

**YOU WILL NEED**

- counters or coins

**EXPLORE...**

- Three students are playing a game. Two of the students flip a coin, and the third student records their scores. Student 1 gets a point if the result is two heads, student 2 gets a point if the result is two tails, and student 3 gets a point if the result is a head and a tail. The first student to get 10 points wins. Explain whether you would prefer to be student 1, student 2, or student 3.

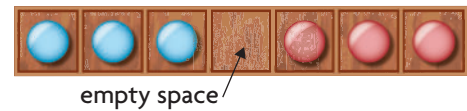
**GOAL**

Determine, explain, and verify a reasoning strategy to solve a puzzle or to win a game.

**INVESTIGATE the Math**

To solve a leapfrog puzzle, coloured counters are moved along a space on a board. The goal is to move each set of coloured counters to the opposite side of the board.

Board at start



A counter can move into the empty space.



A counter can leapfrog over another counter into the empty space.



Board at end



- ?** What is the minimum number of moves needed to switch five counters of each colour?



- Develop a group strategy to switch the blue and red counters using as few moves as possible.
- Execute your strategy, counting each move you make.
- How many moves did you need to complete the switch?

**Reflecting**

- How did you know that you had completed the switch in the fewest number of moves?
- Did you use inductive or deductive reasoning to solve the puzzle? Explain.
- Predict the minimum number of moves needed to solve the puzzle if you had six counters of each colour. Explain how you made your prediction.
- Did you use inductive or deductive reasoning in step F? Explain.

## APPLY the Math

### EXAMPLE 1

### Using reasoning to determine possible winning plays

Frank and Tara are playing darts, using the given rules. Their scores are shown in the table below. To win, Frank must reduce his score to exactly zero and have his last counting dart be a double.

#### Rules

- Each player's score starts at 501.
- The goal is to reduce your score to zero.
- Players alternate turns.
- Each player throws three darts per turn.

Frank		Tara	
Turn Score	Total Score	Turn Score	Total Score
	501		501
100	401	85	416
95	306	85	331
140	166	140	191
130	36	91	100

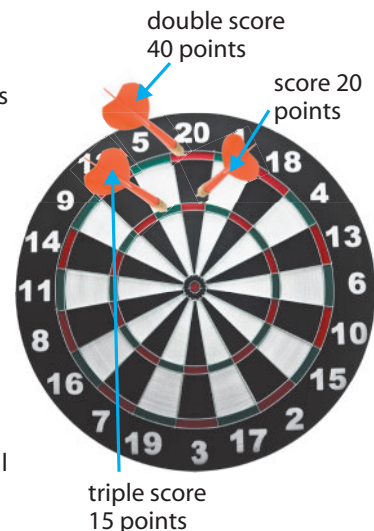
dart in this ring scores triple points

dart in this ring scores double points



dart in inner bull scores 50 points (the inner bull also counts as a double)

dart in outer bull scores 25 points



**Total score for turn: 75 points**

What strategies for plays would give Frank a winning turn?

#### Frank's Solution

$$2(18) = 36$$

I could win with a single dart in double 18.

$$18 + 2(9) = 36$$

If I hit 18 instead of double 18, then I could use my second dart to try for double 9.

$$18 + 9 = 27$$

9 would be left.

If I hit 9 instead of double 9 with my second dart, then I couldn't win this turn. That's because I can't score 9 with a double.

#### Your Turn

- Describe two other ways that Frank could win the game on his turn.
- If Frank does not win on his turn, describe a strategy that Tara could use to win on her next turn.

**EXAMPLE 2****Using deductive and inductive reasoning to determine a winning strategy**

Nadine and Alice are playing a toothpick game. They place a pile of 20 toothpicks on a desk and alternate turns. On each turn, the player can take one or two toothpicks from the pile. The player to remove the last toothpick is the winner. Nadine and Alice flip a coin to determine the starting player.



