c) $_5P_5 = 5!$

$_5P_5 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$_5P_5 = 120$

d) $_{12}P_{10} = \dfrac{12!}{(12-10)!}$

$_{12}P_{10} = \dfrac{12!}{2!}$

$_{12}P_{10} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$

$_{12}P_{10} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

$_{12}P_{10} = 239\,500\,800$

10. a) a: $n+5 \geq 0$ AND $n+4 \geq 0$

$n \geq -5$ $\quad$ $n \geq -4$

The expression is defined for $n \geq -4$, where $n \in$ I.

b: $n+4 \geq 0$ AND $n+2 \geq 0$

$n \geq -4$ $\quad$ $n \geq -2$

The expression is defined for $n \geq -2$, where $n \in$ I.

c: $n-4 \geq 0$ AND $n-5 \geq 0$

$n \geq 4$ $\quad$ $n \geq 5$

The expression is defined for $n \geq 5$, where $n \in$ I.

d: $n+2 \geq 0$ AND $n \geq 0$

$n \geq -2$

The expression is defined for $n \geq 0$, where $n \in$ I.

b) a: $n \geq 0$ AND $n-2 \geq 0$

$n \geq 2$

The expression is defined for $n \geq 2$, where $n \in$ I.

b: $n-1 \geq 0$ AND $n-3 \geq 0$

$n \geq 1$ $\quad$ $n \geq 3$

The expression is defined for $n \geq 3$, where $n \in$ I.

11. n = 20 and r = 6

$_{20}P_6 = \dfrac{20!}{(20-6)!}$

$_{20}P_6 = \dfrac{20!}{14!}$

$_{20}P_6 = \dfrac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14!}{14!}$

$_{20}P_6 = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15$

$_{20}P_6 = 27\,907\,200$

Rennie can load his CD player in 27 907 200 different ways.

12. n = 14 and r = 2

$_{14}P_2 = \dfrac{14!}{(14-2)!}$

$_{14}P_2 = \dfrac{14!}{12!}$

$_{14}P_2 = \dfrac{14 \cdot 13 \cdot 12!}{12!}$

$_{14}P_2 = 14 \cdot 13$

$_{14}P_2 = 182$

There are 182 ways that Manny and 2 other players can line up to receive the championship trophy.

13. Agree. e.g., The number of ways to choose a president and a vice-president from a group of five students is $\dfrac{5!}{(5-2)!} = 20$. I could also use the Fundamental Counting Principle because there are five choices for president and four choices remaining for vice-president: 5 · 4 = 20.

Lesson 4.4: Permutations When Objects Are Identical, page 266

1. a) $\dfrac{7!}{3!2!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1) \cdot (2 \cdot 1)}$

$\dfrac{7!}{3!2!} = \dfrac{7 \cdot 3 \cdot 2 \cdot 5 \cdot 4 \cdot 3}{(3 \cdot 2 \cdot 1)}$

$\dfrac{7!}{3!2!} = 7 \cdot 5 \cdot 4 \cdot 3$

$\dfrac{7!}{3!2!} = 420$

b) $\dfrac{8!}{2!2!2!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2 \cdot 1 \cdot 2!2 \cdot 1}$

$\dfrac{8!}{2!2!2!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 3$

$\dfrac{8!}{2!2!2!} = 5040$

c) $\dfrac{10!}{4!3!2!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

$\dfrac{10!}{4!3!2!} = 10 \cdot 9 \cdot 7 \cdot 5 \cdot 4$

$\dfrac{10!}{4!3!2!} = 12600$

d) $\dfrac{12!}{2!4!5!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$\dfrac{12!}{2!4!5!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 7$

$\dfrac{12!}{2!4!5!} = 83160$

2. Let *A* represent the arrangement of 6 flags:

$$A=\frac{6!}{2!2!2!}$$
$$A=\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1\cdot2\cdot1}$$
$$A=6\cdot5\cdot3$$
$$A=90$$

There are 90 different signals that can be made from the 6 flags hung in a vertical line.

3. Let *C* represent the number of ways:

$$C=\frac{6!}{3!\cdot3!}$$
$$C=\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1}$$
$$C=5\cdot4$$
$$C=20$$

There are 20 different ways three coins land as heads and three coins land as tails.

