

### Math in Action, page 56

- e.g., the interest rate for a 5-year GIC is 1.267% per annum, and the interest rate for a 5-year mortgage is 5.19%, compounded semi-annually.
- The future value of the investment is \$54 230.386... or \$54 230.39, and the interest earned is \$4230.39.
- The future value of the mortgage (with no regular loan payments) is \$64 599.917... or \$64 599.92, and the interest charged is \$14 599.92.
- The difference between the interest paid by the bank and the interest earned by the bank is \$10 369.53, which represents the amount earned by the bank.
- The greater the difference between the amount of interest that a bank pays on an investment and the amount of interest that the bank charges on a loan, the more money the bank will make.

### Lesson 1.6: Solving Investment Portfolio Problems, page 64

1. a) A GIC would allow Stan to earn interest on the \$300 he has from his summer job. The high interest savings account would allow him to earn interest on his weekly savings.

b) **GIC:** The principal is \$300.

The annual interest rate is 5%.

The compounding period is annual, or once per year.

The term (in years) is 1 year.

*The future value is unknown.*

The GIC will be worth \$315 at the end of one year.

**Savings account:** The regular payment amount is \$15.

The payment frequency is weekly, or 52 times per year.

The number of payments is 52.

The payments are made at the end of each payment period.

The annual interest rate is 2.9%.

The compounding frequency is weekly, or 52 times per year.

*The future value is unknown.*

The savings account will be worth \$791.20 at the end of one year.

Total saved:  $315 + 791.20 = 1106.20$

Difference:  $1750 - 1106.20 = 643.80$

Stan will not have enough money in one year to buy the guitar. He needs \$643.80 more.

*The regular payment amount is unknown.*

The payment frequency is weekly, or 52 times per year.

The number of payments is 52.

The payments are made at the end of each payment period.

The annual interest rate is 2.9%.

The compounding frequency is weekly, or 52 times per year.

Stan needs to deposit \$27.21 into his account each week to meet his goal.

2. **Investment account:** The regular payment amount is \$50.

The payment frequency is weekly, or 52 times per year.

The number of payments is 104.

The payments are made at the end of each payment period.

The annual interest rate is 4.1%.

The compounding frequency is weekly, or 52 times per year.

*The future value is unknown.*

The savings account will be worth \$5416.92 at the end of two years.

Difference:  $10\ 000 - 5416.92 = 4583.08$

Robin and Leslie need an additional \$4583.08 in two years.

**CSB:** *The principal is unknown.*

The annual interest rate is 6%.

The compounding period is annual, or once per year.

The term (in years) is 2 years.

The future value is \$4583.08.

Robin and Leslie's parents should give them \$4078.92.

### 3. CSBs

Year	$P$ (\$)	$i$	$n$	$A$ (\$)
1	1000	0.034		1351
2	1000	0.034		1306
3	1000	0.034		1263
4	1000	0.034		1222
5	1000	0.034		1181
6	1000	0.034		1143
7	1000	0.034		1105
8	1000	0.034		1069
9	1000	0.034		1034
10	1000	0.034		1000
				11 677

The CSBs are worth \$11 677.32.

**Trust account:** The principal is \$3000.

The annual interest rate is 4.3%.

The compounding period is quarterly, or 4 times per year.

The term (in years) is 42 years.

*The future value is unknown.*

The trust account is worth \$18 083.03.

**GIC:** The principal is \$10 000

The annual interest rate is 3.95%.

The compounding period is semi-annual, or 2 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

The GIC is worth \$14 786.80.

Total:  $11\ 677.32 + 18\ 083.03 + 14\ 786.80 = 44\ 547.15$

**Five-year bond:** The principal is \$44 547.15

The annual interest rate is 5.1%.

The compounding period is annual, or once per year.

The term (in years) is 5 years.

*The future value is unknown.*

The five-year bond will be worth \$57 125.96.

4. a) **Investment account:** The regular payment amount is \$50.

The payment frequency is monthly, or 12 times per year.

The number of payments is 36.

The payments are made at the end of each payment period.