Is there a strategy Alice can use to ensure that she wins the game?

**Alice's Solution**

I need to make sure that there are one or two toothpicks left after Nadine's last turn.

I will win the game if I can take the last toothpick. If I work backward, I might see a pattern I can use to win.

To make sure this happens, I have to leave three toothpicks on the desk for Nadine.

If I leave three toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, I can take the two that are left and win. If she takes two, I can take the last one and win.

To make sure this happens, I have to leave six toothpicks on the desk for Nadine.

If I leave six toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, I can take two, which would leave three. If she takes two, I can take one and leave her with three.

To make sure this happens, I have to leave nine toothpicks on the desk for Nadine.

If I leave nine toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, then I can take two, which would leave six. If she takes two, I can take one and leave her with six.

I can see that I need to leave 12, 15, and 18 toothpicks for Nadine.

There is a pattern to the number of toothpicks I must leave for Nadine: 3, 6, 9, 12, 15, 18.

I will win if I go first and take two toothpicks. Each turn after that, I need to pick one or two so that I leave Nadine with a number of toothpicks that is a multiple of 3.

If Nadine goes first and knows this strategy, I can't win. If she goes first and doesn't know this strategy, however, I can win by arranging to leave her a number of toothpicks that is a multiple of 3.

**Your Turn**

- Which part of Alice's strategy involved deductive reasoning? Explain.
- Which part of Alice's strategy involved inductive reasoning? Explain.

## In Summary

### Key Idea

- Both inductive reasoning and deductive reasoning are useful for determining a strategy to solve a puzzle or win a game.

### Need to Know

- Inductive reasoning is useful when analyzing games and puzzles that require recognizing patterns or creating a particular order.
- Deductive reasoning is useful when analyzing games and puzzles that require inquiry and discovery to complete.

## CHECK Your Understanding

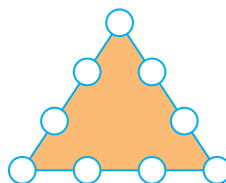
1. In the leapfrog puzzle, what would be the minimum number of moves needed to exchange 10 red counters with 10 blue counters? Explain how you know.
2. Frank and Tara are playing another game of darts. Tara's game score is 66. List three different strategies she could use to win on her turn.
3. In the toothpick game, suppose that players are allowed to take one, two, or three toothpicks. Determine a strategy you could use to ensure that you win if you do not have the first turn.

## PRACTISING

4. Rearrange three golf balls so that the arrowhead points down instead of up.



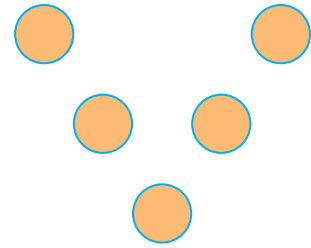
5. a) Draw a diagram like the one to the right. Place the numbers 1 through 9 in the circles so that the sum of the numbers on each side of the triangle is 17.  
b) Explain the strategy you used.



6. Examine this square. It has a magic sum.
  - a) Describe the patterns you see.
  - b) Selva noticed that when he added three numbers that were not in the same row or column, the sum was 36 (the magic sum). This number is 3 times the number in the middle square. Create your own magic square using the patterns you identified. Do Selva's observations hold in your square?
  - c) Prove algebraically that Selva's observations hold in any square that is created using these patterns.

5	9	13
8	12	16
11	15	19

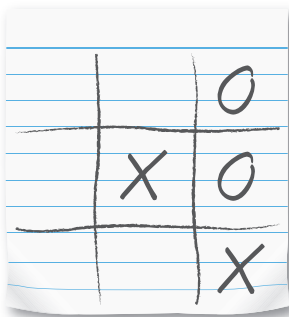
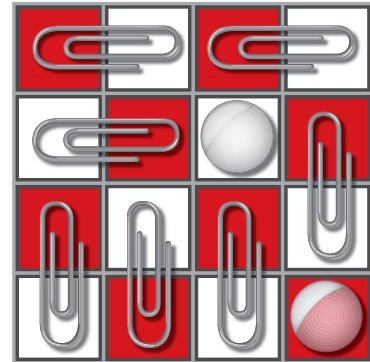
7. Place the numbers 1 to 5 in a V shape, as shown, so that the two arms of the V have the same total.



