

Using the financial application on a graphing calculator, the number of payments is 29.151..., or 30. The amount of interest paid is \$187.163... or \$187.17.
It would take 30 months to pay off Credit card B.

It would take James 43 months, or 3 years 7 months, to pay off all of his balances.

b) Line of credit:

The present value is $2775.20 + 1200.35 + 687.38$, or \$4662.93.

The regular payment amount is \$570.

The payment frequency is 12 times a year.

The number of payments is unknown.

The payments are made at the end of the payment periods.

The annual interest rate is 6.8%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the number of payments is 8.399..., or 9. The amount of interest paid is \$125.054... or \$125.06.

It would take James 9 months to become debt-free if he consolidated his loan and credit card balances.

c) Without consolidation:

Interest paid = $52.562... + 480.024... + 187.163...$

Interest paid = \$719.750...

With consolidation:

Interest paid = \$125.054...

Difference = $719.750... - 125.054...$

Difference = \$594.696...

James would save \$594.70 in interest if he consolidated his debts.

Math in Action, page 119

Sample Solution

Answers will vary, e.g., \$544.3 billion (September 29, 2010).

The debt would be $\frac{\$544\,300\,000\,000}{260\,000}$ or \$15 887.33 per

person.

The first year, \$13.7645 billion in interest would accumulate.

This is \$3 771 097.84 per day.

The present value is \$544.3 billion.

The regular payment amount is \$4 billion.

The payments are made 12 times per year.

The number of payments is unknown.

The payments are made at the end of the payment periods.

The annual interest rate is 2.5%.

The compounding frequency is 12 times per year.

The future value is \$0.

Therefore, the number of payments is 160.180....

It would take 160.18 months, or 13 years 4 months.

Lesson 2.4: Buy, Rent or Lease?, page 129

1. a) e.g., renting cost for 4 months or 120 days:
 $(120)(75) = \$9000$;

benefits: cleaning service, utilities

leasing: costs for 4 months:

$(4)(1600) + (4)(125) + 1600 = \8500 ;

benefits: \$1600 refund if no damage

b) e.g. I would recommend leasing. Even if the deposit is lost, it is less expensive.

2. a) Leasing costs:

Computer cost: $(3)(5548.20) = \$16\,644.60$

Server cost: $(3)(2200) = \$6600$

Printer cost: $(3)(215) = \$645$

Total leasing cost:

$16\,644.60 + 6600 + 645 = \$23\,889.60$

Purchasing costs:

Computer: \$24 000

Resale value: $24\,000(1 - 0.4)^3 = \$5184$

Computer cost: $24\,000 - 5184 = \$18\,816$

Server: \$7200

Resale value: $7200(1 - 0.4)^3 = \$1555.20$

Server cost: $7200 - 1555.20 = \$5644.80$

Printers: \$5200

Resale value: $5200(1 - 0.4)^3 = \$1123.20$

Printer cost: $5200 - 1123.20 = \$4076.80$

Total purchasing cost:

$18\,816 + 5644.80 + 4076.80 = \$28\,537.60$

Leasing: costs: \$23 889.60; benefits: e.g., could upgrade earlier than 3 years; no need to resell equipment

purchasing: costs: \$28 537.60; benefits: e.g., own equipment, which is cheaper from the fourth year onward

b) e.g., I would suggest that this company lease, because the total cost of leasing is less than purchasing the equipment.

3. a) Rental cost: $(45)(14) = \$630$

Buy:

Canoe cost = \$3000

Safety equipment cost = \$160

Paddle cost: $(120)(3) = \$360$

Total cost: $3000 + 160 + 360 = \$3520$

They should rent because it is cheaper.

b) Let x represent the number of days the gear is rented.

Rental cost = Total cost

$45x = 3520$

$x = 78.222...$

Paul and Ali could rent for up to 78 days for less than it would cost to buy.

c) $78.222... \div 30 = 2.607...$

It would take 3 full canoeing seasons for purchasing to be more economical than renting.

d) e.g. They might need to buy new equipment sooner than 3 years.

4. e.g., Using half day option:
Number of half days = $3.5 \div 0.5$
Number of half days = 7

Cost to rent = $700(7)$
Cost to rent = 4900

e.g. She should pay \$700 per half day if she is confident she can finish in 4 days or less, weather permitting, or she should pay \$6000 for the week so she a few extra days for unforeseen problems that may arise.

5. a) Let O represent the original value, and let V represent the new value. Let a represent the appreciation rate, and let n represent the number of years.