The annual interest rate is 2.7%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

The investment account is worth \$1872.72.

**RESP account:** The regular payment amount is \$10.

The payment frequency is monthly, or 12 times per year.

The number of payments is 216.

The payments are made at the end of each payment period.

The annual interest rate is 3.2%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

The RESP account is worth \$2915.79.

Total:  $1872.72 + 2915.79 = 4788.51$

Paula has \$4788.51 when she starts first year.

**b)** Difference:  $4788.51 - 2000 = 2788.51$

The principal is \$2788.51

The annual interest rate is 3.5%.

The compounding period is daily, or 365 times per year.

The term (in years) is 1 year.

*The future value is unknown.*

The savings account is worth \$2887.83 after one year.

Paula will have enough money saved for second year.

**c)** Difference:  $2887.83 - 2000 = 887.83$

The principal is \$887.83

The annual interest rate is 3.5%.

The compounding period is daily, or 365 times per year.

The term (in years) is 1 year.

*The future value is unknown.*

The savings account is worth \$919.45 after one year.

Amount short:  $2000 - 919.45 = 1080.55$

Paula will not have enough money saved for third year.

She needs to save \$1080.55 over the summer.

**5. a)** Gayla's principal:  $(15)(12)(12) = 2160$

Corey principal:  $(18)(12)(10) = 2160$

They both invested \$2160.

**b)** Gayla has the greater savings because her money has been invested for 12 years longer than Corey's money.

**Gayla: Regular payment investment:**

The regular payment amount is \$15.

The payment frequency is monthly, or 12 times per year.

The number of payments is 144.

The payments are made at the end of each payment period.

The annual interest rate is 3.6%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

The investment is worth \$2696.70.

**Lump sum investment:** The principal is \$2696.70.

The annual interest rate is 3.8%.

The compounding period monthly, or 12 times per year.

The term (in years) is 10 year.

*The future value is unknown.*

Gayle's total investment is worth \$3940.97.

**Corey:** The regular payment amount is \$18.

The payment frequency is monthly, or 12 times per year.

The number of payments is 120.

The payments are made at the end of each payment period.

The annual interest rate is 3.8%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

Corey's investment is worth \$2622.73.

Gayle has a larger investment than Corey.

**6. GIC:** The principal is \$10 000.

The annual interest rate is 4.4%.

The compounding period is semi-annual, or 2 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

The GIC is worth \$15 453.18.

**CSBs:**

Year	P (\$)	i	n	A (\$)
1	1000	0.047	5	1258.15
2	1000	0.047	4	1201.67
3	1000	0.048	3	1151.02
4	1000	0.048	2	1098.30
5	1000	0.049	1	1049.00
				5758.14

The CSBs are worth \$5758.14.

**Trust account: Regular payment investment:**

The regular payment amount is \$500.

The payment frequency is annual, or once per year.

The number of payments is 20.

The payments are made at the end of each payment period.

The annual interest rate is 5%.

The compounding frequency is annual, or once per year.

*The future value is unknown.*

The first investment is worth \$16 532.98.

**Lump sum investment:** The principal is \$16 532.98.

The annual interest rate is 5%.

The compounding period is annual, or once per year.

The term (in years) is 20 years.

*The future value is unknown.*

The trust account is worth \$43 866.91.

Total:  $15\,453.18 + 5758.14 + 43\,866.91 = 65\,078.23$

Jayne's investment portfolio is worth \$65 078.23.

Principal:  $10\,000 + 1000(5) + 500(20) = 25\,000$

Interest:  $65\,078.23 - 25\,000 = 40\,078.23$

$$\text{Rate of return} = \frac{40\,078.23}{25\,000}$$

$$\text{Rate of return} = 1.603\dots$$

Jayne's rate of return on her investment is 160.3%.

**7. a) Savings account:** The regular payment amount is \$50.

The payment frequency is monthly, or 12 times per year.

The number of payments is 240.

The payments are made at the end of each payment period.

The annual interest rate is 2.7%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

When Ela is forty, the savings account is worth \$15 888.14.