- How many different solutions are there?
- What do you notice about all the solutions you found?
- How could you convince someone that you have identified all the possible solutions?

8. Draw a 4-by-4 grid that is large enough to place a coin in any square. Your opponent in this game has seven paper clips. Each paper clip is large enough to cover two squares when placed horizontally or vertically. You need to place a coin on each of any two squares so that your opponent is unable to cover the remaining squares with the seven paper clips. Determine a strategy to ensure that you will always win.

The player with the paper clips wins.



- Who started this game of tic-tac-toe: player X or player O? Explain. Assume that both players are experienced at playing tic-tac-toe.
- Sudoku requires both inductive and deductive reasoning skills. The numbers that are used to complete a Sudoku puzzle relate to the size of the grid. For a 6-by-6 grid, the numbers 1 to 6 are used. For a 9-by-9 grid, the numbers 1 to 9 are used. The grid must be filled so that each column, row, or block contains all the numbers. No number can be repeated within any column, row, or block. Solve each of the Sudoku puzzles below.

a)

5			2	6	
		4			
1					6
			1	5	
	3			2	1
			6		

b)

6		4	8	2				
					1	4		
				6		3	5	
				1			4	
8	9	2						1
	1	3	2	9				
4			5					6
5						9	3	

11. Fill in the missing numbers, from 1 to 9, so that the sum of the numbers in each row, column, and diagonal is 15.

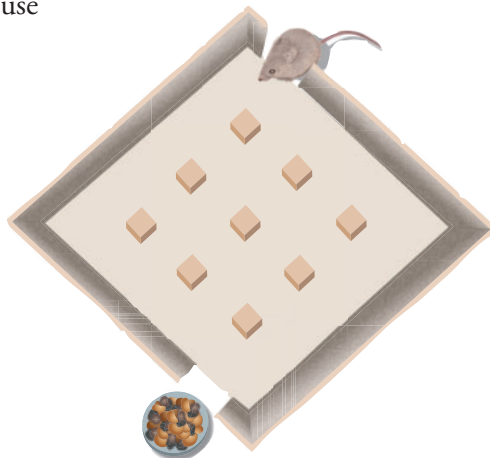
a)

		6
		1
4	3	8

b)

		4
	5	3
		8

12. How many ways can the mouse navigate the maze to reach the trail mix, if the mouse can only travel down?



13. KenKen, like Sudoku, requires both inductive and deductive reasoning skills. Solve this  $6 \times 6$  KenKen puzzle using only the numbers 1 to 6. Do not repeat a number in any row or column. The darkly outlined sets of squares are cages. The numbers in each cage must combine in any order to produce the target number, using the operation shown. For example, the target in the top left cage is  $30 \times$ , which means 30 by multiplication. The two numbers in the cage must be 5 and 6, because no other combination of two factors (from 1 to 6) gives a product of 30. A number may be repeated in a cage as long as it is not in the same row or column.

$30 \times$		$36 \times$	$2 \div$		$18 +$
$3 +$			$7 +$		
	$20 \times$		$5 -$		
$1 -$	$2 -$			$13 +$	
	$7 +$		$2 -$		
$2 \div$			$3 -$		

## Closing

14. Explain how inductive and deductive reasoning can help you develop a strategy to play a game or to solve a puzzle.

## Extending

15. a) Suppose that the goal for tic-tac-toe is changed, so that you have to force your opponent to place three markers in a row, column, or diagonal in order to win. How would your strategy change?  
 b) What role does inductive and deductive reasoning play in helping you develop your new strategy?