From the table, I see that 14 different arrangements might be made.
b) e.g., From the table above, 1 out of the 14 arrangements, from left to right, would be red, white, white, red. Therefore, there is a 1 in 14 chance that the arrangement, from left to right, would be red, white, white, red.

Applying Problem-Solving Strategies, page 270

A. 4044 paths

B. 2(924) + 2(2508) + 2(3498) = 13 860 paths

C. Yes. There are 2(3936), or 7872, paths that lead to no money at all, but 17 904 paths that result in the contestant winning something. The contestant has a better chance of winning something than nothing, so it's a fair game from the contestant's point of view.

Lesson 4.5: Exploring Combinations, page 272

1. a) Let W represent the number of ways:
$W = 3!$
$W = 3 \cdot 2 \cdot 1$
$W = 6$
There are 6 different ways that Brian, Rachelle, and Linh can be chosen for these jobs.

b)

Canned Goods	Dry Goods	Fruits and Vegetables
Brian	Rachelle	Linh
Brian	Linh	Rachelle
Rachelle	Brian	Linh
Rachelle	Linh	Brian
Linh	Rachelle	Brian
Linh	Brian	Rachelle

c) Since all 3 volunteers are being used to help unload the vehicles, there is only one way they can be chosen for this job.
d) Part a) and b) involve permutations and part c) involved combinations. I know because in part a) and b), the order in which the volunteers were selected for the jobs mattered. In part c) the order did not since all the volunteers were being selected to do the same job.

2. e.g., The main difference is that for the permutations, the order of the 4 objects matters, and for the combinations, it does not. For the permutations, you could have multiple arrangements with the same objects since there is more than one way to order a group of four different objects. This is not possible for combinations since you just need one arrangement for each group of 4, regardless of the order.

3. Let C represent the number of dance committees possible:
$C = {}_{10}C_4$
$C = 210$
There are 210 ways that 4 of the members can be chosen to serve on the dance committee.

4. Let C represent the number of combinations:
$C = {}_{12}C_3$
$C = 220$
There are 220 ways 3 of the 12 dogs can be selected to appear.

Lesson 4.6: Combinations, page 280

1. a)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	vanilla
strawberry	chocolate
strawberry	butterscotch
chocolate	vanilla
chocolate	strawberry
chocolate	butterscotch
butterscotch	vanilla
butterscotch	strawberry
butterscotch	chocolate

b)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	chocolate
strawberry	butterscotch
chocolate	butterscotch

c) The number of two-flavour permutations is double the number of two-flavour combinations because each two-flavour combination can be written in two different ways.

2. a) Let C represent the number of committees:

$C = {}_5C_3$

$C = \frac{5!}{3!(5-3)!}$

$C = \frac{5!}{3!\cdot 2!}$

$C = \frac{5\cdot 4\cdot 3!}{3!\cdot 2\cdot 1}$

$C = \frac{5\cdot 4}{2\cdot 1}$

$C = 5\cdot 2$

$C = 10$ There are 10 possible committees.

b) Let C represent the number of committees:

$C = {}_5C_2$

$C = \frac{5!}{2!(5-2)!}$

$C = \frac{5!}{2!\cdot 3!}$

$C = \frac{5\cdot 4\cdot 3!}{2\cdot 1\cdot 3!}$

$C = \frac{5\cdot 4}{2\cdot 1}$

$C = 5\cdot 2$

$C = 10$ There are 10 possible committees.

c) e.g., My answers for parts a) and b) are the same. This occurred because the sum of 2 and 3 is 5.

3. Let T represent the number of possible teams:

$T = {}_{12}C_6$

$T = \frac{12!}{6!(12-6)!}$

$T = \frac{12!}{6!6!}$

$T = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!}$

$T = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$

$T = 2\cdot 11\cdot 5\cdot 3\cdot 2\cdot \frac{7}{5}$

$T = 924$

There are 924 ways 6 people can be selected from a group of 12 to form a dodge-ball team.

