5.4 Mutually Exclusive Events

YOU WILL NEED

- calculator

GOAL

Understand and solve problems that involve mutually exclusive and non-mutually exclusive events.

INVESTIGATE the Math

EXPLORE...

- Carlos drew a single card from a standard deck of 52 playing cards. What is the probability that the card he drew is either an 8 or a black card?

Janek and Violeta are playing a board game. To move on this turn, Janek must roll either doubles or a sum of 7 with the two standard dice.

? What is the probability that Janek will move on this turn?

A. Create an outcome table for the sample space of a pair of dice.

B. Circle each occurrence of throwing doubles in your outcome table. Determine P(throw doubles).

C. Shade each occurrence of rolling a sum of 7 in your table. Determine P(sum of 7).

D. Sum the probabilities of the two events in parts B and C.

E. Use your outcome table to determine the probability that Janek will roll doubles or roll a sum of 7. How does this probability relate to the sum of the probabilities in part D?

F. Illustrate the sample space in a Venn diagram.

G. Are the two events **mutually exclusive**? How does your Venn diagram show this? How does your outcome table show this?

H. Determine P(throw doubles $\cup$ sum of 7) using your Venn diagram.

Reflecting

I. Given that A and B are mutually exclusive events, write a formula you could use to determine $P(A \cup B)$ in terms of $P(A)$ and $P(B)$.

J. Suppose that Janek has to roll either doubles (event A) or a sum of 6 (event B) to move. Could you use the formula you wrote in part I to determine if Janek can move? If not, what formula could you use? Explain.

APPLY the Math

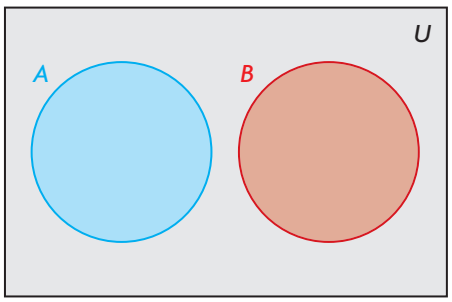
EXAMPLE 1 Reasoning to develop a formula for events that are mutually exclusive

Jamie claims that

$$P(A \cup B) = P(A) + P(B)$$

where A and B represent mutually exclusive events. Is she correct? Explain.

Cyndie's Solution



A and B represent mutually exclusive events, so their sets of favourable outcomes are disjoint. That is, they have no common elements. I drew a Venn diagram to show this.

$$P(A \cup B) = \frac{n(A \cup B)}{n(U)}$$

Since A and B are mutually exclusive, there is no intersection. I did not need to use the **Principle of Inclusion and Exclusion.**

$$P(A \cup B) = \frac{n(A) + n(B)}{n(U)}$$

Since A and B are mutually exclusive, there is no intersection to subtract.

The number of elements in the union of A and B is equal to the number of elements in A plus the number of elements in B.

$$P(A \cup B) = \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)}$$

Since the numerator is the sum of two terms, I can write the fraction as the sum of two fractions with the same denominator.

$$P(A \cup B) = P(A) + P(B)$$

The probability of A is equal to the number of elements in A divided by the number of elements in U.

Similarly, the probability of B is equal to the number of elements in B divided by the number of elements in U.

Jamie is correct and is supported by my reasoning shown above.

Your Turn

Do you think this formula is true for two events that are not mutually exclusive? Explain.

EXAMPLE 2 Determining the probability of events that are not mutually exclusive

Recall the board game that Janek and Violeta were playing. According to a different rule, if a player rolls a sum that is greater than 8 or a multiple of 5, the player gets a bonus of 100 points.

a) Determine the probability that Violeta will receive a bonus of 100 points on her next roll.

b) Write a formula you could use to calculate the probability of two non-mutually exclusive events. Answer part a) again to show that your formula works.

Violeta's Solution

a)

Possible Sums When a Pair of Dice are Rolled						
Die 1/ Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

I created an outcome table for the sample space of the possible sums.

I wrote the sums greater than 8 in red, and I highlighted the multiples of 5.

Three numbers were both red and highlighted. Therefore, the events are not mutually exclusive.

Let A represent rolling a sum greater than 8.
Let B represent rolling a sum that is a multiple of 5.

