The future value of the initial investment is $1340.10.

	Future Value ($)
Initial Investment	1276.28
Option C: Increase compounding frequency to monthly	1283.36
Option B: Increase interest rate by 1%	1338.23
Option A: increase principal by $50	1340.10
Option D: Increase term by 1 year	1340.10

Option A and option D have the greatest, and equal, impact on the future value. Option C has the least impact.

7. a) *The present value is unknown.*
The annual interest rate is 6%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 3 years.
The future value is $10 000.
Desiree should invest $8356.45 to have $10 000 in three years.
b) *The present value is unknown.*
The annual interest rate is 6%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 3 years.
The future value is $10 000.
Desiree should invest $8374.84 to have $10 000 in three years.

8. a) i) *The present value is unknown.*
The annual interest rate is 7.2%.
The compounding period is annual, or once per year.
The term (in years) is 22 years.
The future value is $11 000.
The original investment was $2382.91.
ii) *The present value is unknown.*
The annual interest rate is 7.2%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 22 years.
The future value is $11 000.
The original investment was $2320.40.
b) The ratio would be higher because the compounding is more frequent.
The principal is $11 000.
The annual interest rate is 7.2%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 22 years.
The future value is unknown.
The future value of the investment is $53 365.38
Original investment:
i) $\frac{A}{P} = \frac{11\,000.00}{2382.91}$ **ii)** $\frac{A}{P} = \frac{11\,000.00}{2320.40}$
$\frac{A}{P} = 4.616...$ $\frac{A}{P} = 4.740...$
New investment:
$\frac{A}{P} = \frac{53\,365.38}{11\,000.00}$
$\frac{A}{P} = 4.851...$

The new investment has a higher ratio than the original investment.

9. The principal is $400.
The annual interest rate is unknown.
The compounding period is monthly, or 12 times per year.
The term (in years) is 10.
The future value is $625.
The annual interest rate is 4.47%.

10. $250 will double twice to become $1000.
$\frac{72}{6} \times 2 = 24$
It will take about 24 years for the $250 investment to grow to $1000.
The principal is $250.
The annual interest rate is 6%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is unknown.
The future value is $1000.
It will take 23.45 years, or 23.5 years for the $250 investment to grow to $1000.

Lesson 1.5: Investments Involving Regular Payments, page 55

1. a) The regular payment amount is $200.
The payment frequency is monthly, or 12 times per year.
The number of payments is 600.
The payments are made at the end of each payment period.
The annual interest rate is 4.8%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
The future value of the investment is $498 526.60.
b) The regular payment amount is $1750.
The payment frequency is semi-annual, or 2 times per year.
The number of payments is 40.
The payments are made at the end of each payment period.
The annual interest rate is 5.6%.
The compounding frequency is semi-annual, or 2 times per year.
The future value is unknown.
The future value of the investment is $126 127.32
c) The regular payment amount is $50.
The payment frequency is quarterly or 4 times per year.
The number of payments is 160.
The payments are made at the end of each payment period.
The annual interest rate is 8.4%.
The compounding frequency is quarterly or 4 times per year.
The future value is unknown.
The future value of the investment is $63 820.79.
d) The regular payment amount is $5500.
The payment frequency is semi-annual, or 2 times per year.

The number of payments is 24.
The payments are made at the end of each payment period.
The annual interest rate is 6.5%.
The compounding frequency is semi-annual, or 2 times per year.
The future value is unknown.
The future value of the investment is $195 389.47.

2. a) The regular payment amount is $100.
The payment frequency is monthly, or 12 times per year.
The number of payments is 72.
The payments are made at the end of each payment period.
The annual interest rate is unknown.
I used the financial application on my calculator: The compounding frequency is monthly, or 12 times per year.
The future value is $7800.61.
The interest rate is 2.68%.
b) *The regular payment amount is unknown.*
The payment frequency is semi-annual, or 2 times per year.
The number of payments is 14.
The payments are made at the end of each payment period.
The annual interest rate is 3.5%.
The compounding frequency is semi-annual, or 2 times per year.
The future value is $3927.38.
I used the financial application on my calculator: The payment amount is $250.
c) The regular payment amount is $20 000.
The payment frequency is quarterly or 4 times per year.
The number of payments is unknown.
The payments are made at the end of each payment period.
The annual interest rate is 4.75%.
The compounding frequency is quarterly or 4 times per year.
The future value is $1 054 970.01.
I used the financial application on my calculator: The number of payments is 41.199… so the term is 10.3 years.