4. a) ${}_5C_3 = \frac{5!}{3!(5-3)!}$

${}_5C_3 = \frac{5!}{3!2!}$

${}_5C_3 = \frac{5\cdot 4\cdot 3!}{3!2\cdot 1}$

${}_5C_3 = \frac{5\cdot 4}{2\cdot 1}$

${}_5C_3 = 5\cdot 2$

${}_5C_3 = 10$

b) ${}_9C_8 = \frac{9!}{8!(9-8)!}$

${}_9C_8 = \frac{9!}{8!1!}$

${}_9C_8 = \frac{9\cdot 8!}{8!1}$

${}_9C_8 = \frac{9}{1}$

${}_9C_8 = 9$

c) ${}_6C_4 = \frac{6!}{4!(6-4)!}$

${}_6C_4 = \frac{6!}{4!2!}$

${}_6C_4 = \frac{6\cdot 5\cdot 4!}{4!2\cdot 1}$

${}_6C_4 = \frac{6\cdot 5}{2\cdot 1}$

${}_6C_4 = 3\cdot 5$

${}_6C_4 = 15$

d) ${}_{10}C_0 = \frac{10!}{0!(10-0)!}$

${}_{10}C_0 = \frac{10!}{0!10!}$

${}_{10}C_0 = \frac{1}{1}$

${}_{10}C_0 = 1$

e) ${}_{12}C_6 = \frac{12!}{6!(12-6)!}$

${}_{12}C_6 = \frac{12!}{6!6!}$

${}_{12}C_6 = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!}$

${}_{12}C_6 = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$

${}_{12}C_6 = 2\cdot 11\cdot 5\cdot 3\cdot 2\cdot \frac{7}{5}$

${}_{12}C_6 = 924$

f) ${}_8C_1 = \frac{8!}{1!(8-1)!}$

${}_8C_1 = \frac{8!}{1!7!}$

${}_8C_1 = \frac{8\cdot 7!}{1\cdot 7!}$

${}_8C_1 = \frac{8}{1}$

${}_8C_1 = 8$

5. Let C represent the number of combinations:

$C = {}_{10}C_6$

$C = \dfrac{10!}{6!(10-6)!}$

$C = \dfrac{10!}{6!4!}$

$C = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!4 \cdot 3 \cdot 2 \cdot 1}$

$C = \dfrac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1}$

$C = 5 \cdot 3 \cdot 2 \cdot 7$

$C = 210$

There are 210 ways 6 players can be chosen to start a volleyball game from a team of 10.

6. Let C represent the number of combinations:

$C = {}_{55}C_5$

$C = \dfrac{55!}{5!(55-5)!}$

$C = \dfrac{55!}{5! \cdot 50!}$

$C = \dfrac{55 \cdot 54 \cdot 53 \cdot 52 \cdot 51 \cdot 50!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 50!}$

$C = \dfrac{55 \cdot 54 \cdot 53 \cdot 52 \cdot 51}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$C = 11 \cdot 27 \cdot 53 \cdot 13 \cdot 17$

$C = 3\ 478\ 761$

There are 3 478 761 different combinations of hip-hop songs you can download for free.

7. Let H represent the number of hands:

$H = {}_{52}C_8$

$H = \dfrac{52!}{8!(52-8)!}$

$H = \dfrac{52!}{8! \cdot 44!}$

$H = \dfrac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44!}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 44!}$

$H = \dfrac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$H = 13 \cdot 17 \cdot 10 \cdot 7 \cdot 47 \cdot 45 \cdot 23$

$H = 752538150$

There are 752 538 150 different 8-card hands that can be dealt.

8. a) The problem involves combinations e.g., because it does not state that the order of the starting line matters.

b) Let L represent the number of different lineups, $n = 14$ and $r = 8$ because Connie must be the pitcher of the starting lineup.

$L = {}_nC_r$

$L = {}_{14}C_8$

$L = \dfrac{14!}{8!(14-8)!}$

$L = \dfrac{14!}{8!6!}$

$L = \dfrac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8!6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$L = \dfrac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$L = 7 \cdot 13 \cdot 11 \cdot 3$

$L = 3003$

There are 3003 ways that the coach can choose his starting lineup of 9 players, if Connie must be the pitcher.

