Investment B: P is $5000; r is 5% or 0.05; t is 5.
$A = 5000(1 + (0.05)(5))$
$A = 6250$
Return on investment B: 6250 – 5000 = 1250
Investment C: initial P is $4000; r is 6% or 0.06; t is 1 (for 6 years)
$A = P(1 + (0.06)(1))$

Year	Principal ($)	Year-end Value ($)
1	4000.00	4240.00
2	4240.00	4494.40
3	4494.40	4764.06
4	4764.06	5049.91
5	5049.91	5352.90
6	5352.90	5674.08

Return on investment C: 5674.08 – 4000 = 1674.08
Investment C has the greatest return on investment.

Lesson 1.3: Compound Interest: Future Value, page 30

1. Row 1: $i = \frac{10.2}{2}$, $n = (4)(2)$
$i = 5.1$ $\quad n = 8$

Row 2: $i = \frac{4.1}{12}$, $n = (6)(12)$
$i = 0.341..$ $\quad n = 72$

Row 3: $i = \frac{13.2}{4}$, $n = (7)(4)$
$i = 3.3$ $\quad n = 28$

Row 4: $i = \frac{3.5}{365}$, $n = \left(\frac{9}{12}\right)(365)$
$i = 0.009...$ $\quad n \doteq 274$

2. $A = P(1 + i)^n$
a) $P = 520$; $r = 4.5\%$ compounded monthly,
$i = \frac{0.045}{12} = 0.003\ 75$; $t = 8$ years, $n = 8 \cdot 12 = 96$
$A = 520(1 + 0.00375)^{96}$
$A = 744.829…$
The future value of the investment is $744.83.
744.829… – 520 = 224.829…
The total interest earned on the investment is $224.83.
b) $P = 1400$; $r = 8.6\%$ compounded semi-annually,
$i = \frac{0.086}{2} = 0.043$; $t = 15$ years, $n = 15 \cdot 2 = 30$
$A = 1400(1 + 0.043)^{30}$
$A = 4950.593…$
The future value of the investment is $4950.59.
4950.593… – 1400 = 3550.593…
The total interest earned on the investment is $3550.59.

3. i) a) $\frac{72}{6.8} = 10.588...$
By the rule of 72, the investment will double in 10.59 years.
To determine the actual doubling time:
The principal is $7000. The annual interest rate is 6.8%.
The compounding period is annual, or once per year.
The term (in years) is unknown.
The future value is double $7000, or $14 000.
I used the financial application on my calculator:
The doubling time is 10.54 years, which is very close to the estimate of 10.59 years.

b) $\frac{72}{9.2} = 7.826...$
By the rule of 72, the investment will double in 7.83 years.
To determine the actual doubling time:
The principal is $850. The annual interest rate is 9.2%.
The compounding period is monthly, or 12 times per year.
The term (in years) is unknown.
The future value is double $850, or $1700.
I used the financial application on my calculator:
The doubling time is 7.56 years, which is very close to the estimate of 7.83 years.

c) $\frac{72}{15.6} = 4.615...$
Using the rule of 72, it takes 4.62 years for the investment to double in value.
The principal is $12 500.
The annual interest rate is 15.6%.
The compounding period is weekly, or 52 times per year.
The term (in years) is unknown.
The future value is double $12 500, or $25 000.
I used the financial application on my calculator:
The doubling time is 4.45 years, which is very close to the estimate of 4.62 years.

d) $\frac{72}{2.7} = 26.666...$
Using the rule of 72, it takes 26.67 years for the investment to double in value.
The principal is $40 000.
The annual interest rate is 2.7%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is unknown.
The future value is double $40 000, or $80 000.
I used the financial application on my calculator:
The doubling time is 25.85 years, which is close to the estimate of 26.67 years.