4. Let *R* represent the number of ways:

$$R=\frac{18!}{10!5!3!}$$
$$R=\frac{18\cdot17\cdot16\cdot15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3!}{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot5\cdot4\cdot3\cdot2\cdot1\cdot3!}$$
$$R=17\cdot14\cdot13\cdot11\cdot9\cdot8$$
$$R=2\ 450\ 448$$

There are 2 450 448 ways that this record could have occurred.

5. Let *C* represent the number of ways:

$$C=\frac{9!}{2!3!4!}$$
$$C=\frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1}$$
$$C=9\cdot7\cdot5\cdot4$$
$$C=1260$$

There are 1260 ways that Norm can distribute 1 cookie to each grandchild.

6. a) Let *A* represent the number of arrangements:

A = 5!
A = 5 · 4 · 3 · 2 · 1
A = 120

There are 120 different arrangements that can be made using all the letters.

b) Let *A* represent the number of arrangements:

$$A=\frac{7!}{2!}$$
$$A=\frac{7\cdot6\cdot5\cdot4\cdot3\cdot2!}{2!}$$
$$A=7\cdot6\cdot5\cdot4\cdot3$$
$$A=2520$$

There are 2520 different arrangements that can be made using all the letters.

c) Let *A* represent the number of arrangements:

$$A=\frac{8!}{2!}$$
$$A=\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2!}{2!}$$
$$A=8\cdot7\cdot6\cdot5\cdot4\cdot3$$
$$A=20160$$

There are 20 160 different arrangements that can be made using all the letters.

d) Let *A* represent the number of arrangements:

$$A=\frac{12!}{2!3!}$$
$$A=\frac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1}$$
$$A=11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$$
$$A=39916800$$

There are 39 916 800 different arrangements that can be made using all the letters.

7. a) Let *A* represent the number of arrangements:

$$A=\frac{15!}{5!5!5!}$$
$$A=\frac{15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1\cdot5\cdot4\cdot3\cdot2\cdot1\cdot5\cdot4\cdot3\cdot2\cdot1}$$
$$A=14\cdot13\cdot11\cdot9\cdot7\cdot6$$
$$A=756756$$

There are 756 756 different ways he can arrange the books on the shelf.

b) Group the sets of 5 together.

A = 3 · 2 · 1
A = 6

There are 6 ways he can arrange the books.

8. e.g., A shish kabob skewer has 4 pieces of beef, 2 pieces of green pepper, and 1 piece each of mushroom and onion. How many different combinations are possible?

9. a) Let *R* represent the number of routes:

$$R=\frac{9!}{5!4!}$$
$$R=\frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1\cdot4\cdot3\cdot2\cdot1}$$
$$R=7\cdot6\cdot4\cdot3$$
$$R=126$$

There are 126 routes travelling from point A to point B if you travel only south or east.

b) Let R represent the number of routes:

$$R = \frac{13!}{7!6!}$$

$$R = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$R = 13 \cdot 11 \cdot 4 \cdot 3$$

$$R = 1716$$

There are 1716 routes travelling from point A to point B if you travel only south or east.

10. Let R represent the number of routes:

$$R = \frac{13!}{8!5!}$$

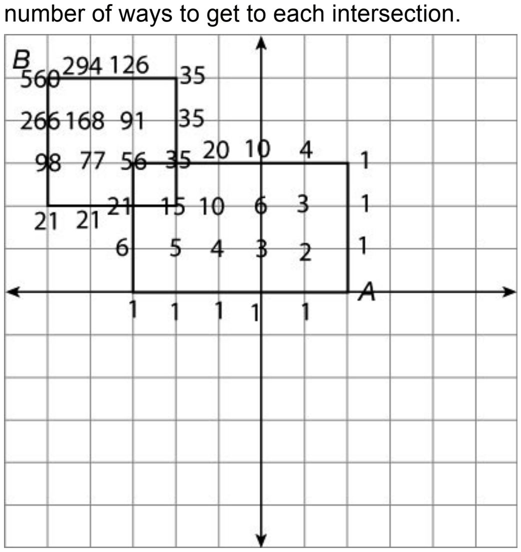
$$R = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$R = 13 \cdot 11 \cdot 9$$

$$R = 1287$$

There are 1287 routes travelling from the house of Jess to the house of her friend if she travels only north or west.

