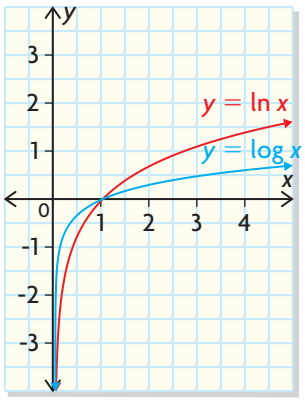
# 7 Chapter Review

## *FREQUENTLY ASKED* Questions

**Study Aid**
- See Lesson 7.4.




**Study Aid**
- See Lesson 7.4, Examples 1 and 2.
- Try Chapter Review Question 9.

**Q: How are the functions $y = \log x$ and $y = \ln x$ similar and how do they differ?**

**A:** Both are logarithmic functions, but they have different bases. The base of $y = \log x$ is 10. This is called the common logarithm. The base of $y = \ln x$ or $y = \log_e x$ is $e$. This is called the natural logarithm. Both functions share the same characteristics:
- one $x$-intercept at $x = 1$
- no $y$-intercept
- the graph extends from quadrant IV to quadrant I
- the domain is restricted, $\{x \mid x > 0, x \in \mathrm{R}\}$
- the range is not restricted, $\{y \mid y \in \mathrm{R}\}$

**Q: How can you predict the characteristics of a logarithmic function from its equation?**

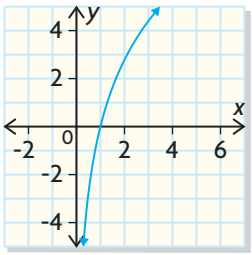
**A:** A logarithmic function of the form

$$y = a \log x \quad \text{or} \quad y = a \ln x$$

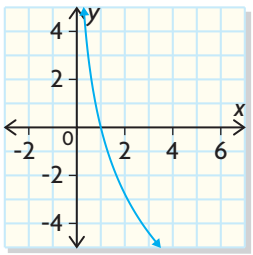
has the following characteristics:
- There is one $x$-intercept, $x = 1$.
- There is no $y$-intercept.
- The value of $a$ indicates whether the graph of the function is increasing or decreasing and determines the function's end behaviour.
  - The graph is increasing when $a > 0$. The graph extends from quadrant IV to quadrant I.
  - The graph is decreasing when $a < 0$. The graph extends from quadrant I to quadrant IV.

  For example:

$y = 4 \log x$




$y = -4 \log x$



**Q: To perform a logarithmic regression on a data set, how do you decide which variable is the independent variable?**

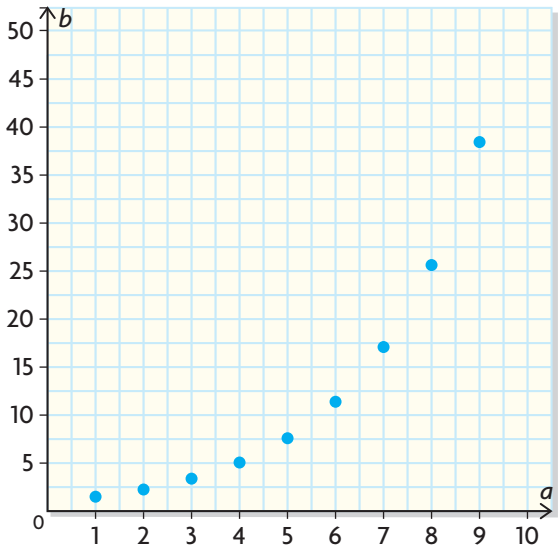
**A:** Start by considering the context in which the data set is presented. Use the quantities in the context to help you decide which quantity depends on the other. This quantity is the dependent variable, while the other quantity is the independent variable.

If you can't decide from the context, create a scatter plot of the data. If you have chosen the independent variable correctly, you should see either an increasing or decreasing trend, with many points scattered near the $y$-axis. These are characteristics of a logarithmic function. If many points are scattered near the $x$-axis, then you have incorrectly chosen the independent variable. Switch the variables, and try again.

