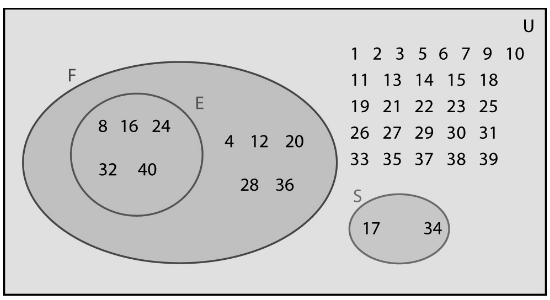
Chapter 3: Set Theory and Logic

Lesson 3.1: Types of Sets and Set Notation, page 154

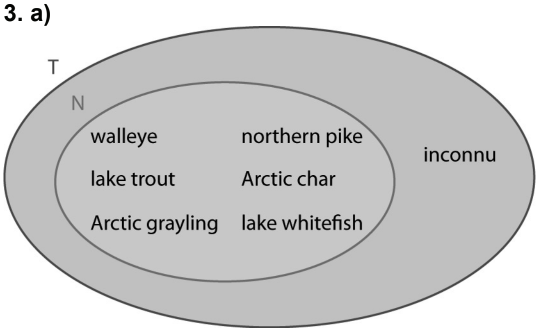
1. a) e.g., Yes, those explanations make sense.
b) The universal set is set *C*.
C = {produce}
O = {orange produce} = {oranges, carrots}
Y = {yellow produce} = {bananas, pineapple, corn}
G = {green produce} = apples, pears, peas, beans}
B = {brown produce} = {potatoes, pears}
c) e.g., $S \subset F$ because all fruits you can eat without peeling are also fruits. $S \subset C$ because all fruits you can eat without peeling are also produce.
d) Sets *S* and *V* are disjoint sets, as are *F* and *V*.
e) Yes. e. g., *C* = {*F* and *V*}, $F' = V$.
f) $n(V) = n(C) - n(F)$
$n(V) = 10 - 5$
$n(V) = 5$
g) oranges, pineapple, bananas, peas, corn, carrots, beans, potatoes

2. a)



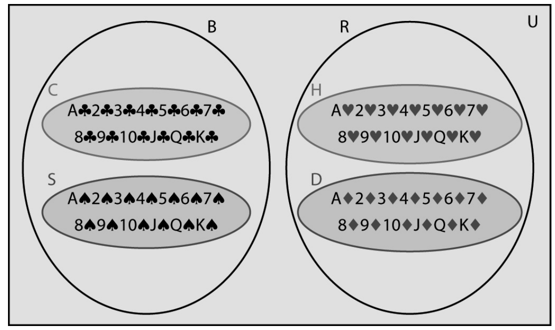
b) Sets *E* and *S* are disjoint sets, as are sets *F* and *S*.
c) i) True. e.g., Multiples of 8 are also multiples of 4.
ii) False. e.g., Not all multiples of 4 are multiples of 8.
iii) True. e.g., All multiples of 4 are multiples of 4.
iv) False. e.g., F' = {all numbers from 1 to 40 that are not multiples of 4}
v) True. e.g., The universal set includes natural numbers from 1 to 40.

3. a)



b) e.g., $N \subset T$ means that all the fish found in Nunavut are also found in the Northwest Territories. $T \not\subset N$ means that not all the fish found in the Northwest Territories are found in Nunavut.

4. a)



b) Subsets of set *B*: $C \subset B$ and $S \subset B$
c) Subsets of set *R*: $H \subset R$ and $D \subset R$
d) Yes, the sets *S* and *C* are disjoint. e.g., A card cannot be both a spade and a club.
e) Yes, the events in sets *H* and *D* are mutually exclusive. e.g., You cannot draw a card that is a heart and a diamond at the same time.
f) Yes, that statement is correct. e.g., Because these sets are disjoint, they contain no common elements. Therefore, when the numbers of elements in each set are added, no element will be counted twice.
$n(S \text{ or } D) = n(S) + n(D)$
$n(S \text{ or } D) = 13 + 13$
$n(S \text{ or } D) = 26$

5. a) e.g., *C* = {all clothes}, *S* = {summer clothes}, *W* = {winter clothes}, *H* = {summer headgear}
b) e.g., In set *C*, but not in set *S* or set *W*, because they would be worn year round.
c) No, set S' is not equal to set *W*. Set S' includes the jacket, but *W* does not.
d) Sets *S* and *W* are disjoint sets. Sets *H* and *W* are disjoint sets.
e) e.g., *C* = {clothes},
H = {headgear} = {cap, sunglasses, toque},
B = {clothing for body} = {shirt, shorts, coat, jacket}, *F* = {footwear} = {sandals, insulated boots}