11. a) e.g., I drew the following diagram to show the number of ways to get to each intersection.



The sum of the numbers on the top right and bottom left corners of each block is equal to the number of routes to the top left corner of each block. There are 560 different routes from A to B, if you travel only north or west.

b) I need to go north twice and west four times, for a total of 6 moves, to travel the first 2 by 4 block of the route. I need to go north once and west once, for a total of 2 moves, to travel the next 1 by 1 block of the route. I need to go north twice and west twice, for a total of 4 moves, to travel the last 2 by 2 block of the route.

Let R represent the number of routes:

$$R = \frac{6!}{4!2!} \cdot 2! \cdot \frac{4!}{2!2!}$$

$$R = \left(\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}\right) \cdot (2 \cdot 1) \cdot \left(\frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}\right)$$

$$R = (5 \cdot 3) \cdot (2) \cdot (6)$$

$$R = 180$$

There are 180 different routes from A to B, if you travel only north or west.

12. Let P represent the number of permutations:

$$P = \frac{8!}{5!3!}$$

$$P = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$

$$P = 7 \cdot 4 \cdot 2$$

$$P = 56$$

There are 56 different permutations of answers that the teacher can create.

13. a) Let P represent the number of permutations:

$P = 7!$

$P = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$P = 5040$

There are 5040 different arrangements possible for the new totem pole.

b) Let P represent the number of permutations:

$$P = \frac{7!}{2!2!}$$

$$P = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1}$$

$$P = 7 \cdot 6 \cdot 5 \cdot 3 \cdot 2 \cdot 1$$

$$P = 1260$$

There are 1260 different arrangements possible for the new totem pole.

14. e.g., ${}_nP_n$ will be too high; it gives the number of arrangements of all n items, but some of the arrangements will be identical because of the a identical items in the group.

15. a) e.g., I am assuming that the coins of the same denomination are considered identical objects. Let A represent the number of arrangements:

$$A = \frac{9!}{4!3!2!}$$

$$A = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 2}$$

$$A = 1260$$

There are 1260 ways the 9 coins can be arranged in a line.

b) e.g., I am assuming that the coins of the same denomination are considered identical objects. Let A represent the number of arrangements:

$$A=\frac{7!}{4!3!}$$
$$A=\frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot3\cdot2}$$
$$A=35$$

There are 35 ways the 9 coins can be arranged in a line.

16. The number of ways to divide the 8 remaining freezies amongst the other 8 children is what I want. Let P represent the number of permutations:

$$P=\frac{8!}{2!5!}$$
$$P=\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot5\cdot4\cdot3\cdot2\cdot1}$$
$$P=7\cdot6\cdot4$$
$$P=168$$

There are 168 ways to distribute the 10 freezies amongst the 10 children.

17. a) Let P represent the number of permutations:

$$P=1\cdot\frac{8!}{3!3!2!}\cdot1$$
$$P=1\cdot\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1\cdot2\cdot1}\cdot1$$
$$P=1\cdot8\cdot7\cdot5\cdot2\cdot1$$
$$P=560$$

There are 560 permutations possible if you must start with A and end with C.

b) e.g., If you start by putting the I's in the first and second positions, and then in the second and third positions, and so on and so forth up until you put them in the ninth and tenth positions, there are 9 different arrangements of the I's just on their own. The number of different arrangements of all the letters in each of these 9 arrangements is the number of ways to organize the other 8 letters. Since the other 8 letters are always the same, the number of permutations of the letters for each arrangement of the I's is the same. Let P represent the number of permutations:

$$P=9\left(\frac{8!}{3!3!}\right)$$
$$P=9\left(\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1}\right)$$
$$P=9(8\cdot7\cdot5\cdot4)$$
$$P=9(1120)$$
$$P=10\,080$$

There are 10 080 permutations possible if the two I's must be together.

18. e.g., BANDITS has 7 different letters, so the number of permutations is 7! BANANAS also has 7 letters, but there are 3 As and 2 Ns so you must divide 7! by 3! · 2! = 12.