I used A and B to represent the two events.

$$P(A) = \frac{10}{36}$$

$$P(B) = \frac{7}{36}$$

I used my outcome table to determine the probability of each event. I did not write $P(A)$ in least terms, because it is easier to work with fractions that have the same denominator.

Since 14 numbers are red or highlighted, or both,

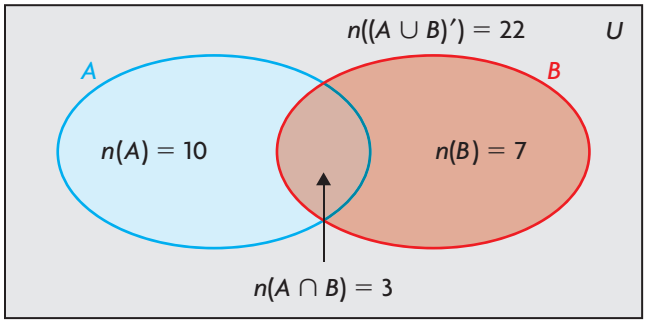
$$P(A \cup B) = \frac{14}{36}$$

$$P(A \cup B) = \frac{7}{18}, \text{ or } 0.388...$$

I determined the probability of A or B using my sample space.

There is about a 39% chance that Violeta will receive the bonus on her next roll.

b) Let U represent the universal set of all possible sums when a pair of dice are rolled.



I drew a Venn diagram to visualize the problem. I wrote the number of elements for each event in the appropriate place. There are three sums of 10 that belong to both event A and event B.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

I knew that the number of elements in the union of two non-disjoint sets is the number of elements in the intersection of the two sets subtracted from the sum of the number of elements in each set.

$$P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(U)}$$

$$P(A \cup B) = \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)} - \frac{n(A \cap B)}{n(U)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

I wrote a formula for the probability of two non-mutually exclusive events.

I used my Venn diagram to determine the number of favourable outcomes.

The total number of outcomes is the number of elements in the universal set.

$$P(A \cup B) = \frac{10}{36} + \frac{7}{36} - \frac{3}{36}$$

$$P(A \cup B) = \frac{14}{36}$$

$$P(A \cup B) = \frac{7}{18}, \text{ or } 0.388...$$

I determined the probability using my formula. I got the same result from the outcome table, which supports the formula.

There is about a 39% chance that Violeta will receive the bonus on her next roll.

Your Turn

a) Would Violeta's formula also work for mutually exclusive events? Explain.

b) Bronwyn determined $P(A \cup B)$ using the following formula:

$$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

Would this formula give the correct answer? Explain.

EXAMPLE 3 Using a Venn diagram to solve a probability problem that involves two events

A school newspaper published the results of a recent survey.

a) Are skipping breakfast and skipping lunch mutually exclusive events?

b) Determine the probability that a randomly selected student skips breakfast but not lunch.

c) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.

C4 B

Eating Habits: Student Survey Results

- 62% skip breakfast
- 24% skip lunch
- 22% eat both breakfast and lunch

Mei Lin's Solution

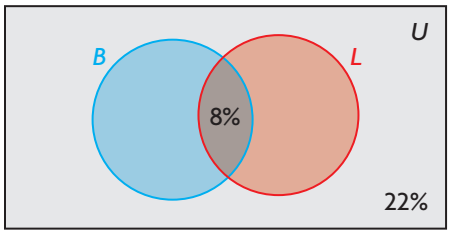
a) If the events of skipping breakfast and skipping lunch are mutually exclusive, then the sum of the probabilities of these events and the sum of the probability of their complement (eat both breakfast and lunch) should total 100%. But, 62% + 24% + 22% is 108%. This means that 8% of the students surveyed were counted twice. The events are not mutually exclusive.

Since the only two options are eating a meal or skipping a meal, the results account for everyone who was surveyed.

My analysis of the survey data shows that some of the students surveyed must skip both breakfast and lunch.

b) Let B represent skipping breakfast.
Let L represent skipping lunch.

I used B and L to represent the two events.



I drew a Venn diagram to show the survey results.

I knew that the probability of skipping breakfast and lunch is 8%.

