1.6 Reasoning to Solve Problems

GOAL

Solve problems using inductive or deductive reasoning.

YOU WILL NEED

- calculator

INVESTIGATE the Math

Emma was given this math trick:

- Choose a number.
- Multiply by 6.
- Add 4.
- Divide by 2.
- Subtract 2.

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true. Here is her response to the problem:

Inductive reasoning:

#	×6	+4	÷2	−2
5	30	34	17	15
−3	−18	−14	−7	−9
0	0	4	2	0
24	144	148	74	72

I followed the steps to work through four examples.

Conjecture: It is 3 times.

Deductive reasoning:

I chose d.

Then I multiplied, added, divided, and subtracted to get an expression.

$\left(\frac{6d + 4}{2}\right) - 2$

It worked.

It simplified to $3d$.

EXPLORE...

- Suppose that you are lost in the woods for hours and come upon a cabin. In the cabin, you find a lantern, a candle, a wood stove with wood in it, and a match. What do you light first?

? How can Emma's communication about her reasoning be improved?

A. With a partner, explain why Emma might have chosen the values she did.

B. What details are missing from the deductive reasoning Emma used to arrive at the expression $3d$?

C. Improve Emma's conjecture, justifications, and explanations.

Reflecting

D. How does it help to understand the mathematics when both symbols and words are used in an explanation?

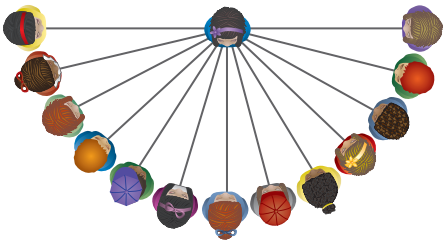
E. Why is it important to explain your reasoning clearly?

APPLY the Math

EXAMPLE 1 Using reasoning to solve a problem

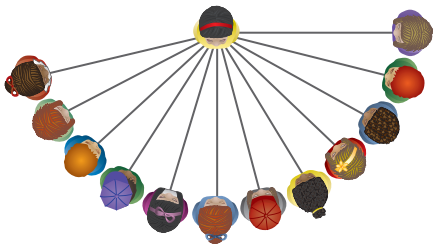
The members of a recently selected varsity basketball team met each other at their first team meeting. Each person shook the hand of every other person. The team had 12 players and 2 coaches. How many handshakes were exchanged?

Kim's Solution



I decided to think about how many times each person shook hands. There were 14 people in total, so person 1 shook hands with each of the other 13 people.

13 handshakes



Person 2 had already shaken hands with person 1. Person 2 shook hands with each of the remaining 12 people.

13 + 12 handshakes

13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 91 handshakes

This pattern of handshakes continued until there were two people left when the last handshake happened.

Your Turn

Discuss, with a partner, whether Kim used inductive or deductive thinking in her solution. How do you know?

EXAMPLE 2 Using reasoning to solve a problem

Sue signed up for games at her school's fun night. Seven other people were assigned to her group, making up four pairs of partners. The other members of her group were Dave, Angie, Josh, Tanya, Joy, Stu, and Linus. When the games started, Dave and his partner were to the left of Stu. Across from Dave was Sue, who was to the right of Josh. Dave's brother's partner, Tanya, was across from Stu. Joy was not on Stu's right.

Name the four pairs of partners.

Vicky's Solution

Dave
Angie
Josh
Tanya
Joy
Stu
Linus
Sue

I drew a rectangle to represent a table. I made a list of the students' names so I could cross them off as I put them in place.

~~Dave~~
Angie
Josh
Tanya
Joy
~~Stu~~
Linus
Sue

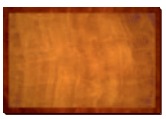
Dave

Stu

The first names I wrote in were Dave and Stu, since they were the first two mentioned. It didn't matter where I started, as long as I kept the relationships of left, right, and across the table. I crossed Dave and Stu off my list.