19. The shortest possible route contains 3 moves diagonally to the right, 3 moves diagonally to the left, and 3 moves down. Let R represent the number of routes;

$$R=\frac{9!}{3!3!3!}$$
$$R=\frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1}$$
$$R=8\cdot7\cdot5\cdot3\cdot2$$
$$R=1680$$

There are 1680 routes from the top rear vertex of the cube to the lower front vertex of the cube.

20. a) e.g., This is the same as arranging the 20 players then dividing by 2! ten times because the order of pairs does not matter. Let T represent the number of pairs:

$$T=\frac{20!}{2!^{10}}$$
$$T\doteq2.375...\times10^{15}$$

There are about 2.38×10^{15} ways to assign 20 players to 10 double rooms.

b) e.g., This is the same as arranging the 20 players then dividing by 4! five times because the order of pairs does not matter. Let T represent the number of pairs:

$$T=\frac{20!}{4!^{5}}$$
$$T\doteq3.055...\times10^{11}$$

There are about 3.06×10^{11} ways to assign 20 players to 5 quadruple rooms.

21. a) e.g., I can make a table to show all of the arrangements that could be made. Position 1 in the table below is the leftmost position, and position 4 is the rightmost position.

	Position 1	**Position 2**	**Position 3**	**Position 4**
1	R	R	R	W
2	R	R	W	R
3	R	R	W	W
4	R	W	R	R
5	R	W	R	W
6	R	W	W	R
7	R	W	W	W
8	W	R	R	R
9	W	R	R	W
10	W	R	W	R
11	W	R	W	W
12	W	W	R	R
13	W	W	R	W
14	W	W	W	R

From the table, I see that 14 different arrangements might be made.
b) e.g., From the table above, 1 out of the 14 arrangements, from left to right, would be red, white, white, red. Therefore, there is a 1 in 14 chance that the arrangement, from left to right, would be red, white, white, red.

Applying Problem-Solving Strategies, page 270

A. 4044 paths

B. 2(924) + 2(2508) + 2(3498) = 13 860 paths

C. Yes. There are 2(3936), or 7872, paths that lead to no money at all, but 17 904 paths that result in the contestant winning something. The contestant has a better chance of winning something than nothing, so it's a fair game from the contestant's point of view.

Lesson 4.5: Exploring Combinations, page 272

1. a) Let W represent the number of ways:
$W = 3!$
$W = 3 \cdot 2 \cdot 1$
$W = 6$
There are 6 different ways that Brian, Rachelle, and Linh can be chosen for these jobs.

b)

Canned Goods	Dry Goods	Fruits and Vegetables
Brian	Rachelle	Linh
Brian	Linh	Rachelle
Rachelle	Brian	Linh
Rachelle	Linh	Brian
Linh	Rachelle	Brian
Linh	Brian	Rachelle

c) Since all 3 volunteers are being used to help unload the vehicles, there is only one way they can be chosen for this job.
d) Part a) and b) involve permutations and part c) involved combinations. I know because in part a) and b), the order in which the volunteers were selected for the jobs mattered. In part c) the order did not since all the volunteers were being selected to do the same job.

2. e.g., The main difference is that for the permutations, the order of the 4 objects matters, and for the combinations, it does not. For the permutations, you could have multiple arrangements with the same objects since there is more than one way to order a group of four different objects. This is not possible for combinations since you just need one arrangement for each group of 4, regardless of the order.

3. Let C represent the number of dance committees possible:
$C = {}_{10}C_4$
$C = 210$
There are 210 ways that 4 of the members can be chosen to serve on the dance committee.

4. Let C represent the number of combinations:
$C = {}_{12}C_3$
$C = 220$
There are 220 ways 3 of the 12 dogs can be selected to appear.

Lesson 4.6: Combinations, page 280

1. a)

Flavour 1	Flavour 2
vanilla	strawberry
vanilla	chocolate
vanilla	butterscotch
strawberry	vanilla
strawberry	chocolate
strawberry	butterscotch
chocolate	vanilla
chocolate	strawberry
chocolate	butterscotch
butterscotch	vanilla
butterscotch	strawberry
butterscotch	chocolate