3. The regular payment amount is $350.
The payment frequency is monthly, or 12 times per year.
The number of payments is 216.
The payments are made at the end of each payment period.
The annual interest rate is 7.2%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
The future value of the investment is $154 030.54.
Interest: 154 030.54 – (216)(350) = 78 430.54
Darlene earned $78 430.54 in interest.

4. Investment A will earn more interest than investment B because the $5000 earns interest for the full five years while four $1000 payments earn interest for less than five years.
Investment A: The principal is $5000.
The annual interest rate is 6%.
The compounding period is annual, or once per year.
The term (in years) is 5.
The future value is unknown.
The future value of investment A is $6691.13.
Interest: 6691.13 – 5000 = 1691.13
Investment A earned $1691.13 in interest.
Investment B: The regular payment amount is $1000.
The payment frequency is annual, or once per year.
The number of payments is 5.
The payments are made at the end of each payment period.
The annual interest rate is 6%.
The compounding frequency is annual, or once per year.
The future value is unknown.
The future value of the investment is $5637.09.
Interest: 5637.09 – (1000)(5) = 637.09
Investment B earned $637.09 in interest.
Investment A earns more interest than investment B.

5. The regular payment amount is $600.
The payment frequency is quarterly or 4 times per year.
The number of payments is 20.
The payments are made at the end of each payment period.
The annual interest rate is 8%.
The compounding frequency is quarterly or 4 times per year.
The future value is unknown.
Fraser will have $14 150.77 when he is 21.
Interest: 14 150.77 – (600)(20) = 2150.77
Fraser earned $2150.77 in interest.

6. *The regular payment amount is unknown.*
The payment frequency is monthly, or 12 times per year.
The number of payments is 24.
The payments are made at the end of each payment period.
The annual interest rate is 6%.
The compounding frequency is monthly, or 12 times per year.
The future value is $5000.
I used the financial application on my calculator: Zoey made regular deposits of $196.60.

7. a) i) *The regular payment amount is unknown.*
The payment frequency is monthly, or 12 times per year.
The number of payments is 420.
The payments are made at the end of each payment period.
The annual interest rate is 14.6%.
The compounding frequency is monthly, or 12 times per year.
The future value is $1 000 000.
Jayne needs to invest $76.22 at the end of each month.
ii) *The regular payment amount is unknown.*
The payment frequency is monthly, or 12 times per year.
The number of payments is 420.
The payments are made at the end of each payment period.
The annual interest rate is 6.9%.
The compounding frequency is monthly, or 12 times per year.
The future value is $1 000 000.
Jayne needs to invest $568.60 at the end of each month.
b) Option i):
Interest: 1 000 088.05– (76.22)(420) = 968 075.65

$$\text{Rate of return} = \frac{968075.65}{32012.40}$$

Rate of return = 30.240 645…
Option i) rate of return is 3024.06%.
Option ii):
Interest: 1 000 008.39 – (568.60)(420) = 761 196.39

$$\text{Rate of return} = \frac{761196.39}{238812}$$

Rate of return = 3.187 429…
Option ii) rate of return is 318.74%.
Jayne should choose option i) since the rate of return is almost ten times greater than the rate of return for option ii).

8. a) The regular payment amount is $25.
The payment frequency is monthly, or 12 times per year.
The number of payments is 60.
The payments are made at the end of each payment period.
The annual interest rate is 4.2%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
At the end of five years, the future value of Aaron's account is $1665.90.
b) *The present value is unknown.*
The annual interest rate is 4.2%.
The compounding period is annual, or once per year.
The term (in years) is 5.
The future value is $1665.90.
The present value is $1348.02.
Casey invested $1356.16.
c) Aaron's investment will be worth more at the end of 10 years because he is making more frequent deposits and the interest is compounded more often.

