8.4 The Equations of Sinusoidal Functions

YOU WILL NEED

- graphing technology

EXPLORE…

- Determine the maximum value, minimum value, and range of

$$y = 3 \sin x + 2$$

GOAL

Identify characteristics of the equations of sinusoidal functions.

INVESTIGATE the Math

Cesar and Malachi have to do a project on local Winnipeg history. They have decided to research Grant's Old Mill, located at the Red River Settlement. Their math teacher said that they could describe the motion of a point on the rim of the water wheel using the sinusoidal function

$$y = a \sin b(x - c) + d$$

where a, b, c, and d are real numbers.

Métis leader Cuthbert Grant built this water wheel in 1829. It was the first source of hydro power in Manitoba.

? How does changing the values of *a*, *b*, *c*, and *d* in a sinusoidal function affect the graph of the function?

A. Graph, in radian mode,

$$y = \sin x$$

for the domain $\{x \mid 0 \le x \le 4\pi, x \in R\}$. Then graph the following functions on the same axes.

i) $y = 3 \sin x$ **ii)** $y = 5 \sin x$ **iii)** $y = \frac{1}{2} \sin x$

B. Compare your graphs in part A.

i) How does changing the value of a affect the graph of $y = \sin x$? Show your answer in a sketch.

ii) What does the value of a tell you about a water wheel? Explain.

C. On a new grid, graph, in radian mode,

$$y = \sin x$$

for the domain $\{x \mid 0 \le x \le 4\pi, x \in R\}$. Then graph these functions on the same axes.

i) $y = \sin 2x$ **ii)** $y = \sin 3x$ **iii)** $y = \sin \frac{1}{2}x$

D. Compare your graphs in part C.

i) How does changing the value of b affect the graph of $y = \sin x$? Show your answer in a sketch.

ii) What does the value of b tell you about a water wheel? Explain.

E. On a new grid, graph, in radian mode,

$$y = \sin x$$

for the domain $\{x \mid 0 \leq x \leq 4\pi, x \in \mathrm{R}\}$. Graph the following functions on the same axes.

i) $y = \sin (x - 1.6)$ **ii)** $y = \sin (x - 3.1)$ **iii)** $y = \sin (x + 0.8)$

Communication Tip

The expression $\sin x + c$ indicates that sine operates only on x. To indicate that sine operates on the expression $x + c$, parentheses are used: $\sin (x + c)$.

F. Compare your graphs in part E.

i) How does changing the value of c affect the graph of $y = \sin x$? Show your answer in a sketch.

ii) What does the value of c tell you about a water wheel? Explain.

G. On a new grid, graph, in radian mode,

$$y = \sin x$$

for the domain $\{x \mid 0 \leq x \leq 4\pi, x \in \mathrm{R}\}$. Graph these functions on the same axes.

i) $y = 4 \sin x + 3$ **ii)** $y = 4 \sin x - 1$ **iii)** $y = 4 \sin x + 4$

H. Compare your graphs in part G.

i) How does changing the value of d affect the graph of $y = \sin x$? Show your answer in a sketch.

ii) What does the value of d tell you about a water wheel? Explain.

Reflecting

I. Consider the sinusoidal equation

$$y = a \sin bx + d$$

Explain how to determine each of the following characteristics of the function, based on the values of a, b, and d:

- the amplitude
- the period
- the equation of the midline

J. Consider the equation

$$y = a \sin (x - c) + d$$

How does the value of c affect the graph?

K. Do you think the values of a, b, c, and d will affect a sine graph in the same way if the angles are measured in degrees? Explain.

L. Will the values of a, b, c, and d affect a cosine graph in the same way that they affect a sine graph? Explain.

APPLY the Math

EXAMPLE 1 Determining the characteristics of a cosine function based on its equation

Consider the function

$$y = 2 \cos 4x + 1$$

for $\{x \mid 0° \leq x \leq 360°, x \in R\}$.

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \cos x$.

b) Verify your description by drawing a graph of this function using graphing technology.