~~Dave~~
Angie
~~Josh~~
Tanya
Joy
~~Stu~~
Linus
~~Sue~~

Dave



Sue

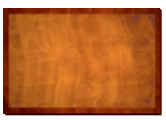
Stu
Josh

I knew that Sue was across from Dave and to the right of Josh. I crossed Sue and Josh off my list.

Linus
Tanya

~~Dave~~
Angie
~~Josh~~
~~Tanya~~
Joy
~~Stu~~
~~Linus~~
~~Sue~~

Dave



Sue

Stu
Josh

The next clue mentioned that Dave's brother and his partner Tanya were across from Stu. The only male name left was Linus, so Linus and Tanya were partners. I crossed their names off my list.

~~Dave~~
~~Angie~~
~~Josh~~
~~Tanya~~
~~Joy~~
~~Stu~~
~~Linus~~
~~Sue~~

Linus
Tanya

Dave
Joy

Sue
Angie

Stu
Josh

If Joy was not on Stu's right, then she must have been on his left. Therefore, she must have been Dave's partner. So, the last person to match was Angie with Sue.

The four pairs of partners were Linus and Tanya, Dave and Joy, Sue and Angie, and Stu and Josh.

The partners sat together, on the same side of the table.

Your Turn

Discuss with a partner whether inductive or deductive reasoning was used for this solution. How do you know?

In Summary

Key Idea

- Inductive and deductive reasoning are useful in problem solving.

Need to Know

- Inductive reasoning involves solving a simpler problem, observing patterns, and drawing a logical conclusion from your observations to solve the original problem.
- Deductive reasoning involves using known facts or assumptions to develop an argument, which is then used to draw a logical conclusion and solve the problem.

CHECK Your Understanding

1. Explain which type of reasoning is demonstrated by each statement.

a) Over the past 12 years, a tree has produced plums every other year. Last year, the tree did not produce plums. Therefore, the tree will produce plums this year.

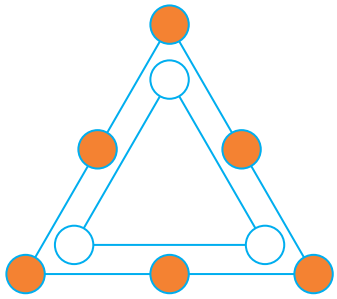
b) Mammals have hair. Dogs are mammals. Therefore, dogs have hair.

c) Every Thursday, a train arrives at 2:30 p.m. Today is Thursday, so the train will arrive at 2:30 p.m.

d) Every even number has a factor of 2. 24 is an even number. Therefore, 24 has a factor of 2.

e) For the pattern 3, 12, 21, 30, 39, the next term is 48.

2. Copy this diagram. Place the digits 1 through 9 in the circles so that the sum of the numbers on the outside triangle is double the sum of the numbers on the inside triangle. Explain whether more than one solution is possible.



PRACTISING

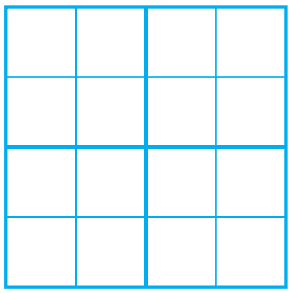
3. Draw the next figure in this sequence.

Figure 1 Figure 2 Figure 3

4. **a)** Substitute numbers for the letters to create an addition problem with a correct answer.
b) How many solutions are possible?

$$\begin{array}{r} y \\ xxx \\ xxx \\ xxx \\ +\ xxx \\ \hline yxxx \end{array}$$

5. **a)** Choose four different colours. Fill in the cells in a copy of this chart, so that each row and column has four different colours and each quadrant also has four different colours.
b) Compare your strategy with a classmate's strategy. How are your strategies the same? How are they different?



6. A farmer wants to get a goat, a wolf, and a bale of hay to the other side of a river. His boat is not very big, so it can only carry him and one other thing. If the farmer leaves the goat alone with the bale of hay, the goat will eat the hay. If he leaves the wolf alone with the goat, the wolf will eat the goat. When the farmer is present, the goat and the hay are safe from being eaten. How does the farmer manage to get everything safely to the other side of the river?