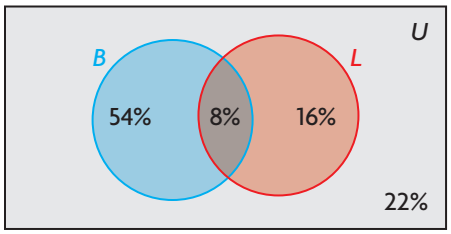
$$P(B \setminus L) = P(B) - P(B \cap L)$$
$$P(B \setminus L) = 62\% - 8\%$$
$$P(B \setminus L) = 54\%$$

The probability of skipping only breakfast is 54%.

I knew that I could determine the probability of skipping breakfast but not lunch by subtracting the probability of skipping both meals from the probability of skipping breakfast.

c) $P(L \setminus B) = P(L) - P(B \cap L)$
$P(L \setminus B) = 24\% - 8\%$
$P(L \setminus B) = 16\%$

I used the same reasoning to determine the probability of skipping lunch but not breakfast.



I updated my Venn diagram.

$P(B \cup L) = P(B \setminus L) + P(L \setminus B) + P(B \cap L)$
$P(B \cup L) = 54\% + 16\% + 8\%$
$P(B \cup L) = 78\%$

According to this survey, there is a 78% chance that a student selected at random will skip breakfast or lunch.

I determined the probability of skipping at least one of breakfast or lunch by adding the three areas of my Venn diagram.

"Skips at least one of breakfast or lunch" means the same as skips one or the other, or both. Since 22% eat both breakfast and lunch, then 100% − 22% or 78% skip one or the other, or both. This supports my solution.

Your Turn

Suppose that the school cafeteria introduces a different lunch menu. Now only 14% of the students skip lunch, but 62% of the students continue to skip breakfast. Determine the maximum percent and minimum percent of students who now eat both breakfast and lunch. Provide your reasoning.

EXAMPLE 4 Making a decision based on probability

Reid's mother buys a new washer and dryer set for $2500 with a 1-year warranty. She can buy a 3-year extended warranty for $450. Reid researches the repair statistics for this washer and dryer set and finds the data in the table below. Should Reid's mother buy the extended warranty? Justify your decision.

Appliance	*P*(repair within extended warranty period)	Average Repair Cost
washer	22%	$400
dryer	13%	$300
both	3%	$700

Reid's Solution

$W = \{\text{washer needs repair}\}$
$D = \{\text{dryer needs repair}\}$
$P(W \cap D) = 3\%$

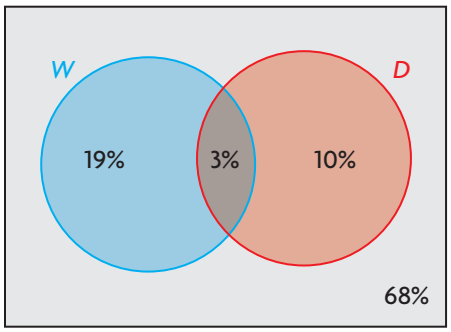
The probability that both appliances will need repair is 3%.

I determined the probability that only the washer will need repair.

$$P(W \setminus D) = P(W) - P(W \cap D)$$
$$P(W \setminus D) = 22\% - 3\%$$
$$P(W \setminus D) = 19\%$$

I determined the probability that only the dryer will need repair.

$$P(D \setminus W) = P(D) - P(W \cap D)$$
$$P(D \setminus W) = 13\% - 3\%$$
$$P(D \setminus W) = 10\%$$



I drew a Venn diagram to visualize the probability of needing repair.

$$P(W \cup D) = P(W \setminus D) + P(D \setminus W) + P(W \cap D)$$
$$P(W \cup D) = 19\% + 10\% + 3\%$$
$$P(W \cup D) = 32\%$$

I determined the probability that at least one appliance will need repair by adding the individual probabilities.

There is a 32% chance that repair will be needed within the extended warranty period. This probability is low, so I am leaning toward not buying the extended warranty.

Repair breakdown:

Appliance	*P*(repair)	Average Repair Cost
washer	19%	$400
dryer	10%	$300
both	3%	$400 + $300 or $700

I decided to do a cost analysis for supporting evidence.

I analyzed the results, along with the average repair costs.