9. a) Yes, I do agree.

e.g.,

LS	RS
${}_6C_2$	${}_6C_4$
$\dfrac{6!}{2!(6-2)!}$	$\dfrac{6!}{4!(6-4)!}$
$\dfrac{6!}{2! \cdot 4!}$	$\dfrac{6!}{4! \cdot 2!}$
$\dfrac{6 \cdot 5 \cdot 4!}{2 \cdot 1 \cdot 4!}$	$\dfrac{6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1}$
$\dfrac{6 \cdot 5}{2 \cdot 1}$	$\dfrac{6 \cdot 5}{2 \cdot 1}$
$3 \cdot 5$	$3 \cdot 5$
15	15

LS = RS

b) e.g., Some other cases with the same relationship as part a) are ${}_8C_1 = {}_8C_7$, ${}_6C_0 = {}_6C_6$, and ${}_{12}C_7 = {}_{12}C_5$. I notice that if you have two combinations with the same n, and the sum of the r's for those combinations is equal to n, then the value of the combinations will be the same.

c) e.g., $\dbinom{n}{r} = \dbinom{n}{n-r}$

10.

Let T represent the number of combinations for the teachers:

$T = {}_5C_2$

$T = \frac{5!}{2!(5-2)!}$

$T = \frac{5!}{2!3!}$

$T = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!}$

$T = \frac{5 \cdot 4}{2 \cdot 1}$

$T = 5 \cdot 2$

$T = 10$

Let S represent the number of combinations for the students:

$S = {}_8C_3$

$S = \frac{8!}{3!(8-3)!}$

$S = \frac{8!}{3!5!}$

$S = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!}$

$S = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$

$S = 4 \cdot 2 \cdot 7$

$S = 56$

Let G represent the number of graduation committees:

$G = S \cdot T$

$G = 56 \cdot 10$

$G = 560$

There are 560 graduation committees that the principal has to choose from.

11. a) Let C represent the number of committees:

$C = {}_{10}C_5$

$C = \frac{10!}{5!(10-5)!}$

$C = \frac{10!}{5!5!}$

$C = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!}$

$C = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$C = 2 \cdot 3 \cdot 2 \cdot 7 \cdot 3$

$C = 252$

There are 252 committees that can be formed if there are no conditions.

b)

Let W represent the number of combinations for the women:

$W = {}_6C_3$

$W = \frac{6!}{3!(6-3)!}$

$W = \frac{6!}{3! \cdot 3!}$

$W = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1 \cdot 3!}$

$W = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}$

$W = 2 \cdot 5 \cdot 2$

$W = 20$

Let M represent the numbe of combinations for the mer

$M = {}_4C_2$

$M = \frac{4!}{2!(4-2)!}$

$M = \frac{4!}{2! \cdot 2!}$

$M = \frac{4 \cdot 3 \cdot 2!}{2 \cdot 1 \cdot 2!}$

$M = \frac{4 \cdot 3}{2 \cdot 1}$

$M = 2 \cdot 3$

$M = 6$

Let C represent the number of committees:

$C = W \cdot M$

$C = 20 \cdot 6$

$C = 120$

There are 120 committees that can be formed if there must be exactly 3 women.

c) Let C represent the number of committees:

$C = {}_6C_1$

$C = \frac{6!}{1!(6-1)!}$

$C = \frac{6!}{1! \cdot 5!}$

$C = \frac{6 \cdot 5!}{1 \cdot 5!}$

$C = \frac{6}{1}$

$C = 6$

There are 6 committees that can be formed if there must be exactly 4 men.

d) Let C represent the number of committees:

$C = {}_6C_5$

$C = \frac{6!}{5!(6-5)!}$

$C = \frac{6!}{5! \cdot 1!}$

$C = \frac{6 \cdot 5!}{5! \cdot 1}$

$C = \frac{6}{1}$

$C = 6$

There are 6 committees that can be formed if there can be no men.

e) e.g., **Case 1:** 3 men and 2 women

${}_4C_3 \cdot {}_6C_2 = \frac{4!}{3! \cdot 1!} \cdot \frac{6!}{2! \cdot 4!}$

${}_4C_3 \cdot {}_6C_2 = 60$

Case 2: 4 men and 1 woman

${}_4C_4 \cdot {}_6C_1 = \frac{4!}{4! \cdot 0!} \cdot \frac{6!}{1! \cdot 5!}$

${}_4C_4 \cdot {}_6C_1 = 1 \cdot 6$

${}_4C_4 \cdot {}_6C_1 = 6$

Number of committees = 60 + 6

Number of committees = 66

66 5-person committees can be formed if there must be at least 3 men.