T.J.'s Solution

a) $y = a \cos b(x - c) + d$

$y = 2 \cos 4x + 1$

$a = 2, b = 4, c = 0, d = 1$

I determined *a*, *b*, *c*, and *d*.

The amplitude of the graph is a, which is 2.
The equation of the midline is $y = d$, or $y = 1$.

I determined the amplitude and the equation of the midline.

Minimum value $= d - a$ Maximum value $= d + a$
Minimum value $= 1 - 2$ Maximum value $= 1 + 2$
Minimum value $= -1$ Maximum value $= 3$

I determined the minimum and maximum values.

The range of the graph is $\{y \mid -1 \leq y \leq 3, y \in R\}$.

I wrote the range using these values.

Since $b = 4$, the graph completes four cycles in 360°.

$$\text{Period} = \frac{360°}{b}$$

$$\text{Period} = \frac{360°}{4}$$

$$\text{Period} = 90°$$

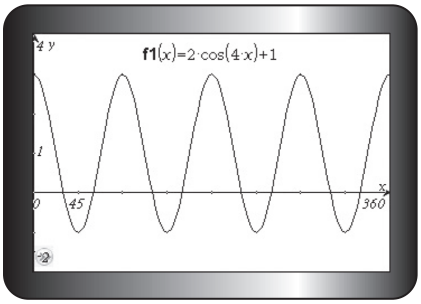
I thought of *b* as an indicator of speed. The graph completes a cycle four times more quickly than the graph of $y = \cos x$.

Since $c = 0$, the graph has a y-intercept at the maximum value, or $y = 3$.

The graph has not been translated horizontally, since the graph of $y = \cos x$ also has a y-intercept at its maximum value.

I knew that the graph has not been translated horizontally, because $c = 0$.

b) $y = 2 \cos 4x + 1$



I entered the equation into my graphing calculator. I set the domain as $\{x \mid 0° \leq x \leq 360°, x \in R\}$ and decided on a scale for x using an interval of 45°, which is half of the period I had determined.

I set the y-axis to show 1 less than the minimum and 1 more than the maximum.

I examined my graph. The minimum is -1 and the maximum is 3, so the amplitude is 2. There are four complete periods in my window, so I confirmed that the period is 90°.

The graph starts at a maximum. This is a cosine function, so the graph has not been translated horizontally.

My description is correct.

Your Turn

Consider the graph of

$$y = 5 \cos \frac{1}{2}x - 3$$

where x is measured in degrees.

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \cos x$.

b) Verify your description by drawing the graph using graphing technology.

EXAMPLE 2 Determining the characteristics of a sine function based on its equation

Consider the function

$$y = 3 \sin 2(x - 45°)$$

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \sin x$.

b) Verify your description by drawing the graph using graphing technology.

c) How would the graph of $y = 3 \cos 2(x + 45°)$ be the same? How would it be different?

Communication *Tip*

The degree symbol on 45 indicates that the independent variable x, is measured in degrees. If no degree symbol is present in an equation, assume that x is measured in radians, unless stated otherwise.

Vanessa's Solution

a) $y = a \sin b(x - c) + d$

$y = 3 \sin 2(x - 45°)$

$a = 3,\ b = 2,\ c = 45°,\ d = 0$

I determined a, b, c, and d.

The amplitude of the graph is a, which is 3.

The equation of the midline is $y = d$, or $y = 0$; that is, the x-axis.

I described the graph using these values.

I wrote the range using the minimum and maximum values.

Minimum value $= d - a$ Maximum value $= d + a$

Minimum value $= 0 - 3$ Maximum value $= 0 + 3$

Minimum value $= -3$ Maximum value $= 3$

The range of the graph is $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$.

Since $b = 2$, the graph completes two cycles in 360°.