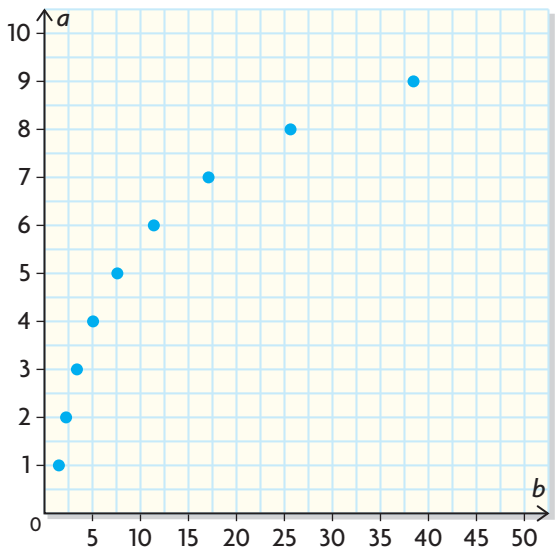
For example, the data below shows what happens when you create a scatter plot with either $a$ or $b$ as the independent variable. Notice that many points are scattered near the $y$-axis in the second scatter plot. To perform a logarithmic regression, $b$ must be treated as the independent variable.

| ***a*** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| ***b*** | 1.50 | 2.25 | 3.38 | 5.06 | 7.59 | 11.39 | 17.09 | 25.63 | 38.44 |

Scatter plot for $a$ as the independent variable




Scatter plot for $b$ as the independent variable




**Study *Aid***

- See Lesson 7.5, Examples 1 and 2.
- Try Chapter Review Question 11.

## PRACTISING

### Lesson 7.1

**1.** **a)** Describe how the end behaviour of exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, differs from the end behaviour of polynomial functions of degree $\leq 3$.

**b)** What characteristics do all exponential functions of the form stated in part a) have in common?

### Lesson 7.2

**2.** **a)** Use the equation of this function to predict the characteristics of its graph:

$$y = 9\left(\frac{1}{3}\right)^x$$

**b)** What could you change in the parameters of the equation to make the function an increasing function? Explain.

**3.** For each function, state:

**i)** the $x$-intercepts

**ii)** the $y$-intercept

**iii)** its end behaviour

**iv)** the domain and range

**v)** whether the function is increasing or decreasing

**a)** $y = 125(0.78)^x$

**b)** $y = 0.12(0.85)^x$

**c)** $y = (3)^x$

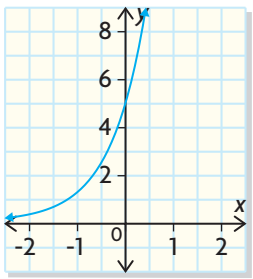
**d)** $y = 0.85(5)^x$

**4.** Match each function to its corresponding graph. Provide your reasoning.

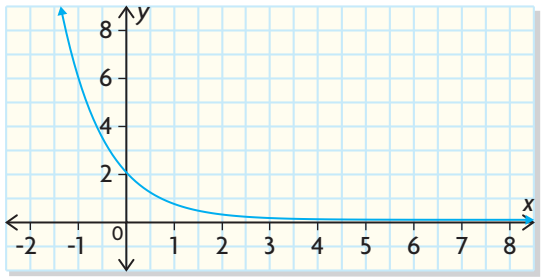
**a)** $y = 5(4)^x$

**b)** $y = 2\left(\frac{1}{3}\right)^x$

**i)**




**ii)**




### Lesson 7.3

**5.** On August 16, 1896, gold was discovered near Dawson City, Yukon Territory. The population of Dawson City experienced rapid growth after the discovery. Below is population data from April 1896 to January 1897.

| Date | Dawson City Population |
|---|---|
| April 1, 1896 | 1 000 |
| July 1, 1896 | 3 000 |
| October 1, 1896 | 9 000 |
| January 1, 1897 | 27 000 |

**a)** Determine the equation of the exponential regression function models the population growth.

**b)** State the domain and range in this context.

**c)** Estimate the population of Dawson City in mid-May and in mid-August of 1896. Explain how you determined your answers.

Dawson City is situated at the junction of the Dawson and Klondike rivers.

**6.** Since 2005, a naturalist group has been tracking the deer population near the town of Hudson Bay, Saskatchewan.

| Years After 2005 | Deer Population |
|---|---|
| 1 | 218 |
| 2 | 237 |
| 3 | 260 |
| 4 | 282 |
| 5 | 307 |
| 6 | 336 |
| 7 | 360 |

**a)** Construct a scatter plot for the data.
**b)** Determine the equation of the exponential regression function that models the data.
**c)** Assuming the same growth rate, predict the deer population 10 years after 2005.
**d)** In which year would you expect the population to have doubled?