12. e.g., Let's say I want to assign students to the room with 5 beds first. Let *A* represent the number of ways to assign the 12 students to the 5 beds:

$A = {}_{12}C_5$

$A = \frac{12!}{5!(12-5)!}$

$A = \frac{12!}{5!\cdot 7!}$

$A = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7!}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 7!}$

$A = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1}$

$A = 3\cdot 11\cdot 2\cdot 3\cdot 4$

$A = 792$

Now there are 12 – 5 or 7 students left to assign. Let's assign students to the room with 4 beds next. Let *B* represent the number of ways to assign the 7 students to the 4 beds:

$B = {}_7C_4$

$B = \frac{7!}{4!(7-4)!}$

$B = \frac{7!}{4!\cdot 3!}$

$B = \frac{7\cdot 6\cdot 5\cdot 4!}{4!\cdot 3\cdot 2\cdot 1}$

$B = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1}$

$B = 7\cdot 5$

$B = 35$

Now there are 7 – 4 or 3 students left to assign to the room with 3 beds. Since all of these students will be assigned to the room, there is only one combination for them. Let *C* now represent the number of different assignments:

$C = A \cdot B \cdot 1$

$C = 792 \cdot 35$

$C = 27\ 720$

There are 27 720 ways the 12 students can be assigned to these rooms.

13. a) i) 5 objects, 3 in each combination
ii) 10 objects, 2 in each combination
iii) 5 objects, 3 in each combination
b) e.g., **i)** How many ways can you choose 3 coins from a bag containing a penny, a nickel, a dime, a quarter, and a loonie?

14. a) i)

$_0C_0 = \frac{0!}{0!(0-0)!}$

$_0C_0 = \frac{0!}{0!\cdot 0!}$

$_0C_0 = \frac{1}{1}$

$_0C_0 = 1$

ii)

$_1C_0 = \frac{1!}{0!(1-0)!}$

$_1C_0 = \frac{1!}{0!\cdot 1!}$

$_1C_0 = \frac{1}{1}$

$_1C_0 = 1$

$_1C_1 = \frac{1!}{1!(1-1)!}$

$_1C_1 = \frac{1!}{1!\cdot 0!}$

$_1C_1 = \frac{1}{1}$

$_1C_1 = 1$

$_1C_0$, $_1C_1$ = 1, 1

iii)

$_2C_0 = \frac{2!}{0!(2-0)!}$

$_2C_0 = \frac{2!}{0!\cdot 2!}$

$_2C_0 = \frac{1}{1}$

$_2C_0 = 1$

$_2C_1 = \frac{2!}{1!(2-1)!}$

$_2C_1 = \frac{2!}{1!\cdot 1!}$

$_2C_1 = \frac{2\cdot 1!}{1\cdot 1!}$

$_2C_1 = \frac{2}{1}$

$_2C_1 = 2$

$_2C_2 = \frac{2!}{2!(2-2)!}$

$_2C_2 = \frac{2!}{2!\cdot 0!}$

$_2C_2 = \frac{1}{1}$

$_2C_2 = 1$

$_2C_0$, $_2C_1$, $_2C_2$ = 1, 2, 1

iv)

$_3C_0 = \frac{3!}{0!(3-0)!}$

$_3C_0 = \frac{3!}{0!\cdot 3!}$

$_3C_0 = \frac{1}{1}$

$_3C_0 = 1$

$_3C_1 = \frac{3!}{1!(3-1)!}$

$_3C_1 = \frac{3!}{1!\cdot 2!}$

$_3C_1 = \frac{3\cdot 2!}{1\cdot 2!}$

$_3C_1 = \frac{3}{1}$

$_3C_1 = 3$

$$_3C_2 = \frac{3!}{2!(3-2)!} \qquad _3C_3 = \frac{3!}{3!(3-3)!}$$
$$_3C_2 = \frac{3!}{2!\cdot 1!} \qquad _3C_3 = \frac{3!}{3!\cdot 0!}$$
$$_3C_2 = \frac{3\cdot 2!}{2!\cdot 1} \qquad _3C_3 = \frac{1}{1}$$
$$_3C_2 = \frac{3}{1} \qquad _3C_3 = 1$$
$$_3C_2 = 3$$

