7.1 Exploring the Characteristics of Exponential Functions

YOU WILL NEED

- graphing technology

GOAL

Investigate the characteristics of the graphs of exponential functions.

EXPLORE the Math

exponential function

A function of the form

$$y = a(b)^x$$

where $a \neq 0$, $b > 0$, and $b \neq 1$.

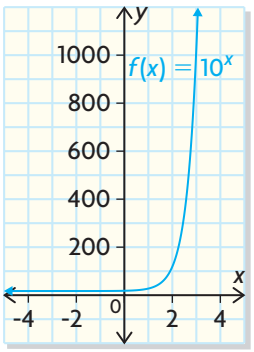
Francine and Kent wondered how the characteristics of **exponential functions** differ from the characteristics of polynomial functions. Francine created a table of values and a graph for each of the functions shown below to investigate similarities and differences.

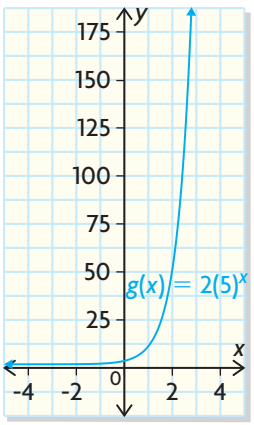
$f(x) = 10^x$

x	$f(x)$
-3	$10^{-3} = 0.001$
-2	$10^{-2} = 0.01$
-1	$10^{-1} = 0.1$
0	$10^{0} = 1$
1	$10^{1} = 10$
2	$10^{2} = 100$
3	$10^{3} = 1000$

$g(x) = 2(5)^x$

x	$g(x)$
-3	$2(5)^{-3} = 0.016$
-2	$2(5)^{-2} = 0.08$
-1	$2(5)^{-1} = 0.4$
0	$2(5)^{0} = 2$
1	$2(5)^{1} = 10$
2	$2(5)^{2} = 50$
3	$2(5)^{3} = 250$

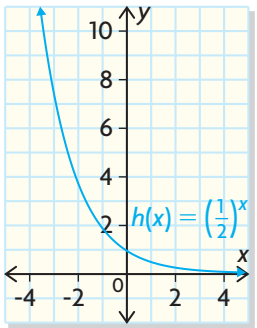


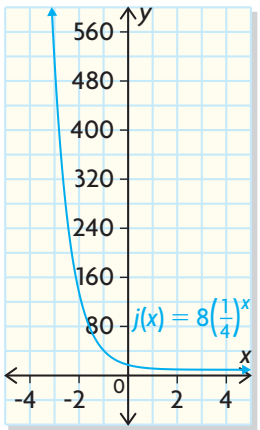


$h(x) = \left(\frac{1}{2}\right)^x$ $\qquad$ $j(x) = 8\left(\frac{1}{4}\right)^x$

x	$h(x)$
-3	$\left(\frac{1}{2}\right)^{-3} = 8$
-2	$\left(\frac{1}{2}\right)^{-2} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^{0} = 1$
1	$\left(\frac{1}{2}\right)^{1} = 0.5$
2	$\left(\frac{1}{2}\right)^{2} = 0.25$
3	$\left(\frac{1}{2}\right)^{3} = 0.125$

x	$j(x)$
-3	$8\left(\frac{1}{4}\right)^{-3} = 512$
-2	$8\left(\frac{1}{4}\right)^{-2} = 128$
-1	$8\left(\frac{1}{4}\right)^{-1} = 32$
0	$8\left(\frac{1}{4}\right)^{0} = 8$
1	$8\left(\frac{1}{4}\right)^{1} = 2$
2	$8\left(\frac{1}{4}\right)^{2} = 0.5$
3	$8\left(\frac{1}{4}\right)^{3} = 0.125$





Kent investigated other exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, using his graphing calculator. Both students considered these characteristics of the functions:

- the number of x-intercepts
- the y-intercept
- the end behaviour
- the domain
- the range

? What are the characteristics of the graphs of exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$?