ii) a) The principal is $7000. The annual interest rate is 6.8% so $i = 0.068$. The compounding period is annual, or once per year. The term (in years) is 35.
The future value is unknown.
$A = P(1 + i)^n$
$A = 7000(1 + 0.068)^{35}$
$A = 69\ 999.007\ldots$
The future value of the investment is $69 999.01.
69 999.007… – 7000 = 62 999.007…
The total interest earned is $62 999.01.
b) The principal is $850. The annual interest rate is 9.2% so $i = 0.092$. The compounding period is monthly. The term (in years) is 20.
$n = 20 \cdot 12 = 240$
The future value is unknown.
$A = P(1 + i)^n$
$A = 850(1 + 0.092)^{240}$
$A = 5314.630\ldots$
The future value of the investment is $5314.63.
5314.630… – 850 = 4464.630…
The total interest earned is $4464.63.
c) The principal is $12 500.
The annual interest rate is 15.6% so $i = 0.156$.
The compounding period is weekly, or 52 times per year.
The term (in years) is 5.
$n = 5 \cdot 52 = 260$
The future value is unknown.
$A = P(1 + i)^n$
$A = 12\ 500(1 + 0.156)^{260}$
$A = 27\ 236.581\ldots$
The future value of the investment is $27 236.58.
27 236.58 – 12 500 = 14 736.58
The total interest earned is $14 736.58.
d) The principal is $40 000.
The annual interest rate is 2.7% so $i = 0.027$.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 8.
$n = 8 \cdot 2 = 16$
The future value is unknown.
$A = P(1 + i)^n$
$A = 40\ 000(1 + 0.027)^{16}$
$A = 49\ 572.410\ldots$
The future value of the investment is $49 572.41.
49 572.41 – 40 000 = 9572.41
The total interest earned is $9572.41.

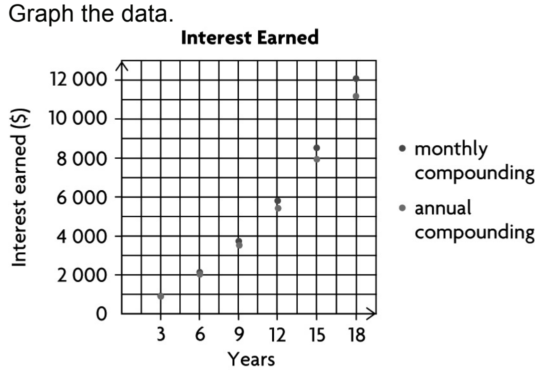
4. a)

Principal ($)	3000	3000
Interest Rate per Annum	0.09	0.09
Periods per Year	1	12
Value at End of Year		
0	3000.00	3000.00
3	3885.09	3925.93
6	5031.30	5137.66
9	6515.68	6723.37
12	8437.99	8798.51
15	10 927.45	11 514.13
18	14 151.36	15 067.91

The current value of the investment with interest compounded annually is $14 151.36.
The current value of the investment with interest compounded monthly is $15 067.91.
b) Calculate the interest.

Years	Annual compounding	Monthly compounding
0	0	0
3	885.09	925.93
6	2031.30	2137.66
9	3515.68	3723.37
12	5437.99	5798.51
15	7927.45	8514.13
18	11151.36	12067.91

Graph the data.



c) As compounding frequency increases, interest rate growth increases.

5. a) $\frac{72}{4.8} = 15$
Using the rule of 72, it will take 15 years for Parker to double his investment.
Using a financial application:
The principal is $6000.
The annual interest rate is 4.8%.
The compounding period is annual, or once per year.
The term (in years) is unknown.
The future value is double $6000, or $12 000.
The doubling time is 14.78 years, which is close to the estimate of 15 years.
b) $\frac{72}{7.2} = 10$
15 – 10 = 5
Using the rule of 72, Parker would be able to buy his motorcycle 5 years sooner.
Using a financial application:
The principal is $6000.
The annual interest rate is 7.2%.
The compounding period is annual, or once per year.
The term (in years) is unknown.
The future value is double $6000, or $12 000.
The doubling time is 9.97 years, which is close to the estimate of 10 years.
14.78 – 9.97 = 4.81
Parker would be able to buy his motorcycle 4.81 years sooner.

6. The principal is $250 000.
The annual interest rate is 3.8%.
The compounding period is semi-annual, or 2 times per year. The term (in years) is 1.
$n = 1 \cdot 2 = 2$
The future value is unknown.
$A = P(1 + i)^n$
$A = 250\,000(1 + 0.038)^2$
$A = 259\,590.250\ldots$
The value of the trust fund after one year is $259 590.25.
250 000 – 259 590.25 = 9590.25
The trust fund has $9590.25 available each year.