$V = O(1 + a)^n$
 $V = 7800(1.015)^n$
When $n = 15$, $V = 9751.81$
When $n = 16$, $V = 9898.09$
When $n = 17$, $V = 10\,046.56$

Susie will need to keep the print for about 17 years for its value to exceed \$10 000.

b) $V = O(1 + a)^n$
 $V = 7800(1.015)^n$

The doubled value is equal to \$15 600.

When $n = 45$, $V = 15\,242.86$
When $n = 46$, $V = 15\,471.50$
When $n = 47$, $V = 15\,703.58$

Susie will need to keep the print for about 47 years for the value to double.

6. Jake:

Cost over 5 years = $1400 \cdot 12 \cdot 5$
Cost over 5 years = \$84 000

Archie:

Down payment = $189\,900(0.1)$
Down payment = \$18 990

The present value is $189\,900(0.9)$ or \$170 910.

The regular payment amount is unknown.

The payment frequency is 26 times a year.

The number of payments is $26 \cdot 20$, or 520.

The payments are made at the end of the payment periods.

The annual interest rate is 4%.

The compounding frequency is 2 times a year.

The final value is \$0.

Using the financial application on a graphing calculator, the bi-weekly mortgage payments are \$476.215... or \$476.22.

The present value is \$170 910.
The regular payment amount is \$476.22.
The payment frequency is 26 times per year.
The number of payments is $26 \cdot 5$ or 130.
The payments are made at the end of the payment periods.
The annual interest rate is 4%.
The compounding period is 2 times a year.
The future value is unknown.

Using the financial application on a graphing calculator, the future value is \$139 927.295... or \$139 927.30.

Resale value of house = $189\,900(1 - 0.02)^5$
Resale value of house = \$171 654.559...

Profit = $171\,654.559... - 139\,927.295...$
Profit = \$31 727.264...

Cost over 5 years = $18\,990 + (130)(476.215...) - 31\,727.264...$

Cost over 5 years = $18\,990 + 61\,908.00 - 31\,727.264...$

Cost over 5 years = \$49 170.737...

Jake's costs for the 5 years is \$84 000, while Archie's costs are \$49 170.74.

7. a) Cost to rent = \$340 per month

Cost to buy new = \$1302.80

Cost to buy used = $1302.80(0.6)$

Cost to buy used = \$781.68

b) Cost to rent = $340(3)$

Cost to rent = \$1020

Cost to buy new = \$1302.80

Cost to buy used = \$781.68

e.g., Joe should buy the used scaffolding, because it is much less expensive. If he only wishes to use new equipment, then it would be more economical to rent the scaffolding.

8. a) $A = P(1 + i)^n$

$$A = 16\,500 \left(1 + \frac{0.124}{12} \right)^{24}$$

$$A = 16\,500(1.0103...)^{24}$$

$$A = 16\,500(1.279...)^{24}$$

$$A = 21\,117.198...$$

Therefore, the total cost of buying the equipment with a line of credit is \$21 117.198... or \$21 117.20.

b) Cost of leasing = $1000 + 480(24)$

Cost of leasing = \$12 520

c) Value after 2 years = $16\,500(1 - 0.3)^2$

Value after 2 years = \$8085

d) The total cost of buying equipment after reselling is $21\,117.20 - 8085 = \$13\,032.20$.

e.g. Lease, if she plans to use the equipment for only 2 years, or purchase equipment if she plans to continue to use the equipment beyond the 2 years.

9. a) Cost of lease = $1000 + 455.56(18)$

Cost of lease = \$9200.08

A lease would cost the centre \$9200.08.

$$\text{b) } A = P(1+i)^n$$

$$A = 11200 \left(1 + \frac{0.007}{12} \right)^{18}$$

$$A = 11200(1.000\dots)^{18}$$

$$A = 11200(1.010\dots)$$

$$A = 11318.184\dots$$

The cost of purchasing on credit is \$11 318.18..., or \$11 318.18.

Trade-in value = \$5000

Total cost = 11 318.18 – 5000

Total cost = \$6318.18

The centre will have to pay \$11 318.18 to buy the equipment, but when the trade-in value is included, it will only cost them \$6318.18, which is less than the cost to lease.

c) Days per week rented = 4

Total days rented = 4(52 weeks per year)(1.5 years)

Total days rented = 312

Revenue earned = 312(35)

Revenue earned = \$10 920

Revenue earned per month = 10 920/18

Revenue earned per month = \$606.67

Revenue – cost of lease = 10 920 – 9200.08

Revenue – cost of lease = \$1719.92

Revenue – cost of buying = 10 920 – 6318.18

Revenue – cost of buying = \$4601.82

This revenue can cover the total costs of both options, but the monthly revenue will not cover the monthly cost of purchasing the equipment on credit.

d) e.g., If the manager is sure she will stay with the same store after 18 months, she should buy. Otherwise, she should lease to keep her options open.

