frac{n(n-1)(n-2) + 3n(n-1) + 6n}{6}$$

$$= \frac{n(n^2 - 2n - n + 2 + 3n - 3 + 6)}{6}$$

$$= \frac{n(n^2 + 5)}{6}$$

$$= \frac{n^3 + 5n}{6}$$

22. e.g.,

LS	RS
${}_{n+1}C_r$	${}_nC_r + {}_nC_{r-1}$
$\frac{(n+1)!}{r!(n+1-r)!}$	$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\left[n-(r-1)\right]!}$
	$\frac{\left[n-(r-1)\right]n!}{r!\left[n-(r-1)\right]!} + \frac{r(n!)}{r!\left[n-(r-1)\right]!}$
	$\frac{(n+1-r)n! + r(n!)}{r!(n+1-r)!}$
	$\frac{n!(n+1-r+r)}{r!(n+1-r)!}$
	$\frac{n!(n+1)}{r!(n+1-r)!}$
	$\frac{(n+1)!}{r!(n+1-r)!}$

LS = RS

Therefore, ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$.

Lesson 4.7: Solving Counting Problems, page 288

1. a) This situation involves combinations because the order of the 3 toppings on the pizza does not matter.
b) This situation involves permutations because the three spots for the candidates who are selected are all different so order matters.
c) This situation involves permutations because for a group of 3 numbers, there are different ways to roll those three numbers because of the different colours of the dice.
d) This situation involves combinations because the 5 children who are selected are all in the same position. No information is stated in the question about positions the children may play, so I can only assume that they are not playing in specific positions.

2. e.g., Situation A involves combinations and situation B involves permutations. For situation A, order does not matter since the 3 people who are selected will all be considered equals. For situation B, this is not the case. Each of the 3 people who are selected will have a different position with a different amount of power and different roles.

3. a) ${}_3C_3 = \frac{3!}{3! \cdot 0!}$

$${}_3C_3 = 1$$

There is 1 way that Maddy can bid on 3 items if she bids on only her 3 favourite items.

b) ${}_8C_3 = \frac{8!}{3! \cdot 5!}$

$${}_8C_3 = 56$$

There are 56 ways that Maddy can bid on 3 items if she bids on any 3 of the 8 items.

4. $\left({}_{13}C_1\right)^4 = \left(\frac{13!}{1! \cdot 12!}\right)^4$

$$\left({}_{13}C_1\right)^4 = (13)^4$$

$$\left({}_{13}C_1\right)^4 = 28\,561$$

There are 28 561 different four-card hands with one card from each suit.

5. a) ${}_{200}P_5 = \frac{200!}{195!}$

$${}_{200}P_5 = \frac{200 \cdot 199 \cdot 198 \cdot 197 \cdot 196 \cdot 195!}{195!}$$

$${}_{200}P_5 = 200 \cdot 199 \cdot 198 \cdot 197 \cdot 196$$

$${}_{200}P_5 = 304\ 278\ 004\ 800$$

There are 304 278 004 800 ways that the top five cash prizes can be awarded if each ticket is not replaced when drawn.

b) $(200)^5 = 320\ 000\ 000\ 000$
There are 320 000 000 000 ways that the top five cash prizes can be awarded if each ticket is replaced when drawn.

6. $_2C_1 \cdot {_6C_2} \cdot {_4C_2}$

$$= \left(\frac{2!}{1!(2-1)!}\right)\cdot\left(\frac{6!}{2!(6-2)!}\right)\cdot\left(\frac{4!}{2!(4-2)!}\right)$$
$$= 2\cdot\left(\frac{6\cdot 5\cdot 4!}{2!\cdot 4!}\right)\cdot\left(\frac{4\cdot 3\cdot 2!}{2!\cdot 2!}\right)$$
$$= 2\cdot\left(\frac{6\cdot 5}{2}\right)\cdot\left(\frac{4\cdot 3}{2}\right)$$
$$= 2\cdot 15\cdot 6$$
$$= 180$$

There are 180 ways that the 5 starting positions on the basketball team can be filled.