7. Bank A: The principal is $20 000.
The annual interest rate is 6.6%.
The compounding period is annual, or once per year.
The term (in years) is 2.
$n = 2 \cdot 1 = 2$
The future value is unknown.
$A = P(1 + i)^n$
$A = 20\,000(1 + 0.066)^2$
$A = 22\,727.120\ldots$
The future value of the investment is $22 727.12.
22 727.12 – 20 000 = 2727.12
Rate of return: $\frac{2727.12}{20\ 000} = 0.136\ 356...$
The rate of return on the Bank A investment is 13.64%.
Bank B: The principal is $20 000.
The annual interest rate is 6.55%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 2.
$n = 2 \cdot 2 = 4$
The future value is unknown.
$A = P(1 + i)^n$
$A = 20\,000(1 + 0.0655)^4$
$A = 22\,751.540\ldots$
The future value of the investment is $22 751.54.
22 751.54 – 20 000 = 2751.54
Rate of return: $\frac{2751.54}{20\ 000} = 0.137\ 577...$
The rate of return on the Bank B investment is 13.76%.
Bank C: The principal is $20 000.
The annual interest rate is 6.5%.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 2. $n = 2 \cdot 4 = 8$
The future value is unknown.
$A = P(1 + i)^n$
$A = 20\,000(1 + 0.065)^8$
$A = 22\,752.779\ldots$
The future value of the investment is $22 752.78.
22 752.78 – 20 000 = 2752.78
Rate of return: $\frac{2752.78}{20\ 000} = 0.137\ 639..$
The rate of return on the Bank C investment is 13.76%.
The rates from greatest to least return on investment are 6.5%, compounded quarterly (13.76%, Bank C), 6.55%, compounded semi-annually (13.75%, Bank B), and 6.6%, compounded annually (13.63%, Bank A).

8. For $1000 to grow to $16 000, it would double four times.

a) $\frac{72}{6} \times 4 = 48$
Using the rule of 72, it will take 48 years for $1000 to grow to $16 000 at an interest rate of 6% compounded annually.

b) $\frac{72}{12} \times 4 = 24$
Using the rule of 72, it will take 24 years for $1000 to grow to $16 000 at an interest rate of 12% compounded annually.

9. The interest grows by a bit more than $50 each year so I will guess that the principal was $800.
Use this value to determine the interest rate.
The principal is $800.
The annual interest rate is unknown.
The compounding period is annual, or once per year.
The term (in years) is 3.
The future value $966.36.
I used the financial application on my calculator: the annual interest rate is 6.5%.
Test these values to see if they produce the correct value of the investment in year 2.
The principal is $800.
The annual interest rate is 6.5%.
The compounding period is annual, or once per year.
The term (in years) is 2.
The future value is unknown.
I used the financial application on my calculator: the future value of the investment is $907.380…
If the principal is $800 and the annual interest rate is 6.5%, the value of the investment after two years is $907.28, which is the value given in the table.
The interest rate of 6.5% and principal of $800 are correct.

10. First four years: The principal is $40 000.
The annual interest rate is 4.8%.
The compounding period is semi-annual, or twice per year.
The term (in years) is 4. $n = 4 \cdot 2 = 8$
The future value is unknown.
$A = P(1 + i)^n$
$A = 40\,000(1 + 0.048)^8$
$A = 48\,357.032\ldots$
The value of the investment after four years is $48 357.03.
Last two years: The principal is $48 357.03.
The annual interest rate is 6%.
The compounding period is annual, or once per year.
The term (in years) is 2.
$n = 2 \cdot 1 = 2$
The future value is unknown.
$A = P(1 + i)^n$
$A = 48\,357.032\ldots(1 + 0.06)^2$
$A = 54\,333.962\ldots$
The total value of the investment after six years is $54 333.96.

11. First four years: The principal is $1500.
The annual interest rate is 9%.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 4.
$n = 4 \cdot 2 = 8$
The future value is unknown.
$A = P(1 + i)^n$
$A = 1500(1 + 0.09)^8$
$A = 2133.150\ldots$
The value of the investment after four years is $2133.15.
Last two years: The principal is $2133.15.
The annual interest rate is 11%.
The compounding period is monthly, or 12 times per year.
The term (in years) is 2.
$n = 2 \cdot 12 = 24$
The future value is unknown.
$A = P(1 + i)^n$
$A = 2133.15(1 + 0.11)^{24}$
$A = 2655.407\ldots$
The total value of the investment after six years is $2655.41.

