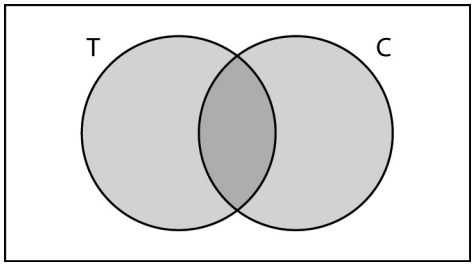
Math in Action, page 194

e.g.,

• I decided to research texting in relation to driving safely.

Search Words	Number of Hits
texting and driving	3 830 000
texting while driving	976 000
“texting while driving”	599 000
“texting while driving in Canada”	8

• I had way too many hits for texting and driving. I figured out that the issue is texting while driving, so I tried that combination. Putting quotes around it netted even fewer results. Since I live in Canada, I was interested in what’s happening here, so I added Canada to my search. Then I tried “texting while driving in Canada”. That really cut down the hits to a manageable number.



Let *T* represent “texting while driving” sites, and *C* represent Canada sites. The overlap of the two circles represents the sites that contain both “texting while driving” and Canada.

• The search engine’s Advanced Search feature allows you to exclude any sites that contain certain words from your search.

Lesson 3.5: Conditional Statements and Their Converse, page 203

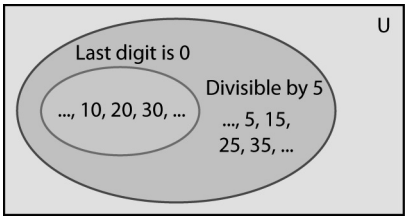
1. a) Hypothesis, p = I am swimming in the ocean.
Conclusion, q = I am swimming in salt water.
b) Yes, the conditional statement is true, because all oceans contain salt water.
c) Converse: If I am swimming in salt water, then I am swimming in the ocean.
The converse is false, because I could be swimming in a salt-water pool, or a salt-water lake.

2. a) Yes, the conditional statement is true. Four is divisible by 2, so any number that is divisible by 4 is also divisible by 2.
b) Converse: If a number is divisible by 2, then it is divisible by 4. The converse is false.
c) e.g., A counterexample of the converse is the number 2, which is divisible by 2, but not 4.

3. a) If a triangle is equilateral, then it has 3 equal sides.
b) If a triangle has 3 equal sides, then it is equilateral.
c) Both statements are true, because the definition of an equilateral triangle is a triangle that has 3 equal sides.
d) Yes, the statement is biconditional, because both the conditional statement and its converse are true.

4. a) If we cannot get what we like, then let us like what we get.
b) Hypothesis: We cannot get what we like.
Conclusion: Let us like what we get.

5. a) The statement is false. A counterexample is the number 25. It is divisible by 5, but it does not end in a 0.
b) If a number ends in a 0, then it is divisible by 5.
c) The converse is true. The Venn diagram shows that all multiples of ten are also multiples of 5, but not all multiples of 5 are multiples of 10.



6. a) The conditional statement is true, because Canada is in North America. The converse is false. Counterexample: You might live in Mexico and still be in North America. The statement is not biconditional.
b) The statement is true, because Ottawa is the capital of Canada. The converse is also true. Biconditional statement: You live in the capital of Canada if and only if you live in Ottawa.

7. Biconditional. e.g.,

$\sqrt{x^2} = x$	x is not negative	$\sqrt{x^2} = x \Rightarrow x$ is not negative
true	true	true
false	false	true
false	true	true
true	false	false

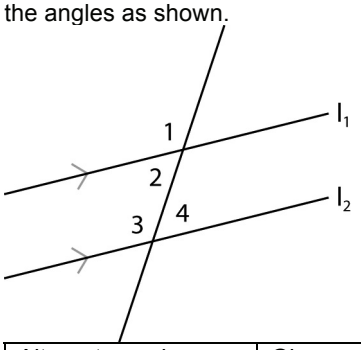
Both the conditional statement and its converse are always true, so the statement is biconditional.