10. a) Rent - Monetary Costs:

Total cost = \$70 000

Rent - Other Costs:

Renting a car can be quite inconvenient if the company is not near the car rental location. Also, renting does not give the company equity.

Purchase - Monetary Costs:

Down payment = 32 000(0.05)

Down payment = \$1600

The present value is 32 000 – 1600, or \$30 400.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is 24.

The payments are made at the end of the payment periods.

The annual interest rate is 3.6%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the regular payment amount is \$1314.712..., or \$1314.72.

The cost of financing is

1314.712...(24) = \$31 553.089... or \$31 553.09.

Resale value = 32 000(1 – 0.4)²

Resale value = \$11 520

Total cost of one car = 1600 + 31 553.09 – 11 520

Total cost of one car = \$21 633.09

Total cost of two cars = 21 633.09(2)

Total cost of two cars = \$43 266.18

Purchase - Other Costs:

Purchasing a vehicle comes with many other costs, including insurance, which can be quite expensive. The company must also now include the expense of depreciation in their financials.

Lease - Monetary Costs:

Total cost of one car = 2000 + 770(24)

Total cost of one car = \$20 480

Total cost of two cars = 20 480(2)

Total cost of two cars = \$40 960

Lease - Other Costs:

Leasing requires money up front, which could be costly for a small company. Even though at the end of the two years, the company has no equity, the company must still pay for things such as mechanic visits, which the company wouldn't need to do if it rented.

b) e.g., I would recommend that the company lease both cars. The monetary cost for this option is the lower of all of the options. The closest in cost is purchasing, but this is reliant on gaining back about \$23 000 from reselling the vehicle, which may not be possible.

11. a) e.g., Lease, because it seems cheaper than renting and there is no need to purchase, because the tractor is only required for 9 months.

b) Buying:

The present value is \$18 600.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is 9.

The payments are made at the end of the payment periods.

The annual interest rate is 5.6%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing

calculator, the monthly payment amount is

\$2115.188... or \$2115.19.

The total cost is \$2115.188... (9) = \$19 036.694...

or \$19 036.70.

Renting:

Number of days in Mar.-Nov. = 275

Total cost = 60(275)

Total cost = \$16 500

Leasing:

Total cost = $2000 + 1345(9)$

Total cost = \$14 105

e.g., I would suggest that the company leases, because the total cost is the lowest of the three choices.

c) e.g., Renting is the most flexible option; the tractor may not be needed for the full 9 months. Renting is better than buying if the depreciated value of the tractor is less than \$2536 in 9 months.

12. a) Down payment: $250\,000(0.1) = \$25\,000$

Assume regular monthly payments:

The present value is $250\,000(0.90)$ or \$225 000.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is $(12 \cdot 30)$, or 360.

The payments are made at the end of the payment periods.

The annual interest rate is 3.5%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the monthly mortgage payments are \$1010.350..... or \$1010.36.

The present value is \$225 000.

The regular payment amount is \$1010.350....

The payment frequency is 12 times per year.

The number of payments is $(12 \cdot 5)$, or 60.

The payments are made at the end of the payment periods.

The annual interest rate is 3.5%.

The compounding period is 12 times a year.

The future value is unknown.

Using the financial application on a graphing calculator,

the future value is \$201 818.413... or

\$201 818.42.

Resale value of house = $250\,000(1 + 0.031)^5$

Resale value of house = \$291 228.139...

Profit = $291\,228.139... - 201\,818.413...$

Profit = \$89 409.725...

Profit over 5 years = $89\,409.725... - 25\,000$

$- (60)(1010.350...)$

Cost over 5 years = \$3788.692...

Kalik will make \$3788.69 on the house as an investment.

b) Kalik's profit is his interest earned

$$I = A - P$$

$$3788.69 = A - 25\,000$$

$$28\,788.69 = A$$

Kalik's bond must be worth \$28 788.69 in 5 years.

$$A = P(1+i)^n$$

$$28\,788.69 = 25\,000 \left(1 + \frac{i}{12}\right)^{60}$$

$$1.151... = \left(1 + \frac{i}{12}\right)^{60}$$

$$\sqrt[60]{1.151...} = \sqrt[60]{\left(1 + \frac{i}{12}\right)^{60}}$$

$$1.002... = 1 + \frac{i}{12}$$

$$i = 0.02825...$$

Kalik needs an interest rate of 2.83% compounded monthly to make the bond a comparable investment to the house.

c) e.g. The bond is less risky; if he bought the house, he would not need to pay rent.