**First 10-year GIC:** The principal is \$15 888.14.

The annual interest rate is 4.2%.

The compounding period is monthly, or 12 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

When Ela is fifty, the GIC is worth \$24 163.41.

**Second 10-year GIC:** The principal is \$24 163.41.  
The annual interest rate is 4.3%.  
The compounding period is monthly, or 12 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

When Ela is sixty, the GIC is worth \$37 116.85.

**First 10-year bond:** The principal is \$500.

The annual interest rate is 3.9%.

The compounding period is annual, or once per year.

The term (in years) is 10 years.

*The future value is unknown.*

When Ela is fifty, the bond is worth \$733.04.

**Second 10-year bond:** The principal is \$733.04.

The annual interest rate is 3.9%.

The compounding period is annual, or once per year.

The term (in years) is 10 years.

*The future value is unknown.*

When Ela is sixty, the bond is worth \$1074.69.

Total:  $37\,116.85 + 1074.69 = 38\,191.54$

When Ela turns sixty, the total value of his portfolio is \$38 191.54.

**b) Principal:**  $50(240) + 500 = 12\,500$

Interest:  $38\,191.54 - 12\,500 = 25\,691.54$

$$\text{Rate of return} = \frac{25\,691.54}{12\,500.00}$$

Rate of return = 2.055...

Ela's rate or return on his investment is 205.5%.

**8. a) Cost of trip:**  $2000 + 300(12) = 5600$

The principal is \$1200.

The annual interest rate is 5%.

The compounding period is annual, or once per year.

The term (in years) is 4 years.

*The future value is unknown.*

The GIC is worth \$1458.61 at maturity.

Difference:  $5600 - 1458.61 = 4141.39$

No, Jo will not have enough money for her trip in four years. She will be \$4141.39 short.

**b) If she makes weekly deposits for four years, that is 208 deposits. Divide the amount she needs to have by 208.**

$$\frac{4141.39}{208} = 19.910\dots$$

e.g., I estimate that Jo will need to make deposits of an amount less than \$19.91 to meet her goal.

*The regular payment amount is unknown.*

The payment frequency is weekly, or 52 times per year.

The number of payments is 208.

The payments are made at the end of each payment period.

The annual interest rate is 2.3%.

The compounding frequency is weekly, or 52 times per year.

The future value is \$4141.39.

Jo needs to make weekly deposits of \$19.01 to meet her goal.

**9. Option A: GIC:** The principal is \$15 000.

The annual interest rate is 4.1%.

The compounding period is annual, or once per year.

The term (in years) is 10 years.

*The future value is unknown.*

The GIC is worth \$22 418.09.

**Savings account:** The principal is \$3638.

The annual interest rate is 3.9%.

The compounding period is weekly, or 52 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

The savings account is worth \$5372.47.

Total:  $22\,418.09 + 5372.47 = 27\,792.04$

Option A will be worth \$27 790.56 in ten years.

**Option B:** The principal is \$18 638.

The annual interest rate is 3.9%.

The compounding period is daily, or 365 times per year.

The term (in years) is 10 years.

*The future value is unknown.*

Option B will be worth \$27 527.39 in ten years.

Option A is worth \$263.17 more than option B.

Derek should chose option A.

**10.** e.g., An investment portfolio may consist of single payment investments or regular payment investments. Some investments may lock in funds for a period of time limiting access to it but often offer a higher interest rate. Investments offering a lower interest rate usually allow access to the funds. A higher investment amount usually tends to offer higher interest rates. The greater the principal, term, interest rate, and compounding frequency, the faster the investment will grow.

**11. a)** Betty has \$200 000 now, so this is the present value. She will have no money in the account in ten years, so the future value is zero. Solve for the regular payment amount.

*The regular payment amount is unknown.*

The present value is \$200 000.

The payment frequency is monthly, or 12 times per year.

The number of payments is 120.

The payments are made at the end of each payment period.

The annual interest rate is 5%.

The compounding frequency is monthly, or 12 times per year.

The future value is 0.

I used the financial application on my calculator:

Betty will receive 120 monthly payments of \$2121.31.