The statement can be written as: $\sqrt{x^2} = x$ if and only if x is non-negative.

8. a) Conditional statement: If a glass is half-empty, then it is half full. This statement is true.
Converse: If a glass is half full, then it is half-empty. The converse is true. The statement is biconditional, because both the conditional statement and its converse are true.
Biconditional statement: A glass is half-empty if and only if it is half full.
b) Conditional statement: If a polygon is a rhombus, then it has equal opposite angles. The statement is true.
Converse: If a polygon has equal opposite angles, then it is a rhombus. The converse is false.
Counterexample: A rectangle has equal opposite angles. The statement is not biconditional.

c) Conditional statement: If a number is a repeating decimal, then it can be expressed as a fraction. The statement is true.
Converse: If a number can be expressed as a fraction, then it is a repeating decimal.
The converse is false. Counterexample: The decimal number 0.3 can be expressed as the fraction $\frac{3}{10}$ but 0.3 is not a repeating decimal.
The statement is not biconditional.

9. **a)** Conditional statement: If *AB* and *CD* are parallel, then the alternate angles are equal.
Converse: If the alternate angles are equal, then *AB* and *CD* are parallel.
b) Proof of conditional statement:
I drew two lines crossed by a transversal and numbered the angles as shown.



Alternate angles are equal.	Given.
$\angle 2 = \angle 4$	Alternate angles.
$\angle 2$ and $\angle 1$ are supplementary.	They form a straight line.
$\angle 4$ and $\angle 3$ are supplementary.	They form a straight line.
$\angle 1 = \angle 3$	Supplements of equal angles are also equal.
AB || *CD*	Corresponding angles are equal.

Therefore, the conditional statement is true.
Proof of converse:
I used the same diagram.

AB || *CD*	Given.
$\angle 1 = \angle 3$	If lines are parallel, corresponding angles are equal.
$\angle 2$ and $\angle 1$ are supplementary.	They form a straight line.
$\angle 4$ and $\angle 3$ are supplementary.	They form a straight line.
$\angle 1 = \angle 3$	Supplements of equal angles are also equal.
$\angle 1$ and $\angle 3$ are alternate angles.	

Therefore, the converse is true.
c) The original statement is true, because both the statement and the converse are true, so the statement is biconditional.

10. a) Converse: If your pet is a dog, then it barks. The statement and its converse are true, so the statement is biconditional.
b) Converse: If your pet wags its tail, then it is a dog.
The converse is false. A cat wags its tail. The statement is not biconditional.

11. a) True.
$$\begin{aligned} x + y &= z \\ x + y - y &= z - y \\ x &= z - y \end{aligned}$$
b) True.
$$\begin{aligned} p - q &= r \\ p - q + q &= r + q \\ p &= r + q \end{aligned}$$

12. e.g., If a number appears in the same row, column, or large square as the shaded square, then it is not in the shaded square. The numbers 1, 4, 5, 6, and 8 must go in column 4. If I were to put 1, 4, 5, or 8 in this square, then I could not put 6 in any other square in column 4. I conclude that 6 is the only number that can go in this square. As a result, 5 can only go in the square above, 4 can only go in the square below, 8 can only go in the top square, and 1 must go in the remaining square. The numbers in the column should be, from top to bottom: 8, 3, 9, 5, 6, 4, 1, 7, 2.

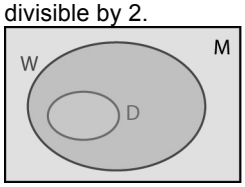
13. a) i) If a figure is a square, then it has four right angles.
Ii) If a figure has four right angles, then it is a square.
iii) The statement is true. The converse is false. The figure could be a rectangle.
iv) The statement is not biconditional.
b) i) If a triangle is a right triangle, then $a^2 + b^2 = c^2$.
ii) If, for a triangle, $a^2 + b^2 = c^2$, then it is a right triangle.
iii) The statement is true. The converse is true.
iv) A triangle is a right triangle if and only if $a^2 + b^2 = c^2$.
c) i) If a quadrilateral is a trapezoid, then it has two parallel sides.
ii) If a quadrilateral has two parallel sides, then it is a trapezoid.
iii) The statement is true. The converse is false. A regular hexagon has two sides that are parallel.
iv) The statement is not biconditional.