13. a) The present value is \$2000.

The regular payment amount is \$125.

The payment frequency is 12 times a year.

The number of payments is unknown.

The payments are made at the end of the payment periods.

The annual interest rate is 4.8%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the number of payments is 16.567..., or 17.

In total it will cost Gerald $125(16.567...)$ =

2070.999... or \$2071.00 to open his business

b) Total cost = \$2070.999...

Amount of revenue needed:

$$5000 + 2070.999... = \$7070.999...$$

Amount charged per client:

$$7070.999.../10 = \$707.099...$$

In order to earn \$5000 over the summer, Gerald should charge each customer \$707.10.

14. e.g. Renting, buying or leasing a car for 3 years. Renting may be the best option if only occasional use is needed. Either buying or leasing may be the cheapest option, depending on finance, but leasing may give the flexibility to change cars later.

15. Assuming payment frequency of 12

Buy:

The present value is $1800 + 1600 + 2800$, or \$6200.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is 18.

The payments are made at the end of the payment periods.

The annual interest rate is 8.7%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the monthly payment amount is \$368.653... or \$368.66. The total cost is $\$368.653 \dots (18) = \$6635.762 \dots$ or \$6635.77.

Let x represent the number of days the lenses are rented for.

$$40x = 6635.762 \dots$$

$$x = 165.894 \dots$$

Number of days per month = $x/18$

Number of days per month = 9.216...

Carmen could rent the lenses for 9 days a month for the cost of buying.

16. e.g., Equipping an office with a computer network for 2 years.

Option 1: Buying the equipment for \$7500 financed at 5.5% interest, compounded monthly, paid back over 2 years, with depreciation of 40% per year on resale.

Option 2: Leasing the equipment at \$230 per month. Cost of buying is \$5237.28; cost of leasing is \$5520.

Buying is slightly cheaper, but leasing avoids the costs and uncertainties of resale.

History Connection, page 133

A. e.g., British Columbia; the Criminal Code of Canada states that any rate of interest above 60%/a is illegal.

B. e.g., In 2006, the B.C. Supreme Court issued a decision on a case against a moneylender stating that any charges, flat rate or otherwise, were to be interpreted as interest. Also in 2006, the federal government amended the criminal code to allow provinces to regulate the money-lending business.

Chapter Self-Test, page 134

1. a) $A = P(1+i)^n$

$$A = 5000 \left(1 + \frac{0.0725}{4} \right)^8$$

$$A = 5000(1+0.018 \dots)^8$$

$$A = 5000(1.018 \dots)^8$$

$$A = 5000(1.154 \dots)$$

$$A = 5772.697 \dots$$

$$I = A - P$$

$$I = 5772.697 \dots - 5000$$

$$I = 772.697 \dots$$

\$5772.70 will be due at the maturity date, with \$772.70 being interest.

b) $A = P(1+i)^n$

$$A = 5000(1+0.0725)^2$$

$$A = 5000(1.0725)^2$$

$$A = 5000(1.150 \dots)$$

$$A = 5751.281 \dots$$

$$I = A - P$$

$$I = 5751.281 \dots - 5000$$

$$I = 751.281 \dots$$

\$5751.28 will be due at the maturity date, with \$751.28 being interest.

c) The present value is \$5000.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is 24.

The payments are made at the end of the payment periods.

The annual interest rate is 7.25%.

The compounding frequency is 4 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the regular monthly payments will be \$224.331..., or \$224.34.

The total amount paid is

$$\$224.331 \dots (24) = \$5383.957 \dots \text{ or } \$5383.96.$$

The interest paid is \$383.96.

Ryan will pay \$5383.96 altogether, and he will pay \$383.96 in interest.

2. a) $P = \frac{A}{(1+i)^n}$

$$P = \frac{15000}{\left(1 + \frac{0.055}{12} \right)^{48}}$$

$$P = \frac{15000}{1.245 \dots}$$

$$P = 12043.834 \dots$$

Allison can borrow at most \$12 043.83.

b) The present value is \$12 043.83.

The regular payment amount is unknown.

The payment frequency is 12 times a year.

The number of payments is $5 \cdot 12$ or 60.

The payments are made at the end of the payment periods.

The annual interest rate is 5.5%.

The compounding frequency is 12 times a year.

The future value is \$0.

Using the financial application on a graphing calculator, the regular payment amount is \$230.051. Allison will be able to repay the loan in 5 years if she pays \$230.05 a month. She will pay \$1759.24 in interest.