I determined the period.

$$\text{Period} = \frac{360°}{b}$$

$$\text{Period} = \frac{360°}{2}$$

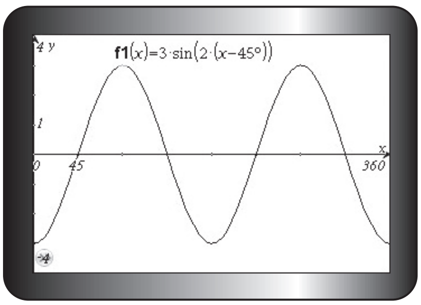
$$\text{Period} = 180°$$

The graph has been translated 45° to the right, since $c = 45°$.

I knew that the value of c indicates the horizontal translation.

b) $y = 3 \sin 2(x - 45°)$

I entered the equation into my graphing calculator. I set the domain as $\{x \mid 0 \le x \le 360°\}$. I set the range to show 1 less than the minimum and 1 more than the maximum.



I examined my graph. The minimum is -3, and the maximum is 3. Therefore, the amplitude is 3.

There are two complete periods in my window, so I confirmed that the period is 180°.

The graph starts at a minimum, so it has been translated horizontally. It first intersects the midline at 45°, so it has been translated horizontally by 45°.

My description is correct.

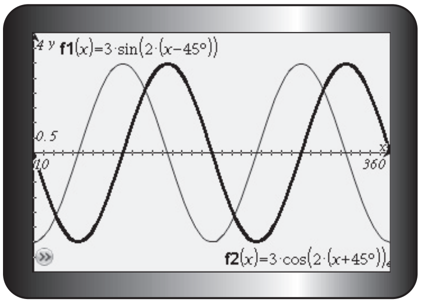
c) The graphs of the functions

$y = 3 \sin 2(x - 45°)$ and
$y = 3 \cos 2(x + 45°)$

should have the same amplitude and period, since a and b are the same.

The graph of the second function is translated by $-45°$, or $45°$ to the left of $y = \cos x$.

Since the value of c is opposite the value of c in the first function, the graph is translated to the left.



I graphed the second equation.

The second graph has been translated $45°$ to the right, as compared to a standard graph of the cosine function. Otherwise, the two graphs have the same characteristics. My description is correct.

Your Turn

Consider the function

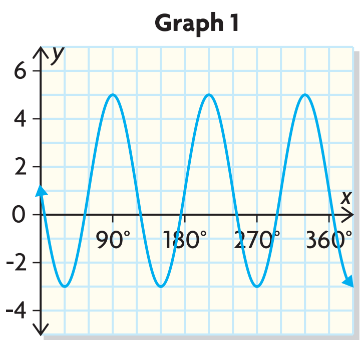
$$y = 4 \cos 3(x - 60°)$$

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \cos x$.
b) Confirm your description by drawing the graph using graphing technology.

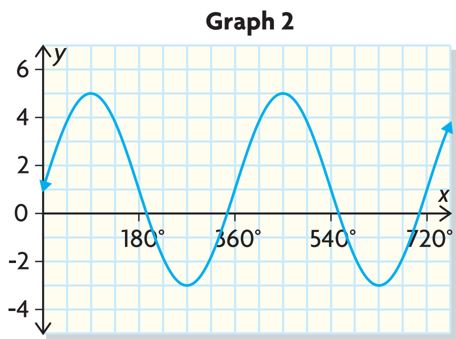
EXAMPLE 3 Matching equations to graphs

Match each graph with the corresponding equation below.

a)



b)



i) $y = 4\cos(x - 90°) + 1$

ii) $y = 5\sin 3(x - 60°)$

iii) $y = 4\sin 3(x - 60°) + 1$

iv) $y = 4\cos 3(x - 60°) + 1$

Anya's Solution

a) $\text{Amplitude} = \dfrac{\text{maximum value} - \text{minimum value}}{2}$

$\text{Amplitude} = \dfrac{5 - (-3)}{2}$

$\text{Amplitude} = 4$

The amplitude of equation ii) is 5, so equation ii) is eliminated.

I could not tell if the graph represents a sine function or a cosine function. I had to examine the characteristics of the graph.

I determined the amplitude using values from the graph.