7. $$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 2\cdot 2\cdot 2\cdot 2}$$
$$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = 10\cdot 9\cdot 7\cdot 6\cdot 5\cdot 3\cdot 2\cdot 1$$
$$\frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!} = 113\,400$$

There are 113 400 ways that the five different pairs of identical teddy bears can be arranged.

8. Case 1: 3 flags are used

$$_5P_3 = \frac{5!}{(5-3)!}$$
$$_5P_3 = \frac{5!}{2!}$$
$$_5P_3 = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1}$$
$$_5P_3 = 5\cdot 4\cdot 3$$
$$_5P_3 = 60$$

Case 2: 4 flags are used

$$_5P_4 = \frac{5!}{(5-4)!}$$
$$_5P_4 = \frac{5!}{1!}$$
$$_5P_4 = 5!$$
$$_5P_4 = 120$$

Case 3: 5 flags are used
$_5P_5 = 5!$
$_5P_5 = 120$

Let S represent the number of different signals that can be sent using at least three of the flags:
$S = 60 + 120 + 120$
$S = 300$
There are 300 different signals that can be sent using at least three of the flags.

9. e.g., First make a table to show the number of ways the two cabin cruisers can be arranged next to each other.

	CC 1	**CC 2**
Arrangement 1	1	2
Arrangement 2	2	3
Arrangement 3	3	4
Arrangement 4	4	5
Arrangement 5	5	6
Arrangement 6	2	1
Arrangement 7	3	2
Arrangement 8	4	3
Arrangement 9	5	4
Arrangement 10	6	5

For each of these arrangements, the number of ways the six boats can dock is the number of ways that the other four boats can dock. Let D represent the total number of ways that the boats can dock:
$D = 10 \cdot 4!$
$D = 10 \cdot 24$
$D = 240$
There are 240 ways that the six boats can dock.

10. e.g., Each row of seats is different, and within a row, the seats are assumed to be different. Therefore, there are 10 different people being seated in 10 different spots. Let A represent the number of seating arrangements:
$A = 10!$
$A = 3\ 628\ 800$
There are 3 628 800 ways that the 10 players can sit in the van.

11. a) $\frac{5!}{2!} = 60$
There are 60 different arrangements that are possible for the letters if there are no conditions.
b) $3! = 6$
There are 6 different arrangements that are possible for the letters if each arrangement must start and end with an N.

12. e.g., When there is an even amount of numbers, half of them will be odd. In this case there are 100 possible numbers that each number can be. Therefore, $\frac{100}{2}$, or 50 of them are odd. Since I want each number to only be 1 of these 50 odd numbers, the number of sequences S is:
$S = 50 \cdot 50 \cdot 50$
$S = 125\ 000$
There are 125 000 completely odd sequences.

13. $\frac{11!}{5!\cdot 6!} = 462$ You can take 462 different routes.

14. e.g., Let's assign people to the 5-person car first. Let J represent the number of ways to assign the people to this car:

$J = {}_{16}C_5$

$J = \dfrac{16!}{5!11!}$

$J = 4368$

Now there are 16 – 5 or 11 people left to assign to the remaining two vehicles. Let's assign people to the 4-person car next. Let K represent the number of ways to assign the people to this car:

$K = {}_{11}C_4$

$K = \dfrac{11!}{4!7!}$

$K = 330$

Now there are 11 – 4 or 7 people left to assign to the remaining vehicle. There is only 1 way to assign these people to the 7-passenger van because all of them are going to be assigned to it. Now let T represent the total number of assignments:

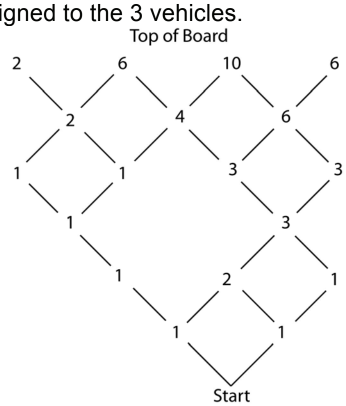
$T = J \cdot K \cdot 1$

$T = 4368 \cdot 330$

$T = 1\ 441\ 440$

There are 1 441 440 ways the 16 people can be assigned to the 3 vehicles.