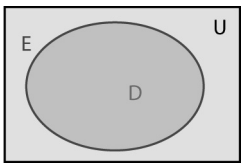
14. Use the finance application on a calculator. *Note:* Mortgages are compounded semi-annually in Canada.
a) i) The number of payments is 25 · 12 or 300.
The interest rate is 6.5%.
The present value is $250 000.
The payment amount is unknown.
The future value is $0.
The payment frequency is 12.
The compounding frequency is 2.
They should pay $1674.559… or $1674.56 per month.
ii) The number of payments is 25 · 24 or 600.
The interest rate is 6.5%.
The present value is $250 000.
The payment amount is unknown.
The future value is $0.
The payment frequency is 24.
The compounding frequency is 2.
They should pay $836.163… or $836.16 bi-monthly.
b) 2 payments/week · 52 weeks/year = 104
The number of payments is unknown.
The interest rate is 6.5%.
The present value is $250 000.
The payment amount is $836.16 ÷ 4 = $209.04.
The future value is $0.
The payment frequency is 104.
The compounding frequency is 2.
They will make 2164.088… or 2164 mortgage payments.

If Michelle and Marc make one payment each month for 300 months, they will pay
$1674.56 · 300 = $502 368 in total.
If they pay two payments each month for 300 months, they will pay
$836.16 · 600 = $501 696 in total.
If they make 2164 payments of $209.04, they will pay
2164 · $209.04 = $452 362.56 in total.

They will save
$502 368 – $452 362.56 = $50 005.44 by paying more frequently, so they should do that if they can.

15. e.g., **a)** *M*: If it is December, then it is winter.
U: If a number is even, then it is divisible by 2.
b) Let *W* represent winter, and *D* represent December.
Let *E* represent even numbers, and *D* represent being divisible by 2.





c) e.g., If the sets are the same (i.e., there is one area in the Venn diagram), then the converse is true. If there are two or more areas in the Venn diagram, then the converse is false.

16. a) e.g., If the first letter is a consonant, the second letter is a vowel.
b) E is the most common letter used in the English language. X is very frequent in the puzzle.
Substitute
J = A and X = E.

```
                E
K S Q   Q S C A X H B M V   T D   T Y
    A                     E E
D K J D   C S N S A U   C X X A
  A       A           E
Q J T D   J   Y T C L V X
      E         E       E
P S P X C D   N X B S H X
    A                           E
Y D J H D T C L   D S   T P Z H S W X
    E
D K X   Q S H V A .
    A     E       A
–   J C C X   B H J C G
```

The last two words are someone's name. What name starts with A, has the same two letters, and then ends with E? Anne. Substitute C = N.

```
            N   E
K S Q   Q S C A X H B M V   T D   T Y
    A     N             N E E
D K J D   C S N S A U   C X X A
  A       A       N     E
Q J T D   J   Y T C L V X
      E N       E       E
P S P X C D   N X B S H X
    A         N                   E
Y D J H D T C L   D S   T P Z H S W X
    E
D K X   Q S H V A .
    A N N E       A N
–   J C C X   B H J C G
```

What could NEE_ be? Need. There are three vowels left: I, O, and U. Two-letter words usually have a I or an O. If T was a vowel, it would probably be an I rather than an O. Substitute A = D and try T = I.

```
            N D E           I     I
K S Q   Q S C A X H B M V   T D   T Y
    A     N       D     N E E D
D K J D   C S N S A U   C X X A
  A I     A     I N     E
Q J T D   J   Y T C L V X
      E N       E       E
P S P X C D   N X B S H X
    A     I N           I           E
Y D J H D T C L   D S   T P Z H S W X
    E           D
D K X   Q S H V A .
    A N N E       A N
–   J C C X   B H J C G
```