Aaron: The regular payment amount is $25.
The payment frequency is monthly, or 12 times per year.
The number of payments is 120.
The payments are made at the end of each payment period.
The annual interest rate is 4.2%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
At the end of five years, the future value of Aaron's account is $3720.33.
Casey: The present value is $1356.16.
The annual interest rate is 4.2%.
The compounding period is annual, or once per year.
The term (in years) is 10.
The future value is unknown.
At the end of five years, the future value of Casey's account is $2046.39.
Aaron's investment is worth more than Casey's investment.

9. The regular payment amount is $500.
The payment frequency is monthly, or 12 times per year.
The number of payments is 60.
The payments are made at the end of each payment period.
The annual interest rate is unknown.
The compounding frequency is monthly, or 12 times per year.
The future value is $35 000.
I used the financial application on my calculator:
The annual interest rate is 6.13%.

10. The regular payment amount is $1000.
The payment frequency is semi-annual, or 2 times per year.
The number of payments is unknown.
The payments are made at the end of each payment period.
The annual interest rate is 7.5%.
The compounding frequency is semi-annual, or twice per year.
The future value is $10 000.
I used the financial application on my calculator: It will take 8.65 payments or 4.33 years for the investment to be worth at least $10 000.

11. Dee: The regular payment amount is $1000.
The payment frequency is monthly, or 12 times per year.
The number of payments is 60.
The payments are made at the end of each payment period.
The annual interest rate is 4%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
At the end of five years, the future value of Dee's account is $66 298.98.

Pete: The regular payment amount is $500.
The payment frequency is monthly, or 12 times per year.
The number of payments is 120.
The payments are made at the end of each payment period.
The annual interest rate is 4%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
At the end of ten years, the future value of Pete's account is $73 624.90.
b) The future value of Pete's investment is higher than Dee's investment because each deposit Pete made had more time to accumulate interest, and additional interest was paid on the accumulated interest.

12. Trey: The regular payment amount is $600.
The payment frequency is quarterly or 4 times per year.
The number of payments is 8.
The payments are made at the end of each payment period.
The annual interest rate is 3.6%.
The compounding frequency is quarterly or 4 times per year.
The future value is unknown.
At the end of two years, the future value of Trey's account is $4953.95.
Sam: The regular payment amount is $2400.
The payment frequency is annual, or once per year.
The number of payments is 2.
The payments are made at the end of each payment period.
The annual interest rate is 3.8%.
The compounding frequency is annual, or once per year.
The future value is unknown.
At the end of two years, the future value of Sam's account is $4891.20.
Difference: 4953.95. – 4891.20 = 62.75
Trey's investment is worth $62.75 more than Sam's investment.

13. Total price of entertainment system:
(2499)(1.13) = 2823.87
The regular payment amount is $225.
The payment frequency is monthly, or 12 times per year.
The number of payments is 12.
The payments are made at the end of each payment period.
The annual interest rate is 3.6%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
In one year, Miguel's account will be worth $2745.00.
$2745.00 – $2823.87 = – $78.87
Miguel will be short $78.87.

14. a) Jill: *The regular payment amount is unknown.*
The payment frequency is monthly, or 12 times per year.
The number of payments is 360.
The payments are made at the end of each payment period.
The annual interest rate is 7.4%.
The compounding frequency is monthly, or 12 times per year.
The future value is $250 000.
Jill needs to invest $189.29 at the end of each month.
$189.29 · 360 = $68 144.40
Vaughn: *The regular payment amount is unknown.*
The payment frequency is monthly, or 12 times per year.
The number of payments is 360.
The payments are made at the end of each payment period.
The annual interest rate is 11.6%.
The compounding frequency is monthly, or 12 times per year.
The future value is $250 000
Vaughn needs to invest $78.16 at the end of each month.
$78.16 · 360 = 28 137.60
$68 144.40 – $28 137.60 = $40 006.80
Jill needs to invest $40 006.80 more than Vaughn over the 30 years.
b) The regular payment amount is $189.29.
The payment frequency is monthly, or 12 times per year.
The number of payments is 360.
The payments are made at the end of each payment period.
The annual interest rate is 11.6%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
Vaughn will have $605 501.19 at the end of 30 years.