$_3C_0$, $_3C_1$, $_3C_2$, $_3C_3$ = 1, 3, 3, 1

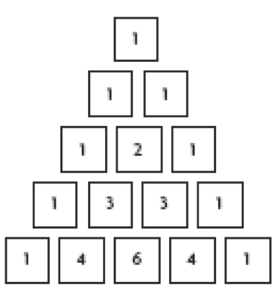
v)
$$_4C_0 = \frac{4!}{0!(4-0)!} \qquad _4C_1 = \frac{4!}{1!(4-1)!}$$
$$_4C_0 = \frac{4!}{0!\cdot 4!} \qquad _4C_1 = \frac{4!}{1!\cdot 3!}$$
$$_4C_0 = \frac{1}{1} \qquad _4C_1 = \frac{4\cdot 3!}{1\cdot 3!}$$
$$_4C_0 = 1 \qquad _4C_1 = \frac{4}{1}$$
$$_4C_1 = 4$$

$$_4C_2 = \frac{4!}{2!(4-2)!} \qquad _4C_3 = \frac{4!}{3!(4-3)!}$$
$$_4C_2 = \frac{4!}{2!\cdot 2!} \qquad _4C_3 = \frac{4!}{3!\cdot 1!}$$
$$_4C_2 = \frac{4\cdot 3\cdot 2!}{2\cdot 1\cdot 2!} \qquad _4C_3 = \frac{4\cdot 3!}{3!\cdot 1}$$
$$_4C_2 = \frac{4\cdot 3}{2\cdot 1} \qquad _4C_3 = \frac{4}{1}$$
$$_4C_2 = 2\cdot 3 \qquad _4C_3 = 4$$
$$_4C_2 = 6$$

$$_4C_4 = \frac{4!}{4!(4-4)!}$$
$$_4C_4 = \frac{4!}{4!\cdot 0!}$$
$$_4C_4 = \frac{1}{1}$$
$$_4C_4 = 1$$

$_4C_0$, $_4C_1$, $_4C_2$, $_4C_3$, $_4C_3$ = 1, 4, 6, 4, 1

b)



c) e.g., The numbers on the left and right sides are all 1s; every other number is the sum of the two numbers above it.

d) sixth row: 1, 5, 10, 10, 5,1
seventh row: 1, 6, 15, 20, 15, 6, 1

e) e.g., The number in each square of Pascal's Triangle is equal to the number of pathways to it from the top square.

15. a) The equation I need to solve is $\frac{n!}{2!(n-2)!} = 15$.

$n \geq 0$ AND $n - 2 \geq 0$
$$n \geq 2$$

$\frac{n!}{2!(n-2)!} = 15$ is defined for $n \geq 2$, where $n \in$ N.

$$\frac{n!}{2!(n-2)!} = 15$$
$$\frac{n!}{(n-2)!} = 15\cdot 2!$$
$$\frac{n!}{(n-2)!} = 15(2)$$
$$\frac{n!}{(n-2)!} = 30$$
$$\frac{n(n-1)(n-2)(n-3)...(3)(2)(1)}{(n-2)(n-3)...(3)(2)(1)} = 30$$
$$n(n-1) = 30$$
$$n^2 - n = 30$$
$$n^2 - n - 30 = 0$$
$$(n+5)(n-6) = 0$$

$n + 5 = 0$ OR $n - 6 = 0$
$n = -5$ $\quad$ $n = 6$

Based on the restrictions, n = –5 cannot be a solution.
Therefore, n = 6 is the solution to the equation.

b) The equation I need to solve is $\frac{n!}{4!(n-4)!}=35$.