12. a) e.g., The higher interest rate is payment in exchange for more time before maturity.
b) i) 10 year option: The principal is $5000.
The annual interest rate is 3.25%.
The compounding period is annual, or once per year.
The term (in years) is 10.
$n = 10 \cdot 1 = 10$
The future value is unknown.
$A = P(1 + i)^n$
$A = 5000(1 + 0.0325)^{10}$
$A = 6884.471\ldots$
The value of the investment after ten years is $6884.47.
Five year twice option: The principal is $5000.
The annual interest rate is 2.65%.
The compounding period is annual, or once per year.
The term (in years) is 5.
$n = 5 \cdot 1 = 5$
The future value is unknown.
$A = P(1 + i)^n$
$A = 5000(1 + 0.0265)^5$
$A = 5698.555\ldots$
The value of the investment after five years is $5698.56
Reinvested: The principal is $5698.56.
The annual interest rate is 2.65%.
The compounding period is annual, or once per year.
The term (in years) is 5.
$n = 5 \cdot 1 = 5$
The future value is unknown.
$A = P(1 + i)^n$
$A = 5698.56(1 + 0.0265)^5$
$A = 6494.706\ldots$
The value of the investment after ten years is $6494.71.
e.g., Interest rates are compounded annually, interest rates remain the same in 5 years, can reinvest all $5698.56 in 5 years.

ii) e.g., **Ten year option:**

Advantage	Disadvantage
GIC interest rate is secure if bank interest rates go down.	Cannot benefit if bank interest rates rise.
Don't need to think about investment for ten years.	Cannot reinvest the money for ten years.

Five year twice option:

Advantage	Disadvantage
Can benefit from higher bank interest rates when reinvesting.	Will earn less interest if bank interest rates drop after five years.
Can reinvest money.	Interest rates need to rise over 3.25% for total investment to earn same interest as ten year option.

13. e.g.,

Similar	Different
Both investments pay interest as a percent of the principal.	Compound interest investments also pay interest on previous interest earned.
Both investments can be calculated using a formula.	Simple interest investments are easier to calculate because they have fewer variables.
Both investments increase in value over time.	Compound interest investments increase in value more rapidly than simple interest investments with the same interest rate.

14. a) The CSB Purleen buys in year 1 is invested for 5 years. The CSB she buys in year 2 is invested for 4 years, and so on. The CSB she buys in year 5 will be invested for 1 year.
Determine the yearly value of the five-year CSB then add the values together.

Principal ($)	500
Interest Rate per Annum	0.029
Periods per Year	2
Value at End of Year	
1	514.61
2	529.64
3	545.11
4	561.03
5	577.42
Total	2727.81

The value of Purleen's investment after five years is $2727.81.

b) Extend the table five more years.

Principal ($)	500
Interest Rate per Annum	0.029
Periods per Year	2
Value at End of Year	
1 to 5	2727.81
6	594.28
7	611.64
8	629.51
9	647.90
10	666.82
Total	5877.96

The value of Purleen's investment after ten years is $5877.96.

15. Terry makes four deposits, but the last deposit does not have any time to earn interest so the total investment time is 6 years. The deposits are two years apart so the account has a new principal every two years.
Determine the value of the six-year deposit at year 0, 2, 4, and 6 and add the deposit of $900.00.

Principal ($)		900
Interest Rate per Annum		0.112
Periods per Year		4
Value at End of Year		
Year	Value	Total with deposit of 900.00
0	0.00	900.00
2	1122.50	2022.50
4	2522.52	3422.52
6	4268.65	5168.65

The value of Terry's investment after six years is $5168.65.

History Connection, page 32

A. e.g., at the start of the 1950s, the interest rate was 2.5%. The interest rate increased slowly over the first half of the decade and, by the end of the decade, it was 5%.
B. The best time to invest would have been at the end of the 1950s, when the interest rates were at their highest for the decade.

Applying Problem-Solving Strategies, page 33

A. I will start saving in 2016.
B. Research results

Investment	**Interest Rate**
Savings account	1.50%
GIC (1-year)	1.75%
GIC (5-year)	2.75%
Canada Savings Bond	0.65%

All these investments compound daily.
C. I chose to invest my $5000 in the 5-year GIC.
D. My investment matured after 5 years: $5725.96

Year	**Savings ($)**
2011	5000.00
2012	5138.40
2013	5280.66
2014	5426.89
2015	5577.19
2016	5725.96