15.



Number of Paths = 2 + 6 + 10 + 6

Number of Paths = 24

There are 24 paths that the red checker can follow.

16. Case 1: 0 hearts and 5 non-hearts: ${}_{13}C_0 \cdot {}_{39}C_5$

Case 2: 1 heart and 4 non-hearts: ${}_{13}C_1 \cdot {}_{39}C_4$

Case 3: 2 hearts and 3 non-hearts: ${}_{13}C_2 \cdot {}_{39}C_3$

Case 4: 3 hearts and 2 non-hearts: ${}_{13}C_3 \cdot {}_{39}C_2$

Let H represent the number of hands with at most 3 hearts:

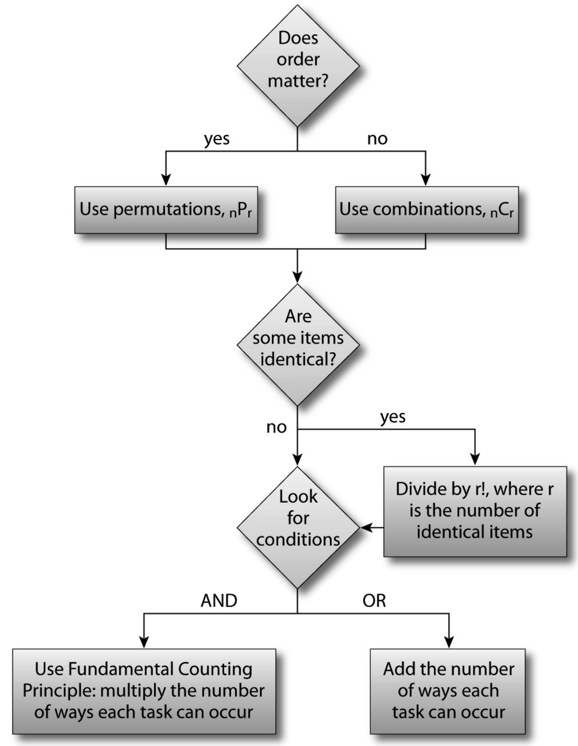
$H = {}_{13}C_0 \cdot {}_{39}C_5 + {}_{13}C_1 \cdot {}_{39}C_4 + {}_{13}C_2 \cdot {}_{39}C_3 + {}_{13}C_3 \cdot {}_{39}C_2$

$H = 1 \cdot 575\ 757 + 13 \cdot 82\ 251 + 78 \cdot 9139 + 286 \cdot 741$

$H = 2\ 569\ 788$

There are 2 569 788 different five-card hands that contain at most three hearts that can be dealt.

17. e.g.,



18. Number of Total Outcomes:

${}_{13}C_6 = \dfrac{13!}{6! \cdot 7!}$

${}_{13}C_6 = 1716$

Number of Outcomes Where 3 Boys and 3 Girls Can Go:

${}_6C_3 \cdot {}_7C_3 = \dfrac{6!}{3! \cdot 3!} \cdot \dfrac{7!}{3! \cdot 4!}$

${}_6C_3 \cdot {}_7C_3 = 700$

Probability (P):

$P = \dfrac{700}{1716} \times 100\%$

$P = 40.792...\%$

There is about a 40.8% chance that there will be three boys and three girls on the trip.

19. e.g., If I have an A as the first letter, there are 4 possibilities for the second letter: A, L, S, or K.

If A is the second letter:

4 possibilities for the third letter: A, L, S, or K

Each one of these has 3 possibilities for the fourth letter. 4(3) = 12

If the second letter is L, S, or K:

3 possibilities for the third letter: A and 2 of L, S, and K (depending on which letter is second)

The A's have 3 possibilities for the fourth letter, and the other two letters have 2 possibilities for the fourth letter. 3 + 2(2) = 7