Appliance	*P*(repair)	Cost or Saving
washer	19%	$450 − $400 or $50
dryer	10%	$450 − $300 or $150
neither	68%	$450 − $0 or $450

I subtracted the repair cost for each appliance from the warranty cost to determine whether the warranty would save money.

For each appliance, the warranty costs more than the repair. The warranty is not worth getting.

On average, it would cost $50 less to repair the washer and $150 less to repair the dryer than it would to buy the extended warranty.

both	3%	$450 − $700 or −$250

If both the washer and the dryer need to be repaired, the repair cost would be more than the extended warranty. However, there is only a 3% probability that both appliances will break down.

If both appliances need repair, then, on average, it would cost $250 more than the extended warranty.

There is a 3% chance that the warranty will save my mother $250 and a 97% chance that it will cost her money. She should not buy the warranty because it is more likely to cost money than to save money.

I presented the results of my analysis.

Your Turn

a) Suppose that the average repair cost was $650 for the washer and $490 for the dryer. Should Reid's mother buy the warranty? Justify your decision.

b) Suppose that the probability of both appliances needing repair was 10%. Would your advice for part a) change? Explain.

EXAMPLE 5 Determining the probability of two events

A car manufacturer keeps a database of all the cars that are available for sale at all the dealerships in Western Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both heated leather seats and a sunroof.

Katherine's Solution

Let A represent the universal set of all model A cars.
Let L represent model A cars with heated leather seats.
Let S represent model A cars with a sunroof.

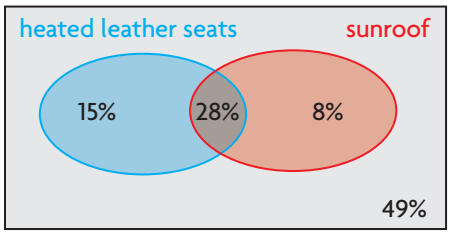
I defined the universal set A and subsets L and S.

I knew that the complement of having neither feature on the car would be having at least one of the two features.

$P(L \cup S) = 100\% - 49\%$
$P(L \cup S) = 51\%$

I used the probability formula for non-mutually exclusive events. I substituted the probabilities I knew and solved for $P(L \cap S)$.

$P(L \cup S) = P(L) + P(S) - P(L \cap S)$
$51\% = 43\% + 36\% - P(L \cap S)$
$28\% = P(L \cap S)$



I drew a Venn diagram to visualize the probabilities in the problem.

The probability of a model A car at a dealership having both heated leather seats and a sunroof is 28%.

Your Turn

The database also reports that 56% of the available model B cars have heated leather seats, 49% have a sunroof, and 27% have neither. What is the probability of a model B car at a dealership having a sunroof but no heated leather seats?

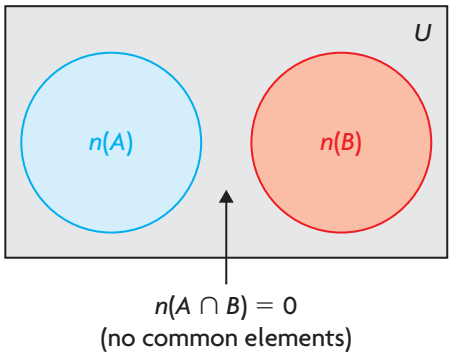
In Summary

Key Ideas

- You can represent the favourable outcomes of two **mutually exclusive events**, A and B, as two disjoint sets.

  You can represent the probability that either A or B will occur by the following formula:

  $$P(A \cup B) = P(A) + P(B)$$



- You can represent the favourable outcomes of two **non-mutually exclusive events**, A and B, as two intersecting sets.

  You can represent the probability that either A or B will occur by this formula:

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

  An alternative formula is

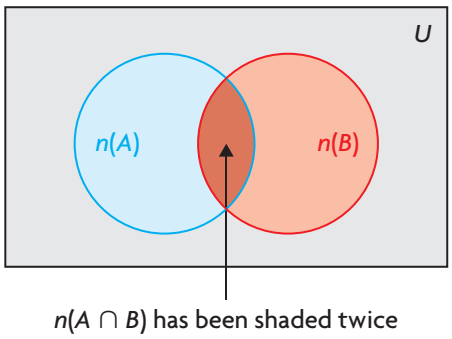
  $$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

  When the two events are mutually exclusive, both formulas are equivalent,

  $$n(A \cap B) = 0$$

  which results in

  $$P(A \cap B) = 0$$

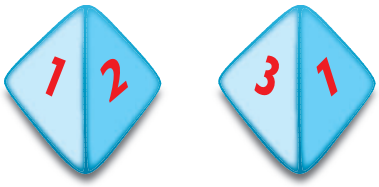


Need to Know

- You can use the Principle of Inclusion and Exclusion, which is used to count the elements in the union of two sets, to determine the probability of non-mutually exclusive events.