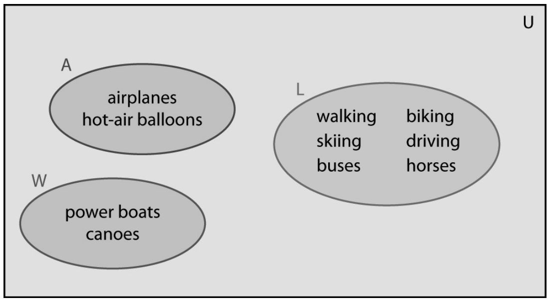
6. $n(X') = n(\text{U}) - n(\text{X})$
$n(X') = 100\,000 - 12$
$n(X') = 99\,988$

7. Not possible; e.g., there may be some elements that are in both *X* and *Y*.

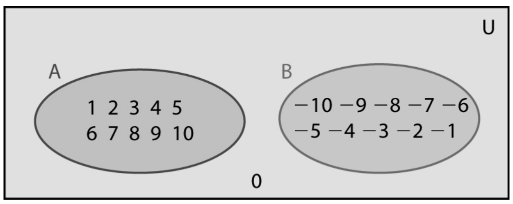
8. $n(U) = n(X) + n(X')$
$n(U) = 34 + 42$
$n(U) = 76$

9. a) *S* = {A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z}
C = {C, O, S}
b) False. e.g., B is not in *S* or *C*.

10. Let *U* represent the universal set. Let *L* represent the set of land transportation. Let *W* represent the set of water transportation. Let *A* represent the set of air transportation.



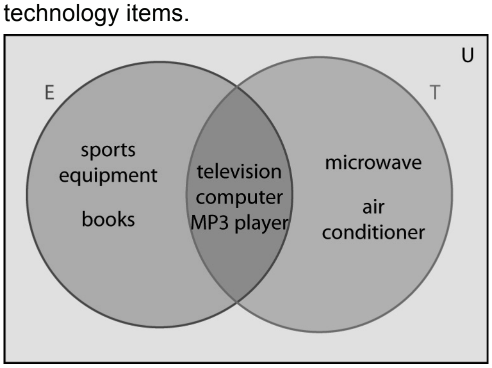
11. a)



b) Sets *A* and *B* are disjoint sets.
c) i) False. e.g., 1 is not in *B*.
ii) False. e.g., –1 is not in *A*.
iii) False. e.g., 0 is in *A′* but not in *B*.
iv) True. e.g., *n*(*A*) = 10, *n*(*B*) = 10.
v) True. e.g., No integer from –20 to –15 is in *U*.

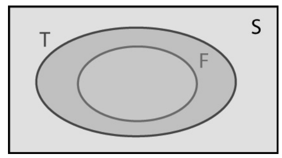
12. a) *S* = {4, 6, 9, 10, 14, 15, 21, 22, 25, 26, 33, 34, 35, 38, 39, 46, 49}
W = {1, 2, 3, 5, 7, 8, 11, 12, 13, 16, 17, 18, 19, 20, 23, 24, 27, 28, 29, 30, 31, 36, 37, 40, 41, 42, 43, 44, 45, 47, 48, 50}
b) e.g., *E* = {even semiprime numbers}
E = {4, 6, 10, 14, 22, 26, 34, 38, 46}
c) $n(W) = n(U) - n(S)$
$n(W) = 50 - 17$
$n(W) = 33$
d) No, it is not possible to determine *n*(*A*). e.g., There is an infinite number of prime numbers, so there is an infinite number of semiprime numbers.

13. e.g., Let *U* represent the universal set. Let *E* represent the set of entertainment items. Let *T* represent technology items.

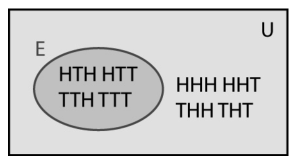


14. Agree; e.g., $A \subset B$ means that set *A* is a part of set *B*, and it could be that set *A* and set *B* are equal. If $A \subset B$, then set *A* will have the same number or fewer elements than set *B*. With numbers, $x \leq y$ means that *x* is less than or equal to *y*. Or, or if $A \subset B$, then $n(A) \leq n(B)$. The number of elements in a subset must be equal to or less than the number of elements in the set.

15. a) $S = \{x \mid -1000 \leq x \leq 1000, x \in \mathrm{I}\}$
$T = \{t \mid t = 25x, -40 \leq x \leq 40, x \in \mathrm{I}\}$
$F = \{f \mid f = 50x, -20 \leq x \leq 20, x \in \mathrm{I}\}$
$F \subset T \subset S$
b)

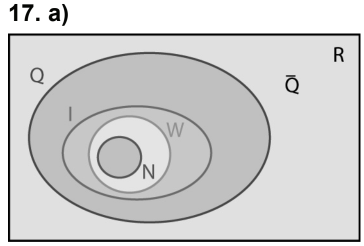


16. a) *U* = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
b) *E* = {HTH, HTT, TTH, TTT}
c) *n*(*U*) = 8, *n*(*E*) = 4
d) Yes, e.g., because each element of *E* is also an element of *U*, and there are some elements of *U* that are not elements of *E*.



e) For example, *E′* is the set of elements of *U* where the second coin turns up heads.
$n(E') = n(U) - n(E)$
$n(E') = 8 - 4$
$n(E') = 4$
E′ = {HHH, HHT, THH, THT} and *n*(*E′*) = 4
f) Yes. e.g., A coin cannot show both heads and tails at the same time.