$n \geq 0$ AND $n-4 \geq 0$

$n \geq 4$

$\frac{n!}{4!(n-4)!}=35$ is defined for $n \geq 4$, where $n \in$ N.

$$\frac{n!}{4!(n-4)!}=35$$
$$\frac{n!}{(n-4)!}=35\cdot 4!$$
$$\frac{n!}{(n-4)!}=840$$
$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)...(3)(2)(1)}{(n-4)(n-5)...(3)(2)(1)}=840$$
$$n(n-1)(n-2)(n-3)=840$$
$$(n^2-n)(n^2-5n+6)=840$$
$$n^4-5n^3+6n^2-n^3+5n^2-6n=840$$
$$n^4-6n^3+11n^2-6n-840=0$$

Write out all of the factors of –840: ±1, ±2, ±3, ±4, ±5, ±6, ±7, ±8, ±10, ±12, ±14, ±15, ±20, ±21, ±24, ±28, ±30, ±35, ±40, ±42, ±56, ±60, ±70, ±84, ±105, ±120, ±140, ±168, ±210, ±280, ±420, ±840. These are all the possible roots of the equation. Substitute them into the equation and if the equation goes to 0, then I have a root of the equation. By trial and error, the roots are –4 and 7. The other roots are not real. Since –4 is out of the domain, the only real solution is $n = 7$.

c) The equation I need to solve is

$4\left[\frac{n!}{2!(n-2)!}\right]=\frac{(n+2)!}{3!(n+2-3)!}$.

$n \geq 0$ AND $n-2 \geq 0$ AND $n+2 \geq 0$

$n \geq 2$ $\quad$ $n \geq -2$

AND $n+2-3 \geq 0$

$n-1 \geq 0$

$n \geq 1$

$4\left[\frac{n!}{2!(n-2)!}\right]=\frac{(n+2)!}{3!(n+2-3)!}$ is defined for $n \geq 2$, where $n \in$ N.

$$4\left[\frac{n!}{2!(n-2)!}\right]=\frac{(n+2)!}{3!(n+2-3)!}$$
$$4\left[\frac{n!}{2(n-2)!}\right]=\frac{(n+2)!}{6(n+2-3)!}$$
$$\frac{2n!}{(n-2)!}=\frac{(n+2)!}{6(n+2-3)!}$$
$$\frac{2n!}{(n-2)!}=\frac{(n+2)!}{6(n-1)!}$$
$$2n(n-1)=\frac{(n+2)(n+1)(n)}{6}$$
$$12n(n-1)=(n+2)(n+1)(n)$$
$$12(n-1)-(n+2)(n+1)=0$$
$$12n-12-(n^2+n+2n+2)=0$$
$$12n-12-n^2-n-2n-2=0$$
$$-n^2+9n-14=0$$
$$-(n-2)(n-7)=0$$

$n-2=0$ or $n-7=0$

$n=2$ $\quad$ $n=7$

Both the roots are within the domain, so there are two solutions, $n = 2$ and $n = 7$.

d) The equation I need to solve is $\frac{6!}{r!(6-r)!}=15$.

$r \geq 0$ AND $6-r \geq 0$

$r \leq 6$

$\frac{6!}{r!(6-r)!}=15$ is defined for $0 \leq r \leq 6$, where $r \in$ I.

$$\frac{6!}{r!(6-r)!}=15$$
$$r!(6-r)!=\frac{6!}{15}$$
$$r!(6-r)!=\frac{720}{15}$$
$$r!(6-r)!=48$$

By substituting each of the integers r for $0 \leq r \leq 6$, I get $r = 2$ or $r = 4$.

16. a) 1, e.g., the player can only win if the six numbers they choose are the same and in the same order as the six numbers drawn.

b) $${}_{66}C_6=\frac{66!}{6!(66-6)!}$$
$${}_{66}C_6=\frac{66!}{6!60!}$$
$${}_{66}C_6=\frac{66\cdot 65\cdot 64\cdot 63\cdot 62\cdot 61\cdot 60!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 60!}$$
$${}_{66}C_6=\frac{66\cdot 65\cdot 64\cdot 63\cdot 62\cdot 61}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$
$${}_{66}C_6=11\cdot 13\cdot 16\cdot 21\cdot 31\cdot 61$$
$${}_{66}C_6=90\,858\,768$$

There are 90 858 768 different ways the player can win.

c) e.g., No. Even if everyone in the city plays, it is very unlikely that anyone will win since each player only has a 1 in 90 858 767 chance of winning.

