Chapter 1: Financial Mathematics: Investing Money

Note: Answers obtained in these exercises may vary based on the technology used.

Lesson 1.1: Simple Interest, page 14

1. a) $A = P(1 + rt)$
P is \$30 000; r is 3.5% or 0.035; t is 8
$A = 30\,000(1 + (0.035)(8))$
$A = 38\,400$
The future value is \$38 400.

b) $A = P(1 + rt)$
P is \$10 000; r is 1.25% or 0.0125; t is 4
$A = 10\,000(1 + (0.0125)(4))$
$A = 10\,500$
The future value is \$10 500.

c) $A = P(1 + rt)$
P is \$25 000; r is 0.5% or 0.005; t is 2
$A = 25\,000(1 + (0.005)(2))$
$A = 25\,250$
The future value is \$25 250.

d) $A = P(1 + rt)$
P is \$12 000; r is 7.4% or 0.074; t is 20
$A = 12\,000(1 + (0.074)(20))$
$A = 29\,760$
The future value is \$29 760.

2. a) $A = P + Prt$
A = \$6500; P = \$5000; $t = 5$

$$6500 = 5000 + (5000)(r)(5)$$

$$1500 = 25\,000r$$

$$\frac{1500}{25\,000} = r$$

$$0.06 = r$$

$6500 = 5000 + (5000)(r)(5)$
$1500 = 25\,000r$
$r = 0.06$
An interest rate of 6% will allow Cam to reach his goal.

b) $A = P(1 + rt)$
P is \$5000; r is 6% or 0.06; t is 3
$A = 5000(1 + (0.06)(3))$
$A = 5900$
Cam will have \$5900.

3. a) $A = P(1 + rt)$
P is \$1000; r is 5% or 0.05; t is 5
$A = 1000(1 + (0.05)(5))$
$A = 1250$
The future value is \$1250.
Determine the interest earned.
$1250 – 1000 = 250$

$$\text{Rate of return} = \frac{250}{1000}$$

Rate of return = 0.25
The rate of return is 25%.

b) A. $A = P(1 + rt)$
P is \$1050; r is 5% or 0.05; t is 5
$A = 1050(1 + (0.05)(5))$
$A = 1312.50$
The future value is \$1312.50.

B. $A = P(1 + rt)$
P is \$1000; r is 6% or 0.06; t is 5
$A = 1000(1 + (0.06)(5))$
$A = 1300$
The future value is \$1300.

C. $A = P(1 + rt)$
P is \$1000; r is 5% or 0.05; t is 5
$A = 1000(1 + (0.05)(5))$
$A = 1250$
The future value is \$1250.

D. $A = P(1 + rt)$
P is \$1000; r is 5% or 0.05; t is 6
$A = 1000(1 + (0.05)(6))$
$A = 1300$
The future value is \$1300.
Option A yields the greatest future value.
Determine the interest earned.
$1312.50 – 1000 = 312.50$

$$\text{Rate of return} = \frac{312.50}{1000}$$

Rate of return = 0.3125
The rate of return for option A is 31.25%.

4. a) $A = P(1 + rt)$
P is \$4000; r is 2.3% or 0.023; t is 5
$A = 4000(1 + (0.023)(5))$
$A = 4460$
After 5 years, the future value is \$4460.

b) $A = P(1 + rt)$
P is \$4000; r is 2.3% or 0.023; t is 10
$A = 4000(1 + (0.023)(10))$
$A = 4920$
After 10 years, the future value is \$4920.

5. $A = P(1 + rt)$
Brad's GIC: P is \$15 000; r is 3.2% or 0.032; t is 6
$A = 15\,000(1 + (0.032)(6))$
$A = 17\,880$
The future value of Brad's investment is \$17 880.
Chris's GIC: P is \$15 000; r is 3.3% or 0.033; t is 5
$A = 15\,000(1 + (0.033)(5))$
$A = 17\,475$
The future value of Chris's investment is \$17 475.
Brad's investment will have a greater future value at maturity. Investing the money for one more year results in more interest earned than does investing the money at a 0.1% higher interest rate.

6. a) $A = P + Prt$
A = \$15 000, P = \$12 000, $t = 10$

$$15\,000 = 12\,000 + (12\,000)(r)(10)$$

$$3000 = 120\,000r$$

$$\frac{3000}{120\,000} = r$$

$$0.025 = r$$

An interest rate of 2.5% is needed for the CSB to grow to \$15 000 in 10 years.

b) $A = P(1 + rt)$
P = \$12 000, r = 2.5% + 1% = 3.5% or 0.035, t = 10
$A = 12\,000(1 + (0.035)(10))$
A = 16 200
The future value of the CSB would be \$16 200.