$\text{Period} = 210° - 90°$

$\text{Period} = 120°$

There are consecutive maximum points at 90° and 210°. I determined the period of the graph using these points.

$\text{Period} = \dfrac{360°}{b}$

$120° = \dfrac{360°}{b}$

$b = 3$

I determined the value of b in the equation $y = a \sin b(x - c) + d$.

The value of b in equation i) is 1, so equation i) is eliminated.

If the graph represents a cosine function, then it has been translated by 90° to the right. Equation iv) represents a cosine function that has been translated 60° to the right, not 90°, so equation iv) is eliminated.

I knew that the graph represents either a cosine function or a sine function. I compared the possible translation of a cosine function to the value of *c* in the equations.

By elimination, equation iii) must be correct.

Verify:

I verified my deduction.

Equation iii) represents a sine function that has been translated 60° to the right, like the graph. It has an amplitude of 4 and a period of 120°. The value of b is 3, and the equation of the midline is $y = 1$. All of these characteristics match the graph. Equation iii) is the equation of this graph.

b) I know that equation iii) is eliminated, since equation iii) matches graph a).

I could not tell if the graph represents a sine function or a cosine function. I had to examine the characteristics of the graph.
I determined the amplitude of the graph.

$$\text{Amplitude} = \frac{\text{maximum value} - \text{minimum value}}{2}$$

$$\text{Amplitude} = \frac{5 - (-3)}{2}$$

$$\text{Amplitude} = 4$$

The amplitude of equation ii) is 5, so equation ii) is eliminated.

$$\text{Period} = 450° - 90°$$

$$\text{Period} = 360°$$

There are consecutive maximum points at 90° and 450°. I determined the period of the graph using these points.

$$\text{Period} = \frac{360°}{b}$$

$$360° = \frac{360°}{b}$$

$$b = 1$$

I determined the value of *b* in the equation of the sinusoidal function.

Only equation i) has $b = 1$, so equation i) must be the equation of this graph.

I verified my deduction.

Verify:

Equation i) represents a cosine function that has been translated 90° to the right, like the graph. It has an amplitude of 4 and a period of 360°. The value of b is 1, and the equation of the midline is $y = 1$. All of these characteristics match the graph.

Your Turn

Write another equation that matches each graph in Example 3.

EXAMPLE 4 Solving a problem using a sinusoidal function

The Far North is called "the Land of the Midnight Sun" for a good reason: during the summer months, in some locations, the Sun can be visible for 24 h a day. The number of hours of daylight in Iqaluit, Nunavut, can be represented by the function

$$y = 8.245 \sin 0.0172(x - 80.988) + 12.585$$

where x is the day number in the year.

a) How many hours of daylight occur in Iqaluit on the following days?
  i) the shortest day of the year
  ii) the longest day of the year

b) In some years, June 21 is the longest day. Suppose that the Sun were to set in Iqaluit at 11:01 p.m. on June 21. At what time did the Sun rise?

c) What is the period of this sinusoidal function? Explain how the period relates to the context of the problem.

d) What does the value of c, 80.988, represent in the context of the problem?

The Sun rising in Iqaluit on the shortest day of the year

Tyler's Solution: Analyzing the equation

a) $y = a \sin b(x - c) + d$
$y = 8.245 \sin 0.0172(x - 80.988) + 12.585$
$a = 8.245$, $b = 0.0172$, $c = 80.988$, $d = 12.585$

I determined a, b, c, and d.

Minimum value $= d - a$
Minimum value $= 12.585 - 8.245$
Minimum value $= 4.340$
Maximum value $= d + a$
Maximum value $= 12.585 + 8.245$
Maximum value $= 20.830$

I determined the minimum and maximum hours of daylight.

$$0.34 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 20.4 \text{ min}$$

I converted the decimal part of the daylight hours to minutes.

The shortest day of the year has 4.34 h of daylight, or about 4 h 20 min.

$$0.83 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 49.8 \text{ min}$$

The longest day of the year has 20.83 h of daylight, or about 20 h 50 min.

b) Sunset is at 11:01 p.m.
Sunrise is 20 h 49.8 min earlier.
12 h earlier is 11:01 a.m.