Reflecting

A. Based on your observations, which characteristics of the graphs of exponential functions are similar to characteristics of polynomial functions you have studied and which are different?

B. Kent claims that all exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, do not have x-intercepts. Do you agree or disagree? Explain.

C. Francine claims that all exponential functions of this form have a restricted range. Do you agree or disagree? Explain.

D. Can you use the end behaviour of exponential functions of this form to help you decide if the function is increasing or decreasing? Explain.

E. Describe any patterns you noticed in the tables of values for the exponential functions you investigated.

In Summary

Key Ideas

- An exponential function has the form $f(x) = a(b)^x$, where x is the exponent and $a \neq 0$, $b > 0$, and $b \neq 1$.
- All exponential functions of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, have the following characteristics:

Number of *x*-Intercepts	0
***y*-Intercept**	a
End Behaviour	Curve extends from quadrant II to quadrant I.
Domain	$\{x \mid x \in \mathrm{R}\}$
Range	$\{y \mid y > 0, y \in \mathrm{R}\}$

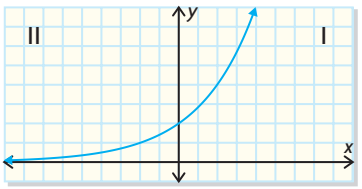
Need to Know

- There are two different shapes of the graphs of an exponential function of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$:

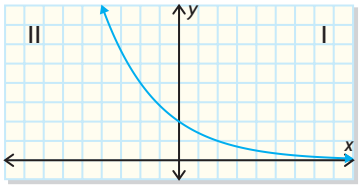
  - Case 1: An increasing function; the curve extends from quadrant II to quadrant I.

  - Case 2: A decreasing function; the curve extends from quadrant II to quadrant I.

Increasing



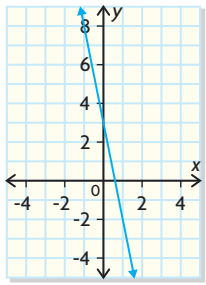
Decreasing



FURTHER Your Understanding

1. Determine whether each graph represents an exponential function. If possible, identify the type of function.

a)



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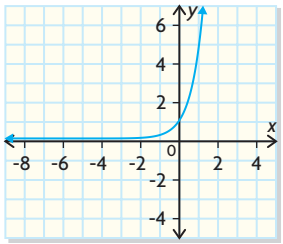

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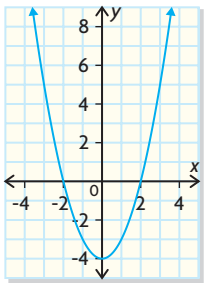

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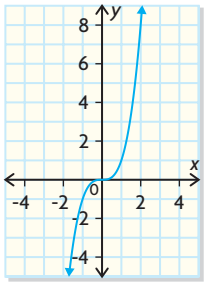

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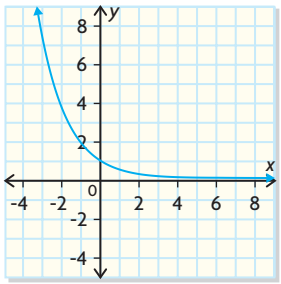

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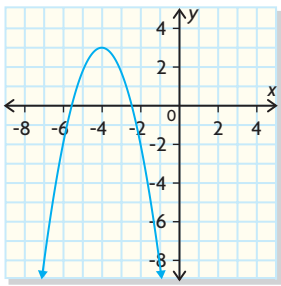

2. For each of the exponential functions you identified in question 1:

- State the number of x-intercepts.
- State the y-intercept.
- State the end behaviour.
- State the domain.
- State the range.

3. Graph each exponential function. Determine the number of x-intercepts, the y-intercept, the end behaviour, the domain, and the range.

a) $y = 10(2)^x$ **b)** $y = 6(2)^x$ **c)** $y = 27\left(\frac{1}{3}\right)^x$ **d)** $y = 4\left(\frac{1}{2}\right)^x$