E. I rolled a 1. My investment's interest rate decreased to 1.75%.
F. I invested in the 5-year GIC once more, since it has the higher interest rate.
I reinvested my savings, $5725.96, and invested $5000 per year since my initial investment for a total of $30 725.96.
G. I reached my goal of 1 million in 2128, when I would be 139.
H. I noticed that when investing in a 5-year GIC, I would not invest more money for another 5 years. When I compared the results of a 1-year GIC over 10 years and a 5-year GIC over 10 years, I noticed that the 5-year GIC performed better than the 1-year GIC only in the 10th year. Therefore, a strategy that would result in the least age to reach my goal would have me invest $5000 per year in a 1-year GIC for the first 5–9 years to increase my principal as fast as possible. Then, I would invest in a series of 5-year GICs until I reached my goal, since the 5-year GIC yields a higher interest rate.
I. The outcome of the die roll affected my investment much more than the strategy I was using, since my interest rate would sometimes dip below 0, meaning my investment lost value.
J. Since savings accounts, GICs, and Canada Savings Bonds do not have negative interest rates, it would make sense not to let the rates go below zero. Moreover, the Bank of Canada adjusts its interest rate based on inflation, to keep inflation between 1% and 2%. Depending on the investment, it would be reasonable not to let rates for the Canada Savings Bond go below 0.5% or exceed 5%. Any die roll that would result in the CSB rate going outside these limits could be considered as "no change".
These modifications could mean that I could reach my investment goal faster, since I would never lose money in any period. On the other hand, it could also result in reaching my investment goal more slowly, since the interest rate on the CSB could never exceed 5%.
K. My modified rules actually decreased the age at which I reached the goal. Following the investment strategy of a 1-year GIC for the first 10 years, then a 5-year GIC thereafter, I reached my goal in 2076, when I would be 87.

To reach my goal at a more reasonable age, the rules could be changed to allow a larger investment each year. Or, the rules could be changed to allow an investment of $5000 in one 5-year GIC each year, instead of every 5 years.

Lesson 1.4: Compound Interest: Present Value, page 40

1. Investment B will require a greater present value to be invested because the compounding frequency in less than for investment A.

$$P=\frac{A}{(1+i)^n}$$

Investment A: A = 10 000, $i=\frac{0.05}{12}$, n = 120

$$P=\frac{10\ 000}{\left(1+\frac{0.05}{12}\right)^{120}}$$

$$P=6071.61$$

The present value of investment A is $6071.61.

Investment B: A = 10 000, i = 0.0125, n = 40

$$P=\frac{10\ 000}{(1+0.0125)^{40}}$$

$$P=6084.13$$

The present value of investment B is $6084.13.
Investment B requires a higher present value.

2. a) Investment A:

$$\frac{A}{P}=\frac{10\ 000}{6071.61}$$

$$\frac{A}{P}=1.647...$$

Investment B:

$$\frac{A}{P}=\frac{10\ 000}{6084.13}$$

$$\frac{A}{P}=1.643...$$

The future value to present value ratio for Investment A is 1.647… and for investment B is 1.643…

b) The investment with annually compounded interest would have a higher ratio because the interest rate is higher and the principal is lower. With a 6% interest rate compounded annually and a future value of $10 000, the present value must be $5583.95. Since the principal is lower than both investment A and B, the ratio will be higher.

3. Row 1: Determine the present value.

$$P=\frac{A}{(1+i)^n}$$

A = 2500, i = 0.078, n = 8
The present value is $1370.85.

Row 2: Determine the annual interest rate.
The present value is $2000.
The annual interest rate is unknown.
The compounding period is semi-annual, or 2 times per year.
The term (in years) is 5.
The future value is $3500.
Using my calculator, the annual interest rate is 11.5%.

Row 3: Determine the present value.

$$P=\frac{A}{(1+i)^n}$$

A = 11 000, i = 0.006, n = 48
The present value is $8254.48.

Row 4: Determine the investment term.
The present value is 609.35.
The annual interest rate is 13.6%.
The compounding period is annual, or once per year.
The term (in years) is unknown.
The future value is $100 000.00.
Using my calculator, the term of the investment is 39.999… or 40 years.

Row 5: Determine the annual interest rate.
The present value is $16 150.00.
The annual interest rate is unknown.
The compounding period is monthly, or 12 times per year.
The term (in years) is 2.
The future value is $23 500.00.
Using my calculator, the annual interest rate is 18.9%.

4. a) $P=\frac{A}{(1+i)^n}$

A = 250 000, i = 0.085, n = 20

$$P=\frac{250\ 000}{(1+0.085)^{20}}$$

$$P=48\ 904.097...$$

Mac should invest $48 904.10 now to have $250 000 in 20 years.

b) 250 000 – 48 904.10 = 201 095.90
The investment will earn $201 095.90 in interest in 20 years.

5. a) The present value is $9000.
The annual interest rate is unknown.
The compounding period is quarterly, or 4 times per year.
The term (in years) is 2.
The future value is $17 000.
Joseppie would need an annual interest rate of 33.1% to meet his goal. This is not reasonable. Current interest rates for savings accounts are 0.5% to 1.25%.