Since the longest day is 20 h 50 min, sunrise must occur this length of time before 11:01 p.m. First, I subtracted 12 h.

11:01 a.m. less 8 h 49.8 min is about 2:11 a.m.
The Sun rose at 2:11 a.m.

I still needed to subtract 8 h 50 min. To make subtraction easier, I thought of 11:01 a.m. as being 10 h and 61 min. I subtracted 8 h 49.8 min.

c) Period $= \frac{2\pi}{b}$

Period $= \frac{2\pi}{0.0172}$

Period $= 365.301...$

I calculated the period. Since no units are indicated, I used radian measure.

The period for $y = \sin x$ is 2π.

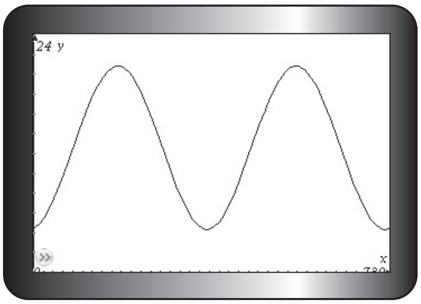
Earth revolves about the Sun about every 365.25 days, so the period and the length of one rotation are about the same.

d) The value of c is 80.988, or about 81 days.
The 81st day of the year is March 22, which is around the first day of spring.

I knew that c represents the horizontal translation of the graph of $y = \sin x$. Since this is the equation of a sine function, the graph crosses the midline at day 81 and is increasing.

Siddiqa's Solution: Analyzing the graph of the equation using technology

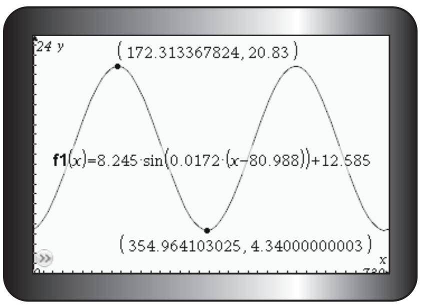
a)



I entered the equation on my graphing calculator. I used a domain of 2 years:
$\{x \mid 0 \le x \le 730, x \in \mathrm{R}\}$

I used a range of 1 day, or 24 h:
$\{y \mid 0 \le y \le 24, y \in \mathrm{R}\}$

I set the mode of my calculator to radians.



I used the maximum and minimum features of my calculator.

The longest day of the year is 20.83 h.
The shortest day of the year is 4.34 h.

I knew that the maximum and minimum values represent the longest and shortest days of the year.

b) I knew that 11:01 p.m. is 23:01 on the 24 h clock.

$0.01 \cancel{\text{min}} \cdot \frac{1 \text{ h}}{60 \cancel{\text{min}}} = 0.016...$

$23.016... - 20.83$ or $2.186...$

$0.186... \cancel{\text{h}} \cdot \frac{60 \text{ min}}{1 \cancel{\text{h}}} = 11.2 \text{ min}$

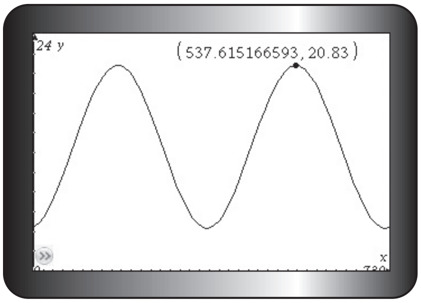
The Sun rose at about 2:11, which is 2:11 a.m.

I used the 24 h clock to make my calculations easier. I wrote 0.01 min in hours.

I subtracted the two times.

I wrote 0.186... in minutes.

c) The first maximum occurs at 172.313... days.