I_ cannot be “in” because N has been used. Try “it” and “is.” Substitute D = T and Y = S.

```
           N D E          I T  I S
K S Q  Q S C A X H B M V  T D  T Y
T  A T  N      D   N E E D
D K J D  C S N S A U  C X X A
  A I T  A S I N    E
Q J T D  J  Y T C L V X
     E N T   E     E
P S P X C D  N X B S H X
  T A  T I N   T   I        E
Y D J H D T C L  D S  T P Z H S W X
T  E         D
D K X  Q S H V A .
   A N N E     A N
–  J C C X  B H J C G
```

T_ has to be “to.” T_E is probably “the,” so T_AT is “that,” which makes sense. Substitute S = O and K = H.

```
H O     O N D E          I T  I S
K S Q  Q S C A X H B M V  T D  T Y
T H A T  N O  O D   N E E D
D K J D  C S N S A U  C X X A
  A I T  A S I N    E
Q J T D  J  Y T C L V X
  O  E N T   E  O  E
P S P X C D  N X B S H X
S T A  T I N   T O  I      O  E
Y D J H D T C L  D S  T P Z H S W X
T H E    O   D
D K X  Q S H V A .
   A N N E     A N
–  J C C X  B H J C G
```

HO_ is probably “how,” so _AIT is “wait,” which makes sense. _O_ENT uses the same letter twice. All I can think of is “moment.” STA_TIN_ probably ends in “ing.” Try Q = W, P = M and L = G.

```
H O W  W O N D E          I T  I S
K S Q  Q S C A X H B M V  T D  T Y
T H A T  N O  O D   N E E D
D K J D  C S N S A U  C X X A
W A I T  A  S I N G   E
Q J T D  J  Y T C L V X
M O M E N T   E  O  E
P S P X C D  N X B S H X
S T A  T I N G  T O  I M    O  E
Y D J H D T C L  D S  T P Z H S W X
T H E  W O    D
D K X  Q S H V A .
   A N N E     A N
–  J C C X  B H J C G
```

SING_E has to be “single.” So STA_TING is “starting” and WONDE_ _ _ _ could be “wonderful.” Substitute V = L, H = R, B = F, and M = U.

```
H O W  W O N D E R F U L  I T  I S
K S Q  Q S C A X H B M V  T D  T Y
T H A T  N O  O D   N E E D
D K J D  C S N S A U  C X X A
W A I T  A  S I N G L E
Q J T D  J  Y T C L V X
M O M E N T   E F O R E
P S P X C D  N X B S H X
S T A R T I N G  T O  I M  R O  E
Y D J H D T C L  D S  T P Z H S W X
T H E  W O R L D
D K X  Q S H V A .
   A N N E  F R A N
–  J C C X  B H J C G
```

FRAN_ is “Frank.” _EFORE is “before.” So NO_OD_ could be “nobody.” What letters are left? C, J, P, Q, V, X, Z. IM_RO_E could be “improve.” Substitute G = K,
N = B, U = Y, Z = P, and W = V.

```
H O W  W O N D E R F U L  I T  I S
K S Q  Q S C A X H B M V  T D  T Y
T H A T  N O B O D Y  N E E D
D K J D  C S N S A U  C X X A
W A I T  A  S I N G L E
Q J T D  J  Y T C L V X
M O M E N T  B E F O R E
P S P X C D  N X B S H X
S T A R T I N G  T O  I M P R O V E
Y D J H D T C L  D S  T P Z H S W X
T H E  W O R L D
D K X  Q S H V A .
   A N N E  F R A N K
–  J C C X  B H J C G
```

17. a) Use the finance application on a calculator.
The number of payments is unknown.
The interest rate is 4%.
The present value is $265 233.48.
The payment amount is $1400 + $250 or $1650.
The future value is $0.
The payment frequency is 12.
The compounding frequency is 12.
It will take them 230.631 or 231 months to pay off the mortgage.

b) At $1400/month: $1400 · 300 = $420 000

At $1650/month: $1650 · 231 = $381 150
$420 000 – $ 351 150 = $38 850
They will save $38 850 over the life of the mortgage by paying $1650 per month instead of $1400.