17. a)



b) e.g., *N′* is the set of all non-natural numbers. *W′* is the set of all non-whole numbers. I′ is the set of non-integer numbers. *Q′* is the set of numbers that cannot be described as a ratio of two integers. $\overline{Q}$ is the set of numbers that can be described as a ratio of two integers.

Set	Complement
N	$N' = \{x \mid x \in R, x \notin N\}$
I	$I' = \{x \mid x \in R, x \notin I\}$
Q	$\overline{Q}$
$\overline{Q}$	Q

c) Sets N and $\overline{Q}$ are disjoint sets. Sets W and $\overline{Q}$ are disjoint sets. Sets I and $\overline{Q}$ are disjoint sets. *Sets* Q and $\overline{Q}$ are disjoint sets.
d) Yes. e.g., Q' is the set of numbers that cannot be described as a ratio of two integers, which is the set of irrational numbers.
e) *W, I, Q, R*
f) No. e.g., The area of a region in a Venn diagram is not related to the number of elements in the set.

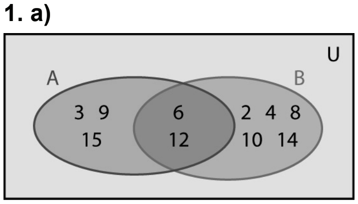
18. a) S = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289}
$n(S) = 17$
E = {4, 16, 36, 64, 100, 144, 196, 256}
$n(E) = 8$
b) $n(S) = 17$, $n(E) = 8$
$n(O) = n(S) - n(E)$
$n(O) = 17 - 8$
$n(O) = 9$
c) $n(U) = 300$, $n(S) = 17$
$n(S') = n(U) - n(S)$
$n(S') = 300 - 17$
$n(S') = 283$

19. a) e.g., $A \subset B$ if all elements of A are also in B. For example, all weekdays are also days of the week, so weekdays is a subset of days of the week.
b) e.g., A' consists of all the elements in the universal set but not in A. For example, all days of the week that are not weekdays are weekend days. So weekend days is the complement of weekdays.

20. e.g., Disagree; since both the subsets are empty, they both contain the same elements and are therefore the same subset.

Lesson 3.2: Exploring Relationships between Sets, page 160

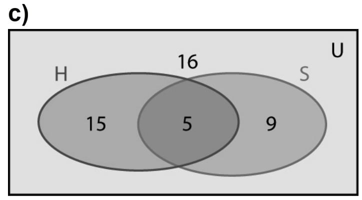
1. a)



b) i) $n(A) = 5$
ii) $n(A \text{ but not } B) = n(A) - n(A \text{ and } B)$
$n(\text{A but not } B) = 5 - 2$
$n(\text{A but not } B) = 3$
iii) $n(B) = 7$
iv) $n(B \text{ but not } A) = n(B) - n(A \text{ and } B)$
$n(B \text{ but not } A) = 7 - 2$
$n(B \text{ but not } A) = 5$
v) $n(A \text{ and } B) = 2$
vi) $n(A \text{ or } B) = n(\text{A but not } B) + n(A \text{ and } B) + n(B \text{ but not } A)$
$n(A \text{ or } B) = 3 + 2 + 5$
$n(A \text{ or } B) = 10$
vii) $n(A) = 5$, therefore $n(A') = 5$

2. a) 8 students are in both the drama club and the band.
b) 11 students are in the drama club only.
6 students are in the band only.
c) Drama: 11 + 8 = 19
Band: 8 + 6 = 14
d) Drama club or band: 11 + 8 + 6 = 25
e) 38 students in grade 12 – 25 in drama club or band = 13 students in neither drama club nor band

3. a) hockey or soccer: 45 – 16 = 29
hockey and soccer: 20 + 14 = 34
overlap: 34 – 29 = 5
5 students like hockey and soccer.
b) only hockey: 20 – 5 = 15
only soccer: 14 – 5 = 9
15 + 9 = 24
24 students like only hockey or only soccer.
c)



4. a) ski or snowboard: 55 – 9 = 46
ski and snowboard: 25 + 32 = 57
Overlap: 57 – 46 = 11
11 guests plan to ski and snowboard.
b) only ski: 25 – 11 = 14
14 guests will only ski.
c) only snowboard: 32 – 11 = 21
21 guests will only snowboard.

5. a) $n(U) - n(U \text{ but not } A \text{ or } B)$: 25 – 4 = 21
$n(A) + n(B)$: 13 + 10 = 23
$n(A \text{ and } B)$: 23 – 21 = 2
$n(A \text{ only})$: 13 – 2 = 11
$n(B \text{ only})$: 10 – 2 = 8
b)