Total for all three second letters that are L, S, or K: 7(3) = 21

Total if A is the first letter:
21 + 12 = 33
Therefore, if the first letter is A, there are 33 possible arrangements.
If the first letter is L, S, or K, there are three possibilities for the second letter: A, and 2 of L, S, and K (the ones that are not the first letter).
If A is the second letter:
3 possibilities for the third letter: A, and 2 of L, S, and K
The A has 3 possibilities for the fourth letter and the other two letters have 2. 3 + 2(2) = 7
If A is not the second letter:
2 possibilities for the third letter
The A has 2 possibilities for the fourth letter and the other letter has 1. 2 + 1 = 3
Total for both second letters that are not A:
3(2) = 6
Total for one of three times where first letter is L, S, or K:
7 + 6 = 13
Total when first letter is L, S, or K:
3(13) = 39
Total arrangements:
39 + 33 = 72
Therefore, 72 four-letter arrangements can be made using all of the letters in the word ALASKA.

20. If I have an O as the first letter, there are 4 possibilities for the second letter, each of which has 3 possibilities for the third letter. 4(3) = 12
Therefore, there are 12 possible arrangements when O is the first letter.
If the first letter is B, K, or S:
There are 3 possibilities for the second letter: O, and two of B, K, and S (the ones that are not the first letter). O has 3 possibilities for the third letter while the other 2 have 2. 3 + 2(2) = 7
Total if the first letter is B, K, or S:
3(7) = 21
Total arrangements:
21 + 12 = 33
Therefore, 33 three-letter arrangements can be made using all of the letters in the word BOOKS.

History Connection, page 290

A. Yes. Each number from 0 to 127 is assigned a different character or symbol on the keyboard. Since the numbers already have an established order, the characters and symbols assigned to these numbers do, as well.
B. Yes. Each number in ASCII (pronounced "askey") must be converted into a string of 0s and 1s to create the binary code, so order matters. Each 0 or 1 is associated with a position in the string. A different permutation of 0s and 1s represents a different number in the ASCII code system.
C. There are 128 numbers in ASCII that must be represented by a string of 0s and 1s. You need to determine the length of the string needed to create 128 different arrangements of 0s and 1s. You can begin by thinking about a string of length of 5.
A box diagram, □□□□□, can help you determine the number of ASCII numbers you can represent.
Within each box you can place a 0 or a 1. There are two choices for each box, since repetition of 0s and 1s is allowed. So for a string length of 5, there are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$ or 32 ASCII numbers that can be represented. Obviously, the string must be longer for 128 numbers. If n represents the string length, and 128 numbers must be represented, then $2^n = 128$. By trial and error, $n = 7$.
A binary string of length 7 is needed to represent each ASCII code.

Chapter Self-Test, page 291

1. a) Let N represent the number of different serial numbers:
$N = 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 3$
$N = 2\,028\,000$
There are 2 028 000 different serial numbers possible, if repetition of characters is allowed.
b) Let N represent the number of different serial numbers:
$N = 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 3$
$N = 1\,296\,000$
There are 1 296 000 different serial numbers possible, if no repetition is allowed.

2. Event A: Drawing a spade
Event B: Drawing a diamond
$n(A \cup B) = n(A) + n(B)$
$n(A \cup B) = 13 + 13$
$n(A \cup B) = 26$
Therefore, there are 26 ways to draw 1 card that is a spade or a diamond.

3. a) $n + 9 \geq 0$
$n \geq -9$
$(n + 10)(n + 9)!$ is defined for $n \geq -9$, where $n \in I$.
$(n + 10)(n + 9)! = (n + 10)[(n + 9)(n + 8)\ldots(3)(2)(1)]$
$(n + 10)(n + 9)! = (n + 10)(n + 9)(n + 8)\ldots(3)(2)(1)$
$(n + 10)(n + 9)! = (n + 10)!$
b) $n - 2 \geq 0$ AND $n \geq 0$
$n \geq 2$
$\frac{(n-2)!}{n!}$ is defined for $n \geq 2$, where $n \in I$.

$$\frac{(n-2)!}{n!} = \frac{(n-2)(n-3)\ldots(3)(2)(1)}{n(n-1)(n-2)(n-3)\ldots(3)(2)(1)}$$

$$\frac{(n-2)!}{n!} = \frac{1}{n(n-1)}$$

$$\frac{(n-2)!}{n!} = \frac{1}{n^2 - n}$$