CHECK Your Understanding

1. Zach is playing a board game. He must roll two four-sided dice, numbered 1 to 4. He can move if he rolls a sum of 2 or a sum of 8.



a) Use A and B to represent the two events that will allow Zach to move. Then draw a Venn diagram to illustrate A and B.
b) Are A and B mutually exclusive or not mutually exclusive?
c) Determine the probability that Zach will roll a sum of 2 or a sum of 8.
d) Determine the probability that Zach will roll doubles or a sum of 6.

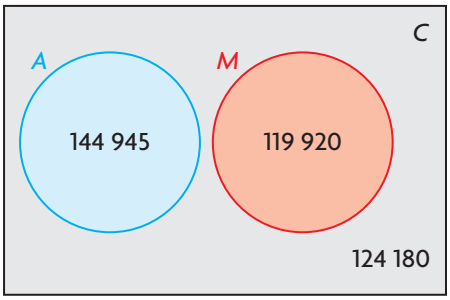
2. Pearl is about to draw a card at random from a standard deck of 52 playing cards. If she draws a face card or a spade, she will win a point.
a) Draw a Venn diagram to represent the two events.
b) Are the events mutually exclusive?
c) Determine the probability of drawing a face card or a spade.

3. The probability that Maria will go to the gym on Saturday is 0.75. The probability that she will go shopping on Saturday is 0.4. The probability that she will do neither is 0.2.
a) Draw a Venn diagram to represent the two events.
b) Are the two events mutually exclusive?
c) Determine the probability that Maria will do at least one of these activities on Saturday.

PRACTISING

4. For each of the following, state whether the events are mutually exclusive. Explain your reasoning.
a) Selecting a prime number or selecting an even number from a set of 15 balls, numbered 1 to 15
b) Rolling a sum of 10 or a sum of 7 with a pair of six-sided dice, numbered 1 to 6
c) Walking to school or getting a ride to school

5. The following Venn diagram shows the declared population of Métis in Canada, where
$A = \{\text{Métis in Alberta and British Columbia}\}$,
$M = \{\text{Métis in Manitoba and Saskatchewan}\}$, and
$C = \{\text{Métis in Canada}\}$.



a) Determine the probability that a person who is Métis lives in Alberta or British Columbia.
b) Determine the probability that a person who is Métis lives in Manitoba or Saskatchewan.
c) Does $P(A \cup M) = P(A) + P(M)$ in this situation? Explain.
d) Determine the odds in favour of a person who is Métis living in one of the four Western provinces.

6. Tanya plays the balloon pop game at a carnival. There are 40 balloons, with the name of a prize inside each balloon. The prizes are 8 stuffed bears, 5 toy trucks, 16 decks of cards, 7 yo-yos, and 4 giant stuffed dogs. Tanya pops a balloon with a dart. Determine the odds in favour of her winning either a stuffed dog or a stuffed bear.

7. Edward rolls two regular six-sided dice. Determine the odds against each event below.
a) The sum is 5 or 9.
b) Both dice are even numbers, or the sum is 8.

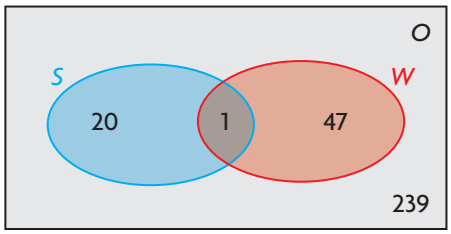
8. The probability that John will study on Friday night is 0.4.
The probability that he will play video games on Friday night is 0.6.
The probability that he will do at least one of these activities is 0.8.
a) Determine the probability that he will do both activities.
b) Are these events mutually exclusive? Explain how you know.