7. Determine the unknown term in this pattern: 17, 22, ___, 35, 43. Explain your reasoning.

8. Suppose that you are marooned on an island where there are only liars and truth-tellers. Liars always tell lies, and truth-tellers always tell the truth. You meet two siblings. The brother says, "My sister told me that she is a liar." Is he a liar or a truth-teller? Explain how you know.

Goats have the reputation that they will eat almost anything. In fact, they will taste just about anything, but they are picky about what they eat. They do eat hay.

9. Bob, Kurt, and Morty are football players. One is a quarterback, one is a receiver, and one is a kicker. The kicker, who is the shortest of the three, is not married. Bob, who is Kurt's father-in-law, is taller than the receiver. Who plays which position?

10. A set of 10 cards, each showing one of the digits from 0 to 9, is divided between five envelopes so that there are two cards in each envelope. The sum of the cards inside each envelope is written on the envelope:

A sum of 8 could be made by these pairs of cards: (8, 0), (7, 1), (6, 2), and (5, 3).

a) Explain which of these pairs of cards cannot possibly be in the envelope marked 8.

b) Describe the reasoning you used to solve this problem.

11. Solve the multiplication problem below. Each letter represents a different digit, and the product is correct.
$abcd \cdot 4 = dcba$

12. At lunchtime, a soccer team meets in the school cafeteria to help organize a tournament. There are 18 players and 2 coaches at the meeting. The tables in the cafeteria are rectangular. Two people can sit on each of the long sides, and one person can sit at each end.

a) What arrangement of tables would enable the team members to sit as close to each other as possible, so that everyone can be heard?

b) Compare your solution with other students' solutions. As a group, decide which is the best solution for the team.

Competitors in the Eco-Challenge race 500 km through the mountains of British Columbia.

13. Early in a bicycle race, Tamara led Kateri by 3 km, while Justine was behind Shreya by 2 km. Shreya was ahead of Kateri by 1 km. By the halfway point, Tamara and Shreya had exchanged places, but they were still the same distance apart. Justine had pulled even with Tamara. Over the last part of the race, Justine dropped 1 km behind Tamara, and Kateri passed Shreya; there were no other changes of position. Who finished third?

14. Use inductive reasoning to determine the number of diagonals that can be drawn in a decagon (a polygon with 10 sides).

15. Max, Karl, Terri, and Suganthy live on the first floor of an apartment building. One is a manager, one is a computer programmer, one is a singer, and one is a teacher.

a) Use the statements below to determine which person is the manager.

- Suganthy and Terri eat lunch with the singer.
- Karl and Max carpool with the manager.
- Terri watches football on television with the manager and the singer.

b) Describe the reasoning you used to solve this problem.

16. There are six pails in a row. The first three pails are filled with water. How can you move only one pail to make the following pattern: full pail, empty pail, full pail, empty pail, full pail, empty pail?

Closing

17. How do you recognize a problem that can be solved using inductive reasoning? How do you recognize a problem that can be solved using deductive reasoning? Is it always possible to tell which kind of reasoning is needed to solve a problem? Explain.

Extending

18. During Sid's vacation, it rained on five days. However, when it rained in the morning, the afternoon was sunny, and every rainy afternoon was preceded by a sunny morning. There were six sunny mornings and nine sunny afternoons. How long was Sid's vacation?

19. Two girls, Arlene and Cathy, and two boys, Leander and Dean, are athletes. One is a long distance runner, one is a softball player, one is a hockey player, and one is a golfer. At lunchtime, they sit around a square table, usually in the same places.

- The runner sits on Arlene's left.
- The hockey player sits across from Leander.
- Cathy and Dean sit next to each other.
- A girl sits on the softball player's left.

Who is the golfer?

20. The labels have been placed on the wrong boxes. You may select one fruit from one box, but you may not look in the box. Based on the fruit you have selected, how can you immediately label all the boxes correctly?