7. a) They will be equal. e.g., The principal, interest rate, and term are equal. Both earn \$300 in interest.
b) No. e.g., With simple interest, there is no advantage to having it paid more often.
c) e.g., They may need the interest money to pay a monthly bill.

8. a) $A = P(1 + rt)$
A. P is \$10 000; r is 5% or 0.05; t is 5
$A = 10\,000(1 + (0.05)(5))$
A = 12 500
The future value of investment A is \$12 500.
B. P is \$10 000; r is 1.4% or 0.014; t is 10
$A = 10\,000(1 + (0.014)(10))$
A = 11 400
The future value of investment B is \$11 400.
C. P is \$10 000; r is 1.9% or 0.019; t is 7
$A = 10\,000(1 + (0.019)(7))$
A = 11 330
The future value of investment C is \$11 330.
D. P is \$10 000; r is 1.05% or 0.0105; t is 8
$A = 10\,000(1 + (0.0105)(8))$
A = 10 840
The future value of investment D is \$10 840.
From greatest maturity value to least: Investment A, Investment B, Investment C, Investment D.
b) No. e.g., the amount of interest not earned in the last 1.5 years is insufficient to change rankings.
A. Investment is paid semi-annually, so $t = 5 - 1.5$ or 3.5.
P is \$10 000; r is 5% or 0.05; t is 3.5
$A = 10\,000(1 + (0.05)(3.5))$
A = 11 750
The future value of investment A is \$11 750.
B. Investment is paid annually, so $t = 10 - 2$ or 8.
P is \$10 000; r is 1.4% or 0.014; t is 8
$A = 10\,000(1 + (0.014)(8))$
A = 11 120
The future value of investment B is \$11 120.
C. Investment is paid quarterly, so t is $7 - 1.5$ or 5.5.
P is \$10 000; r is 1.9% or 0.019; t is 5.5
$A = 10\,000(1 + (0.019)(5.5))$
A = 11 045
The future value of investment C is \$11 045.
D. Investment is paid annually, so t is $8 - 2$ or 6.
P is \$10 000; r is 1.05% or 0.0105; t is 6
$A = 10\,000(1 + (0.0105)(6))$
A = 10 630
The future value of investment D is \$10 630.

9. a) $A = P(1 + rt)$
Desiree: P is \$3600; r is 2.5% or 0.025; t is 0.5
$A = 3600(1 + (0.025)(0.5))$
A = 3645
The future value of Desiree's investment is \$3645.
Latoya: P is \$3500; r is 3% or 0.03; t is 0.5
$A = 3500(1 + (0.03)(0.5))$
A = 3552.50
The future value of Latoya's investment is \$3552.50.
Determine the difference.
3645 – 3552.50 = 92.50
Desiree will have \$92.50 more than Latoya after 6 months.
b) Desiree: 3645 – 3600 = 45
Desiree will earn \$45 in interest.
Latoya: 3552.50 – 3500 = 52.50
Latoya will earn \$52.50 in interest.
Determine the difference:
52.50 – 45 = 7.50
Latoya will earn \$7.50 more in interest than Desiree.
c) Desiree:

$$\text{rate of return} = \frac{45}{3600}$$

rate of return = 0.0125 or 1.25%
Latoya:

$$\text{rate of return} = \frac{52.50}{3500}$$

rate of return = 0.015 or 1.5%
Latoya's rate of return is higher than Desiree's rate of return.

10. $A = P + Prt$
A is \$15 000; P is \$10 000; r *is* 6.5% or 0.065
$15\,000 = 10\,000 + (10\,000)(0.065)(t)$
$5000 = 650t$
t = 7.69
It will take Shaun 8 years to earn enough money to make the down payment.

11. a) $A = P(1 + rt)$
A is \$20 000; r is 3.2% or 0.032; t is 5
$20\,000 = P(1 + (0.032)(5))$
$20\,000 = 1.16P$
P = \$17 241.38
b) $A = P + Prt$
A is \$25 000; P is \$17 241.38; r is 3.2% or 0.032
$25\,000 = 17\,241.38 + (17\,241.38)(0.032)(t)$
$7758.62 = 551.72t$
t = 14.06
It will take about 14.1 years for the GIC to be \$25 000.