**b) The regular payment amount is unknown.**

The present value is \$200 000.

The payment frequency is monthly, or 12 times per year.

The number of payments is 120.

The payments are made at the end of each payment period.

The annual interest rate is 7.5%.

The compounding frequency is monthly, or 12 times per year.

The future value is 0.

I used the financial application on my calculator: Betty would receive 120 monthly payments of \$2374.04 if the interest rate increased to 7.5%.

**12. Regular payment investment:** The regular payment amount is \$150.

The payment frequency is annual, or once per year.

The number of payments is 50.

The payments are made at the end of each payment period.

The annual interest rate is 8.8%.

The compounding frequency is annual, or once per year.

*The future value is unknown.*

The investment is worth \$113 920.88.

**Monthly Income:** *The regular payment amount is unknown.*

The present value is \$113 920.88.

The payment frequency is monthly, or 12 times per year.

The number of payments is 120.

The payments are made at the end of each payment period.

The annual interest rate is 4.8%.

The compounding frequency is monthly, or 12 times per year.

The future value is 0.

I used the financial application on my calculator: Stu's grandmother will receive 120 monthly payments of \$1197.20. She will receive \$143 664.00 altogether in payments.

### Chapter Self-Test, page 68

1. a)  $\frac{72}{6} = 12$

The investment will double in about 12 years.

The principal is \$40 000.

The annual interest rate is 6%.

The compounding period is annual, or once per year.

*The term (in years) is unknown.*

The future value is \$80 000.

I used the financial application on my calculator: Hal's investment will double in 11.9 years.

b)  $A = P + Prt$

A is \$80 000; P is \$40 000; r is 6% or 0.06

$$80\,000 = 40\,000 + (40\,000)(0.06)(t)$$

$$40\,000 = 2400t$$

$$t = 16.666\dots$$

With simple interest, Hal's investment will double in 16.67 years.

**2. Option A:** The principal is \$12 000.

The annual interest rate is 6%.

The compounding period is semi-annual, or 2 times per year.

*The term (in years) is unknown.*

The future value is \$50 000.

I used the financial application on my calculator: The option A investment will grow to \$50 000 in 48.28 periods or 24.14 years.

**Option B:** The principal is \$12 000.

The annual interest rate is 5.1%.

The compounding period is quarterly, or 4 times per year.

*The term (in years) is unknown.*

The future value is \$50 000.

I used the financial application on my calculator:

The option B investment will grow to \$50 000 in 112.64 periods or 28.16 years.

Val should choose option A because she will reach her financial goal about 4 years sooner.

**3. a) Principal:**  $5(12)(20) = \$1200$

Warren invested \$1200.

The regular payment amount is \$5.

The payment frequency is monthly, or 12 times per year.

The number of payments is 240.

The payments are made at the end of each payment period.

The annual interest rate is 5.8%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

Warren's investment is currently worth \$2256.24.

Interest:  $2256.24 - 1200 = 1056.24$

$$\text{Rate of return} = \frac{1056.24}{1200.00}$$

$$\text{Rate of return} = 0.880\dots$$

Warren's rate of return on his investment is 88%.

**b)** Warren invested a total of \$1200. If he made monthly deposits for five years, each deposit would be:

$$\frac{1200}{(5)(12)} = 20$$

The regular payment amount is \$20.

The payment frequency is monthly, or 12 times per year.

The number of payments is 60.

The payments are made at the end of each payment period.

The annual interest rate is 5.8%.

The compounding frequency is monthly, or 12 times per year.

*The future value is unknown.*

Warren's investment would be currently worth \$1388.25.

**c)** *The regular payment amount is unknown.*

The payment frequency is monthly, or 12 times per year.

The number of payments is 60.

The payments are made at the end of each payment period.

The annual interest rate is 5.8%.

The compounding frequency is monthly, or 12 times per year.

The future value is \$2256.24.

I used the financial application on my calculator:

Warren would have needed to make monthly payments of \$32.50.

**4. a) Alex: GIC:** The principal is \$5000.

The annual interest rate is 2.6%.

The compounding period is annual, or once per year.