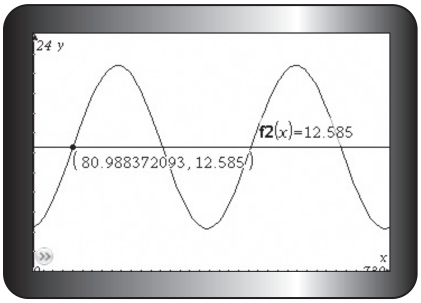
I determined the *x*-coordinates of two consecutive maximum points on my graph. I subtracted to determine the period.

The second maximum occurs at 537.615... days.

Period = second maximum − first maximum
Period = 537.615... − 172.313...
Period = 365.301...

Earth rotates about the Sun about every 365.25 days, so the period and the length of one rotation are about the same.

d) I determined the value of y when $x = 80.988$.



I noticed that 12.585 is the value of d in the given equation.

Based on the equation of the function, $y = d$ or $y = 12.585$ is the equation of the midline.

The point (80.988, 12.585) represents the first day of spring, when the daylight hours are getting longer. This seems reasonable, because the 81st day of the year is in March.

Your Turn

Suppose that the Sun rises at 9:22 a.m. in Iqaluit on the shortest day of the year. At what time does the Sun set?

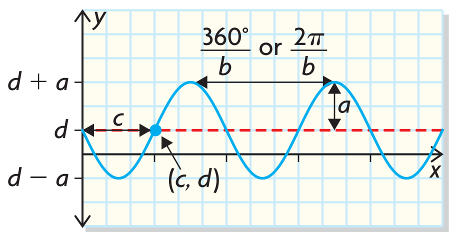
In Summary

Key Idea

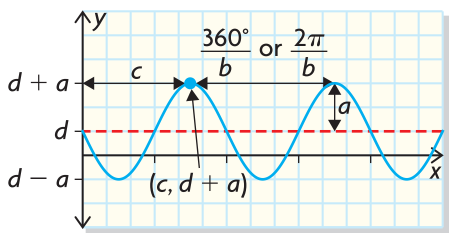
- Any sinusoidal function can be expressed as either a cosine function or a sine function.

Need to Know

- A sinusoidal function of the form
$$y = a \sin b(x - c) + d \text{ or}$$
$$y = a \cos b(x - c) + d$$
has the following characteristics:
  - The value of a is the **amplitude:**
$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$
  - The value of b is the number of cycles in 360° or 2π. The **period** is $\frac{360°}{b}$ or $\frac{2\pi}{b}$.
  - The value of c indicates the horizontal translation that has been applied to the graph of $y = \sin x$ or $y = \cos x$.
  - The **equation of the midline** is
$$y = d$$
where
$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$
  - The **maximum value** is $d + a$, and the **minimum value** is $d - a$.
- In the graph of a sine function, c is the distance from the vertical axis to the first midline point where the function is increasing.



- In the graph of a cosine function, c is the distance from the vertical axis to the first maximum point.



CHECK Your Understanding

1. Arrange these functions in order, from the least amplitude to the greatest amplitude. Provide your reasoning.

a) $y = 3 \sin 4x$ **b)** $y = 4 \sin 3x$ **c)** $y = 2 \sin x + 3$

2. Arrange these functions in order, from smallest range to greatest range. Provide your reasoning.

a) $y = 8 \sin 6x$ **b)** $y = 9 \sin 3x - 1$ **c)** $y = 6 \sin x + 2$

3. Arrange these functions in order, from the least period to the greatest period. Provide your reasoning.

a) $y = 5 \sin 4x$ **b)** $y = 3 \sin 2(x - 4)$ **c)** $y = 2 \cos 0.25x$

4. Describe the horizontal translation of each function.

a) $y = 0.5 \cos 2(x + 45°) + 3$

b) $y = 2 \sin 0.5(x - 180°) + 3$

c) $y = 2 \cos 2(x - 45°) + 3$

PRACTISING

5. Determine the amplitude and the range of each function.

a) $y = 7 \sin 2(x - 5)$ **b)** $y = 13 \cos 0.5(x + 26)$

6. Determine the equation of the midline, the amplitude, and the range of each function.

a) $y = 8 \sin 4(x - 22.5°) + 5$ **b)** $y = 6 \cos 3(x + 15°) - 7$

7. Determine the equation of the midline and the maximum and minimum values of each function.

a) $y = 5 \sin 2(x - 2.5) + 2$ **b)** $y = 3 \cos (x + 1) - 3$

8. Determine the distance in degrees or radians by which $y = \sin x$ or $y = \cos x$ would be horizontally translated to create the graph of each function.