12. a) $A = P + Prt$
A is \$9400; P is \$4700; t is 8

$$9400 = 4700 + (4700)(r)(8)$$
$$4700 = 37\ 600r$$
$$\frac{4700}{37\ 600} = r$$
$$0.125 = r$$

The annual interest rate is 12.5%.
b) $A = P(1 + rt)$
P is \$4700 ; r is 12.5% or 0.125; t is 16
$A = 4700(1 + (0.125)(16))$
$A = 14\ 100$
The value of the investment would be \$14 100.

13. a) The slope of the graph would increase if the interest rate increased. For example, the investment currently earns \$1 of interest every 3 years, so it has a slope of $\frac{1}{3}$. A steeper graph with a slope of $\frac{2}{3}$ would mean the investment is earning \$2 of interest every 3 years. This could only happen if the interest rate increased.
b) e.g., Similar: They have the same interest rate. The situations are represented by a linear relation that slopes upward to the right. The graphs relate money to time. The graphs have the same slope.
Different: They have different principals. The interest graph starts at (0, 0) while the investment graph starts at (0, 3). The investment graph shows the value of the investment and the interest while the interest graph only shows only the interest.

14. The CSB Graham buys in year 1 is invested for 5 years. The CSB he buys in year 2 is invested for 4 years, and so on. The CSB he buys in year 5 will be invested for 1 year. Determine the yearly value of the five-year CSB then add the values together.
$A = P(1 + rt)$
P is \$1000; r is 3.8% or 0.038; t is 1, 2, 3, 4, 5

Year	Year End Value ($)
1	1038
2	1076
3	1114
4	1152
5	1190
Total	5570

Graham's investment will be worth \$5570 after 5 years.
15. Carole's account pays interest daily so she will earn 86 days of interest when she withdraws her money. Convert 86 days to a fraction of a year:

$$\frac{86}{365} = 0.235...$$

$A = P(1 + rt)$
P is \$24 000; r is 5.2% or 0.052; t is 0.235…
$A = 24\ 000(1 + (0.112)(0.235…))$
$A = 24\ 294.049…$
Carole withdrew \$24 294.05 when she closed her savings account.

Lesson 1.2: Exploring Compound Interest, page 19

1. $A = P(1 + rt)$
Eve: P is \$3000; r is 4% or 0.04; t is 5
$A = 3000(1 + (0.04)(5))$
$A = 3600$
Eve's investment is worth \$3600.00 after 5 years.
Larry: The initial principal is \$3000; r is 4% or 0.04; t is 1 since each year has a new principal.
$A = P(1 + (0.04)(1))$
Calculate the interest 5 times, using the value of A as the new value of P for each new year. Use a table to organize the answers.

Year	Principal ($)	Year-end Value ($)
1	3000.00	3120.00
2	3120.00	3244.80
3	3244.80	3374.59
4	3374.59	3509.58
5	3509.58	3649.96

Larry's investment is worth \$3649.96 after 5 years.
Calculate the difference in interest.
3649.96 – 3600 = 49.96
Larry's investment earned \$49.96 more in interest than Eve's because his investment earned interest on the principal and on the accumulated interest.

2. $A = P(1 + rt)$
Account A: P is \$6500; r is 5.1% or 0.051; t is 4
$A = 6500(1 + (0.051)(4))$
$A = 7826.00$
The investment in account A is worth \$7826.00 after four years.
Account B: Initial principal is \$6500; r is 4.8% or 0.048; t is 1.
$A = P(1 + (0.048)(1))$
Calculate the interest 4 times, using the value of A as the new value of P for each new year. Use a table to organize the answers.

Year	Principal ($)	Year-end Value ($)
1	6500.00	6812.00
2	6812.00	7138.98
3	7138.98	7481.65
4	7481.65	7840.77

The investment in account B is worth \$7840.77 after four years. Sydney should choose account B because it will earn more interest.

3. a) e.g., no it is not possible to tell, as the principals, interest rates, and timelines all differ.
b) $A = P(1 + rt)$
Investment A: initial P is \$6000; r is 1.2% or 0.012; t is 1 (for 4 years).
$A = P(1 + (0.012)(1))$

Year	Principal ($)	Year-end Value ($)
1	6000.00	6072.00
2	6072.00	6144.86
3	6144.86	6218.60
4	6218.60	6293.23

Return on investment A: 6293.23 – 6000 = 293.23