9. The following Venn diagram shows the number of Canadian athletes who have won medals at the Olympics from 1996 to 2010. In the diagram below,

S = {athletes who have won two or more medals at the Summer Olympics},

W = {athletes who have won two or more medals at the Winter Olympics}, and

O = {athletes who have won at least one Olympic medal}.



a) Are the two events (winning two or more medals at the Summer Olympics and winning two or more medals at the Winter Olympics) mutually exclusive? Explain.

b) A Canadian athlete who won a medal at the Summer Olympics from 1996 to 2010 is selected at random. Determine the odds in favour of this athlete having won two or more medals.

c) A Canadian athlete who won a medal at the Summer Olympics or the Winter Olympics from 1996 to 2010 is selected at random. Determine the odds in favour of this athlete having won two or more medals.

Olympian Clara Hughes is the only Canadian who has won medals at both the Summer Olympics and Winter Olympics.

10. Create a problem that involves determining the probability of two mutually exclusive events. Give your problem to a classmate to solve.

11. Create a problem that involves determining the probability of two non-mutually exclusive events. Give your problem to a classmate to solve.

12. The probability of a 65-year-old person wearing glasses is 76%. The probability of a 65-year-old person having some hearing loss is 68%. Suppose that 10% of 65-year-olds who have a hearing loss do not wear glasses. A 65-year-old person is selected at random.

a) Determine the probability of this person wearing glasses and not having a hearing loss.

b) Determine the probability of this person not wearing glasses and not having a hearing loss.

13. Suppose that you are about to draw a single card, at random, from a standard deck of 52 playing cards. Determine the probability of each event below.

a) You draw an 8 or a king.

b) You draw a red card or a face card.

14. An Ipsos survey reported that 37% of Prairie households have one or more dogs, 31% have one or more cats, and 47% have neither dogs nor cats. Suppose that a Prairie household is selected at random. Determine each probability.

a) There are cats or dogs in the household.

b) There are cats but no dogs in the household.

c) There are dogs but no cats in the household.

15. On Sunday, the weather forecaster says that there is a 60% chance of snow on Monday and a 40% chance of snow on Tuesday. The forecaster also says that there is a 20% chance of snow on both Monday and Tuesday. Determine the probability that there will be snow on Monday or on Tuesday.

16. Suppose that you are the analyst for a computer manufacturer. Today, you are testing a new computer to see whether it needs a surge protector.

- Your tests show that a spike in voltage has a 0.15% chance of damaging the computer's power supply, a 0.30% chance of damaging other components, and a 0.1% chance of damaging the power supply and other components.
- A surge protector is needed if the probability of a spike in voltage damaging a computer exceeds 0.5%.

Would you recommend that this computer should have a surge protector?

17. Consider this table:

Blood Type	Percent of Canadians with This Blood Type (%)
O+	38
O−	7
A+	34
A−	6
B+	9
B−	2
AB+	3
AB−	1

a) Determine the probability that a randomly selected Canadian will have each type of blood.
i) type O **ii)** a negative blood type **iii)** type A or B

b) A person with type A− blood can donate blood to someone with type A+, A−, AB+, or AB− blood. Suppose that Dani has type A− blood. Determine the probability that she can donate blood to the next person who needs a transfusion.

c) Assume that your local blood bank has a blood supply in the same proportions as shown in the table. A person with type AB− blood can receive type AB−, A−, B−, or O− blood. Richard has type AB− blood. Determine the probability that the most recent donation to the blood bank will be effective for Richard's transfusion.

Closing

18. Describe how determining the probability of two mutually exclusive events is different from determining the probability of two events that are not mutually exclusive. Give an example.

Extending

19. A school newspaper reports on the students' taste in music.
- 20% like only rock.
- 13% like only blues.
- 30% like only rap.
- 10% like rock and blues, but not rap.
- 14% like rock and rap, but not blues.

Determine the probability that a randomly selected student will either like all three types of music or like blues and rap, but not rock.

20. Consider three events, represented by A, B, and C. For each situation, determine a formula for $P(A \cup B \cup C)$.
a) A, B, and C are mutually exclusive.
b) A, B, and C are not mutually exclusive.