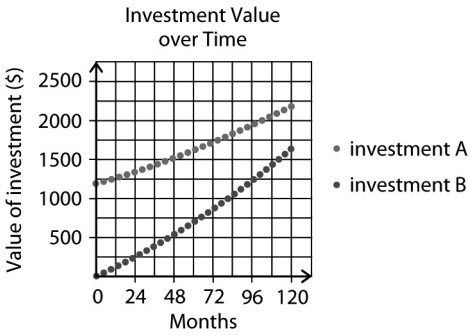
15. The regular payment amount is $300.
The payment frequency is weekly, or 52 times per year.
The number of payments is 104.
The payments are made at the end of each payment period.
The annual interest rate is 10.5%.
The compounding frequency is weekly, or 52 times per year.
The future value is unknown.
Tim will have $34 679.08 at the end of two years.
With the $50 000 he gets from selling his first sailboat, he has $84 679.08.
Difference: 120 000 – 84 679.08 = 35 320.92
Tim will not have enough money to buy his dream sailboat. He needs an additional $35 320.92, assuming the sailboat's price has not increased in the two years.

16. a)

Similar	Different
Both have a total principal of $1200. Both earn 6% annual interest compounded monthly. Both are invested for 10 years.	Option A is a single deposit while option B is a series of payments.

b)

	Option A	Option B
Principal ($)	1200 lump sum	10 per month
Interest Rate per Annum	0.06	0.06
Periods per Year	12	12
Value at End of Year		
0	1200	0
2	1352.59	254.32
4	1524.59	540.98
6	1718.45	864.09
8	1936.97	1228.29
10	2182.28	1638.79



Time (years)	Value of Investment ($)	
	Option A	**Option B**
0	1200	0
2	1352.59	254.32
4	1524.59	540.98
6	1718.45	864.09
8	1936.97	1228.29
10	2182.28	1638.79

The value of Option B grows much faster than the value of Option A, but much of Option B's growth is due to the monthly deposits. Option A earns more interest than option B over the 10 years.

17. No. e.g., The probability that his tips will equal the same amount each month is very unlikely. (However, the problem could be solved using the formula $A = R + R(1 + i)^1 + R(1 + i)^2 + \ldots + R(1 + i)^{n-1}$ and changing R to the value Farah deposits each month.)

18. Amount needed: 5000 – 5000(0.25) = 3750
Farah will need $3750 in her account after two years.
The regular payment amount is unknown.
The payment frequency is monthly, or 12 times per year.
The number of payments is 24.
The payments are made at the end of each payment period.
The annual interest rate is 3.6%.
The compounding frequency is monthly, or 12 times per year.
The future value is $3750.
I used the financial application on my calculator:
Farah needs to deposit $150.93 at the end of each month in order to buy a new computer in two years.

19. Regular payment investment: The regular payment amount is $50.
The payment frequency is semi-annual, or 2 times per year.
The number of payments is 40.
The payments are made at the end of each payment period.
The annual interest rate is 5%.
The compounding frequency is semi-annual, or 2 times per year.
The future value is unknown.
The value of the account after 20 years is $3370.13.
Lump sum investment: The principal is $3370.13.
The annual interest rate is 5%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 10 years.
The future value is unknown.
The future value of the investment is $53 365.38
The final value of the investment is $5522.35.

20. Amount needed: 46 000(1.05) – 4000 = 44 300
The regular payment amount is $550.
The payment frequency is monthly, or 12 times per year.
The number of payments is 60.
The payments are made at the end of each payment period.
The annual interest rate is 9.8%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
In five years. Pat's account will be worth $42 366.01. Pat needs to save $44 300 to afford the car, so he cannot make the purchase. Pat will be short by $1933.99.