17. e.g., The number of sides in a polygon is equal to the number of vertices, the number of vertices = n. From each vertex, v, a diagonal is formed by a line segment intersecting a vertex that is not directly beside v. Thus, the number of vertices that will make a diagonal with v in an n-sided polygon is $n - 2$. Trying $r = 2$ in ${}_nC_r$ with the values from the polygons on the side of the textbook page, there is a pattern: (d = number of diagonals)

n	r	${}_nC_2$	d
3	2	3	0
4	2	6	2
5	2	10	5
6	2	15	9

Notice that $n + d = {}_nC_2$.
Rearranging, $d = {}_nC_2 - n$. Thus, the number of diagonals for an n-sided polygon can be determined using ${}_nC_2 - n$.

18. a) Case 1: 2 boys and 3 girls

$${}_7C_2 \cdot {}_{13}C_3 = \frac{7!}{2!5!} \cdot \frac{13!}{3!10!}$$
$${}_7C_2 \cdot {}_{13}C_3 = 21 \cdot 286$$
$${}_7C_2 \cdot {}_{13}C_3 = 6006$$

Case 2: 3 boys and 2 girls

$${}_7C_3 \cdot {}_{13}C_2 = \frac{7!}{3!4!} \cdot \frac{13!}{2!11!}$$
$${}_7C_3 \cdot {}_{13}C_2 = 35 \cdot 78$$
$${}_7C_3 \cdot {}_{13}C_2 = 2730$$

Case 3: 4 boys and 1 girl

$${}_7C_4 \cdot {}_{13}C_1 = \frac{7!}{4!3!} \cdot \frac{13!}{1!12!}$$
$${}_7C_4 \cdot {}_{13}C_1 = 35 \cdot 13$$
$${}_7C_4 \cdot {}_{13}C_1 = 455$$

Case 4: 5 boys and 0 girls

$${}_7C_5 \cdot {}_{13}C_0 = \frac{7!}{5!2!} \cdot \frac{13!}{0!13!}$$
$${}_7C_5 \cdot {}_{13}C_0 = 21 \cdot 1$$
$${}_7C_5 \cdot {}_{13}C_0 = 21$$

Number of groups = 6006 + 2730 + 455 + 21
Number of groups = 9212
There are 9212 different groups of 5 students with at least 2 boys to choose from.
b) Number of groups with no conditions:

$${}_{20}C_5 = \frac{20!}{5! \cdot 15!}$$

Case 1: 1 boy and 4 girls

$${}_7C_1 \cdot {}_{13}C_4 = \frac{7!}{1!6!} \cdot \frac{13!}{4!9!}$$
$${}_7C_1 \cdot {}_{13}C_4 = 5005$$

Case 2: 0 boys and 5 girls

$${}_7C_0 \cdot {}_{13}C_5 = \frac{7!}{0!7!} \cdot \frac{13!}{5!8!}$$
$${}_7C_0 \cdot {}_{13}C_5 = 1287$$

Number of groups with at least two boys:
${}_{20}C_5 - {}_7C_1 \cdot {}_{13}C_4 - {}_7C_0 \cdot {}_{13}C_5 = 9212$
There are 9212 different groups of 5 students with at least 2 boys to choose from.
c) e.g., I prefer indirect reasoning because fewer calculations are needed.

19. a) e.g., Combinations and permutations both involve choosing objects from a group. For permutations, order matters. For combinations, order does not matter. For example, *abc* and *bac* are different permutations, but the same combination.
b) Divide ${}_nP_r$ by $r!$ to get ${}_nC_r$. For example, ${}_6C_4 = 15$ and ${}_6P_4 = 360$; $\frac{360}{4!} = 15$

20. First, determine the total number of outcomes possible. I'll assume that once a song is selected, it cannot be selected again. The number of outcomes, O, is:

$$O = {}_{71}C_5$$
$$O = \frac{71!}{5!66!}$$
$$O = 13\,019\,909$$

a) Number of times the event could occur:

$${}_{26}C_5 = \frac{26!}{5!21!}$$
$${}_{26}C_5 = 65\,780$$

Probability (P):

$$P = \frac{65\,780}{13\,019\,909} \times 100\%$$
$$P = 0.505...\%$$

There is about a 0.51% chance that the five songs will be from CD 2 and CD 4.
b) Number of times the event could occur:
$12 \cdot 14 \cdot 15 \cdot 12 \cdot 18 = 544\,320$
Probability (P):

$$P = \frac{544\,320}{13\,019\,909} \times 100\%$$
$$P = 4.180...\%$$

There is about a 4.18% chance that one of the five songs will be from each CD.

c) There is only one time where your favourite song from each of the 5 CDs will be played.
Probability (P):

$$P = \frac{1}{13\,019\,909} \times 100\%$$

$$P = 0.000008\%$$

There is about a 0.000008% or 1 in 13 019 909 chance that your favourite song from each of the 5 CDs will be played.