a) $y = 2.5 \sin 3(x - 30°) + 5$ **c)** $y = 2 \sin 8(x - 4.5) - 3$

b) $y = 11 \cos (x + 100°) - 2$ **d)** $y = 4 \cos (x + 3) - 1$

9. Determine the period of each function and the distance by which its graph has been horizontally translated.

a) $y = 3 \sin 4(x - 45°) - 1$ **b)** $y = 2 \sin 0.5(x - 1) + 4$

10. a) Determine the equation of a function whose amplitude and period are both double the amplitude and period of the following function:

$$y = 3 \sin 4x$$

b) Verify your solution using graphing technology. Draw the graphs of both functions on the same axes.

11. **a)** Determine the equation of the function whose amplitude and period are both $\frac{1}{4}$ the amplitude and period of this function:

$$y = 2 \cos x$$

b) Verify your solution using graphing technology. Draw the graphs of both functions on the same axes.

12. Ashley boards the Ferris wheel at the Pacific National Exhibition. When the ride begins, her position can be modelled by the function

$$y = 43 \sin 3.5(x - 0.9) + 47,$$

where y represents the height in feet and x represents the time in minutes.

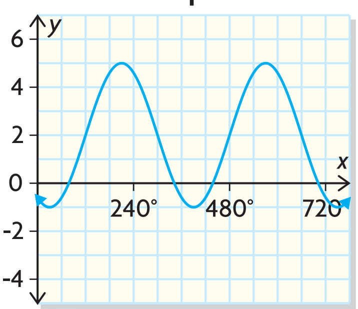
a) Determine the diameter of the Ferris wheel.

b) How long does it take for the Ferris wheel to complete one revolution?

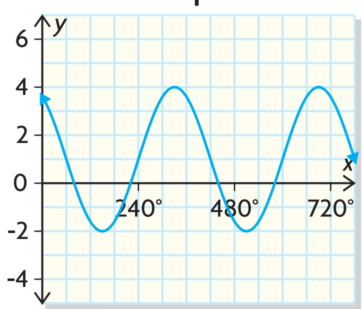
c) How high above the ground is Ashley at the lowest point?

13. Match each graph with the corresponding equation below.

a) **Graph A**



b) **Graph B**



i) $y = 3 \sin (x + 120°) + 1$

ii) $y = 3 \cos (x - 90°) + 1$

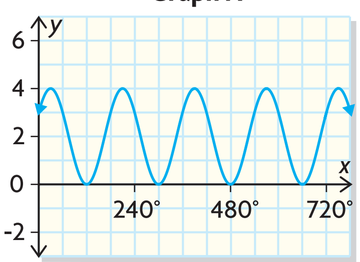
iii) $y = 3 \sin (x - 120°) + 1$

iv) $y = 4 \sin (x - 120°)$

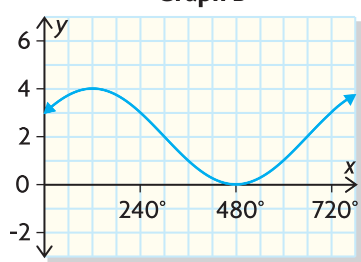
v) $y = 2 \sin (x - 120°) + 2$

14. Match each graph with the corresponding equation below.

a) **Graph A**



b) **Graph B**



i) $y = 2 \cos 2(x + 30°) + 2$

ii) $y = 2 \cos 0.5(x - 120°) + 2$

iii) $y = 3 \sin (x - 30°) + 1$

iv) $y = 2 \sin (x - 30°) + 2$

v) $y = 2 \cos 2(x - 30°) + 2$

15. Consider the following function:

$$y = 3 \cos 5(x + 30°) - 1$$

a) Describe the graph of the function, including the amplitude, the equation of its midline, the range, the period, and the distance of horizontal translation from $y = \cos x$.

b) Confirm your description using technology.