A Saskatchewan mule deer

**7.** Spears and wooden tools used by early inhabitants were found at an archeological dig site close to Cypress Hills, Alberta. Carbon-14 dating was used to determine the age of the tools. The function that models the decay of carbon-14 is

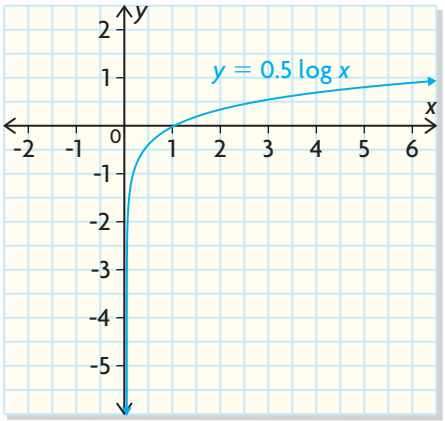
$$A(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

where the initial amount of carbon-14 is 100%, $A(t)$ represents the percent of carbon-14 left in the tools, and $t$ represents the time in years.

**a)** Based on their experience, the archeologists guessed that the tools were about 8000 years old. If their guess was accurate, what percent of the initial carbon-14 would be present in the tools?
**b)** The tools were found to have 41% of the original carbon-14 present. Determine the age of the tools to the nearest hundred years.

### Lesson 7.4

**8.** For the logarithmic function shown below, state the $x$-intercept, the number of $y$-intercepts, the end behaviour, the domain, and the range.




**9.** Predict the $x$-intercept, the number of $y$-intercepts, the end behaviour, the domain, and the range of each function, based on its equation. Verify your predictions using graphing technology.

**a)** $y = 9 \log x$
**b)** $y = -6 \log x$
**c)** $y = 10 \ln x$
**d)** $y = -0.3 \ln x$

**10.** Match each function with its corresponding graph. Provide your reasoning.

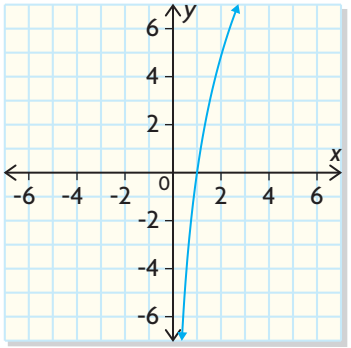
**i)** $y = -\log x$

**ii)** $y = 10^x$

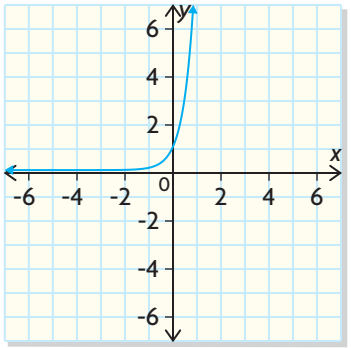
**iii)** $y = 7 \log x$

**iv)** $y = 6\left(\frac{3}{4}\right)^x$

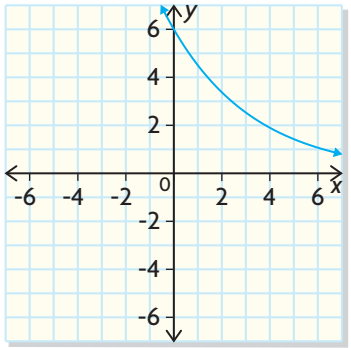
**a)**



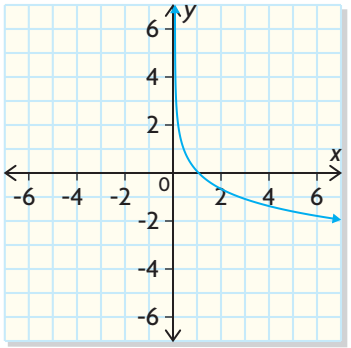

**b)**




**c)**




**d)**




**Lesson 7.5**

**11.** The euphotic zone is the upper 200 m layer in an ocean. Very little sunlight penetrates deeper than 200 m, so most plants live in the euphotic zone. As a result, 70% of all photosynthesis on Earth occurs in the euphotic zone of the oceans. The table below shows light penetration data for a location in the Pacific Ocean.

| Depth (m) | Penetration of Sunlight (%) |
|---|---|
| 0 | 100.00 |
| 20 | 54.37 |
| 40 | 29.57 |
| 60 | 16.08 |
| 80 | 8.74 |
| 100 | 4.76 |

Use the equation of the logarithmic regression function to determine the amount of sunlight that penetrates to a depth of 200 m at this location. Express your answer to the nearest hundredth of a percent.