21. $_nC_3 + {_nC_2} + {_nC_1}$

$$= \frac{n!}{3!(n-3)!} + \frac{n!}{2!(n-2)!} + \frac{n!}{1!(n-1)!}$$

$$= \frac{n!}{6(n-3)!} + \frac{n!}{2(n-2)!} + \frac{n!}{(n-1)!}$$

$$= \frac{n!}{6(n-3)!} + \frac{3n(n-1)(n-3)!}{6(n-3)!} + \frac{6n(n-3)!}{6(n-3)!}$$

$$= \frac{n! + 3n(n-1)(n-3)! + 6n(n-3)!}{6(n-3)!}$$

$$= \frac{(n-3)!\left[n(n-1)(n-2) + 3n(n-1) + 6n\right]}{6(n-3)!}$$

$$= \frac{n(n-1)(n-2) + 3n(n-1) + 6n}{6}$$

$$= \frac{n\left(n^2 - 2n - n + 2 + 3n - 3 + 6\right)}{6}$$

$$= \frac{n\left(n^2 + 5\right)}{6}$$

$$= \frac{n^3 + 5n}{6}$$

22. e.g.,

LS	RS
$_{n+1}C_r$	$_nC_r + {_nC_{r-1}}$
$\frac{(n+1)!}{r!(n+1-r)!}$	$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\left[n-(r-1)\right]!}$
	$\frac{\left[n-(r-1)\right]n!}{r!\left[n-(r-1)\right]!} + \frac{r(n!)}{r!\left[n-(r-1)\right]!}$
	$\frac{(n+1-r)n! + r(n!)}{r!(n+1-r)!}$
	$\frac{n!(n+1-r+r)}{r!(n+1-r)!}$
	$\frac{n!(n+1)}{r!(n+1-r)!}$
	$\frac{(n+1)!}{r!(n+1-r)!}$

LS = RS

Therefore, $_{n+1}C_r = {_nC_r} + {_nC_{r-1}}$.

Lesson 4.7: Solving Counting Problems, page 288

1. a) This situation involves combinations because the order of the 3 toppings on the pizza does not matter.
b) This situation involves permutations because the three spots for the candidates who are selected are all different so order matters.
c) This situation involves permutations because for a group of 3 numbers, there are different ways to roll those three numbers because of the different colours of the dice.
d) This situation involves combinations because the 5 children who are selected are all in the same position. No information is stated in the question about positions the children may play, so I can only assume that they are not playing in specific positions.

2. e.g., Situation A involves combinations and situation B involves permutations. For situation A, order does not matter since the 3 people who are selected will all be considered equals. For situation B, this is not the case. Each of the 3 people who are selected will have a different position with a different amount of power and different roles.

3. a) $_3C_3 = \frac{3!}{3!\cdot 0!}$

$_3C_3 = 1$

There is 1 way that Maddy can bid on 3 items if she bids on only her 3 favourite items.

b) $_8C_3 = \frac{8!}{3!\cdot 5!}$

$_8C_3 = 56$

There are 56 ways that Maddy can bid on 3 items if she bids on any 3 of the 8 items.

4. $\left({_{13}C_1}\right)^4 = \left(\frac{13!}{1!\cdot 12!}\right)^4$

$\left({_{13}C_1}\right)^4 = (13)^4$

$\left({_{13}C_1}\right)^4 = 28\,561$

There are 28 561 different four-card hands with one card from each suit.

5. a) $_{200}P_5 = \frac{200!}{195!}$

$_{200}P_5 = \frac{200\cdot 199\cdot 198\cdot 197\cdot 196\cdot 195!}{195!}$

$_{200}P_5 = 200\cdot 199\cdot 198\cdot 197\cdot 196$

$_{200}P_5 = 304\ 278\ 004\ 800$

There are 304 278 004 800 ways that the top five cash prizes can be awarded if each ticket is not replaced when drawn.