Sierra Noble, of Winnipeg, is a Métis singer, songwriter, and fiddle player. She is one of many young Aboriginal musicians.

16. Fiddle music is traditional in Métis culture. The fiddle plays the melody in the music and tells a story. Many Métis legends are recorded in fiddle music. A fiddle has four strings, tuned to the notes G, D, A, and E. The equations that represent the sound waves of these notes are given below, where x represents time, in seconds.

G: $y = \sin 1231.5x$ A: $y = \sin 2764.6x$

D: $y = \sin 1845.4x$ E: $y = \sin 4142.5x$

a) Determine the period and the frequency (complete cycles per second) of each string. State the frequency to one decimal place.

b) Comment on the relative frequencies of the four strings.

c) If two consecutive strings are played simultaneously, how many periods will have elapsed for the sound wave from each string when the sound waves coincide at the midline and are increasing? Explain.

d) If the first three strings are played simultaneously, how many periods will have elapsed for the sound wave from each string when the sound waves coincide at the midline and are increasing?

17. a) Describe each function by stating the amplitude, the equation of the midline, the range, the period, and the horizontal translation of $y = \sin x$. Verify your descriptions using graphing technology.

i) $y = 3 \sin 2(x - 60°) + 4$

ii) $y = 3 \sin 2(x - 240°) + 4$

iii) $y = 3 \sin 2(x + 120°) + 4$

b) Compare the three graphs. What do you notice? Explain.

c) Graph the function

$$y = 3 \cos 2(x - 105°) + 4$$

Compare this graph with your graphs for part a). What do you notice? Explain.

18. An apple is attached to a spring. The height of the apple as it oscillates up and down can be modelled by the equation

$$h(t) = 4 \sin (8\pi t) + 6.5$$

where $h(t)$ represents the height of the apple in centimetres and t represents the time in seconds.

a) What are the highest and lowest points that the apple reaches?

b) What is the period of the function? What does the period tell you about the apple in this context?

19. The height of a chair on a Ferris wheel is described by the function

$$h(t) = 15 \cos \left(t - \frac{1}{2}\right) + 18$$

where $h(t)$ represents the height of the chair in metres and t represents the time in minutes.

a) What are the maximum and minimum heights you can reach if you are riding the Ferris wheel?

b) What is the period of the function? What does the period tell you about the Ferris wheel in this context?

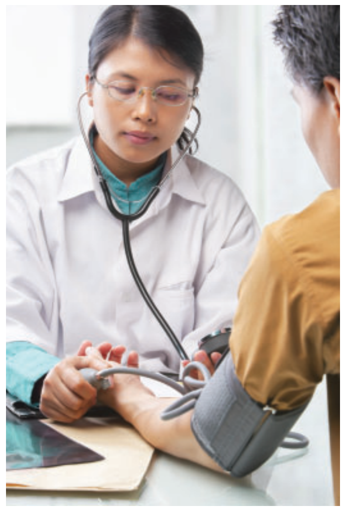
20. A person's blood pressure, $P(t)$, in millimetres of mercury (mm Hg), can be modelled by the function

$$P(t) = -20 \cos (8.4t) + 100$$

where t is the time in seconds.

a) What is the period of the function?

b) What does the value of the period mean in this situation?



Closing

21. Explain how to describe the graph of a sinusoidal function, given its equation. Give an example.

Extending

22. **a)** Graph the functions

$$y = 2 \sin x \text{ and}$$
$$y = -2 \sin x$$

on the same axes. How are your graphs the same? How are they different?

b) What horizontal translation of $y = 2 \sin x$ has the same graph as $y = -2 \sin x$?

c) Do you think what you observed in part b) would always be the same when the values of a are opposites? Explain.