Math in Action, page 56
• e.g., the interest rate for a 5-year GIC is 1.267% per annum, and the interest rate for a 5-year mortgage is 5.19%, compounded semi-annually.
• The future value of the investment is $54 230.386… or $54 230.39, and the interest earned is $4230.39.
• The future value of the mortgage (with no regular loan payments) is $64 599.917… or $64 599.92, and the interest charged is $14 599.92.
• The difference between the interest paid by the bank and the interest earned by the bank is $10 369.53, which represents the amount earned by the bank.
• The greater the difference between the amount of interest that a bank pays on an investment and the amount of interest that the bank charges on a loan, the more money the bank will make.

Lesson 1.6: Solving Investment Portfolio Problems, page 64

1. a) A GIC would allow Stan to earn interest on the $300 he has from his summer job. The high interest savings account would allow him to earn interest on his weekly savings.
b) GIC: The principal is $300.
The annual interest rate is 5%.
The compounding period is annual, or once per year.
The term (in years) is 1 year.
The future value is unknown.
The GIC will be worth $315 at the end of one year.
Savings account: The regular payment amount is $15.
The payment frequency is weekly, or 52 times per year.
The number of payments is 52.
The payments are made at the end of each payment period.
The annual interest rate is 2.9%.
The compounding frequency is weekly, or 52 times per year.
The future value is unknown.
The savings account will be worth $791.20 at the end of one year.
Total saved: 315 + 791.20 = 1106.20
Difference: 1750 – 1106.20 = 643.80
Stan will not have enough money in one year to buy the guitar. He needs $643.80 more.
The regular payment amount is unknown.
The payment frequency is weekly, or 52 times per year.
The number of payments is 52.
The payments are made at the end of each payment period.
The annual interest rate is 2.9%.
The compounding frequency is weekly, or 52 times per year.
Stan needs to deposit $27.21 into his account each week to meet his goal.

2. Investment account: The regular payment amount is $50.
The payment frequency is weekly, or 52 times per year.
The number of payments is 104.
The payments are made at the end of each payment period.
The annual interest rate is 4.1%.
The compounding frequency is weekly, or 52 times per year.
The future value is unknown.
The savings account will be worth $5416.92 at the end of two years.
Difference: 10 000 – 5416.92 = 4583.08
Robin and Leslie need an additional $4583.08 in two years.

CSB: *The principal is unknown.*
The annual interest rate is 6%.
The compounding period is annual, or once per year.
The term (in years) is 2 years.
The future value is $4583.08.
Robin and Leslie's parents should give them $4078.92.

3. CSBs

Year	*P* ($)	*i*	*n*	*A* ($)
1	1000	0.034		1351
2	1000	0.034		1306
3	1000	0.034		1263
4	1000	0.034		1222
5	1000	0.034		1181
6	1000	0.034		1143
7	1000	0.034		1105
8	1000	0.034		1069
9	1000	0.034		1034
10	1000	0.034		1000
				11 677

The CSBs are worth $11 677.32.
Trust account: The principal is $3000.
The annual interest rate is 4.3%.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 42 years.
The future value is unknown.
The trust account is worth $18 083.03.

GIC: The principal is $10 000
The annual interest rate is 3.95%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 10 years.
The future value is unknown.
The GIC is worth $14 786.80.
Total: 11 677.32 + 18 083.03 + 14786.80 = 44 547.15
Five-year bond: The principal is $44 547.15
The annual interest rate is 5.1%.
The compounding period is annual, or once per year.
The term (in years) is 5 years.
The future value is unknown.
The five-year bond will be worth $57 125.96.

4. a) Investment account: The regular payment amount is $50.
The payment frequency is monthly, or 12 times per year.
The number of payments is 36.
The payments are made at the end of each payment period.
The annual interest rate is 2.7%.
The compounding frequency is monthly, or 12 times per year.
The future value is unknown.
The investment account is worth $1872.72.
RESP account: The regular payment amount is $10.
The payment frequency is monthly, or 12 times per year.
The number of payments is 216.
The payments are made at the end of each payment period.
The annual interest rate is 3.2%.