

# Making Conjectures: Inductive Reasoning

## YOU WILL NEED

- calculator
- compass, protractor, and ruler, or dynamic geometry software

## EXPLORE...

- If the first three colours in a sequence are red, orange, and yellow, what colours might be found in the rest of the sequence? Explain.

## conjecture

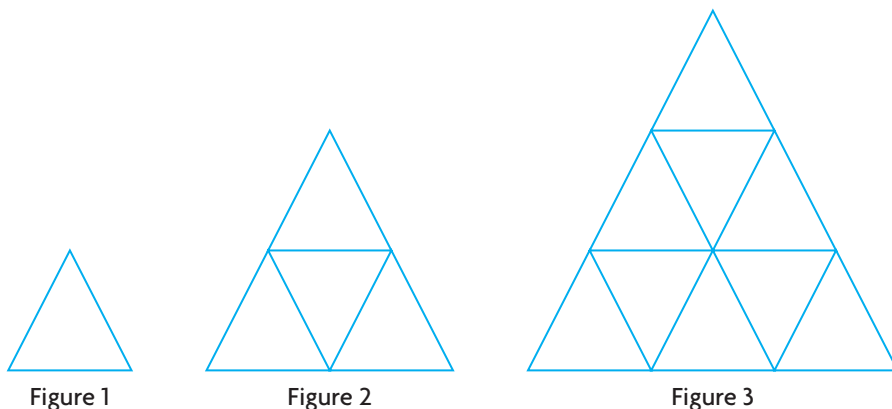
A testable expression that is based on available evidence but is not yet proved.

## GOAL

Use reasoning to make predictions.

## INVESTIGATE the Math

Georgia, a fabric artist, has been patterning with equilateral triangles. Consider Georgia's **conjecture** about the following pattern.



I think Figure 10 in this pattern will have 100 triangles, and all these triangles will be congruent to the triangle in Figure 1.

## ? How did Georgia arrive at this conjecture?

- A. Organize the information about the pattern in a table like the one below.

Figure	1	2	3
Number of Triangles	1	4	

- B. With a partner, discuss what you notice about the data in the table.
- C. Extend the pattern for two more figures.
- D. What numeric pattern do you see in the table?

## Reflecting

## inductive reasoning

Drawing a general conclusion by observing patterns and identifying properties in specific examples.

- E. Is Georgia's conjecture reasonable? Explain.
- F. How did Georgia use **inductive reasoning** to develop her conjecture?
- G. Is there a different conjecture you could make based upon the pattern you see? Explain.

## APPLY the Math

### EXAMPLE 1

### Using inductive reasoning to make a conjecture about annual precipitation

Lila studied the following five-year chart for total precipitation in Vancouver.

Precipitation in Vancouver (mm)												
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
<b>2003</b>	150.5	27.1	133.7	139.8	49.3	12.8	19.8	4.1	40.2	248.2	167.4	113.2
<b>2004</b>	249.6	45.8	132.8	90.2	68.6	49.6	43.6	28.6	53.6	155.4	136.6	160.8
<b>2005</b>	283.6	57.0	92.4	70.0	42.8	54.4	25.2	4.8	39.4	57.8	350.8	146.0
<b>2006</b>	181.4	116.0	214.8	76.2	37.0	80.0	53.0	8.4	73.6	155.2	116.2	210.6
<b>2007</b>	137.6	68.6	75.2	62.2	43.2	43.0	15.8	75.8	30.6	99.6	177.0	197.2

Environment Canada, National Climate Data and Information Archive

What conjecture could Lila make based on the data?

### Lila's Solution

Jul.	Aug.	Sep.
19.8	4.1	40.2
43.6	28.6	53.6
25.2	4.8	39.4
53.0	8.4	73.6
15.8	75.8	30.6

I looked for patterns in the data. I noticed that the summer months seemed to have less precipitation than the other months. I checked the sum of the precipitation in July, August, and September over the five-year period.

**Totals:** 157.4 121.7 237.4

Jan.	Feb.	Mar.
150.5	27.1	133.7
249.6	45.8	132.8
283.6	57.0	92.4
181.4	116.0	214.8
137.6	68.6	75.2

Then I looked for the months with the greatest precipitation, anticipating that the winter months might have greater precipitation. I checked the sums for January, February, and March.

**Totals:** 1002.7 314.5 648.9

Nov.
167.4
136.6
350.8
116.2
177.0

When I examined the information further, I saw that November had the highest value for precipitation: 350.8 mm. I checked the sum for November.

**Total:** 948.0



My conjecture is that fall and winter have more precipitation than spring and summer.

Apr.	May	Jun.	Jul.	Aug.	Sep.
438.4	240.9	239.8	157.4	121.7	237.4
<b>Total:</b> 1435.6 mm					

Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
716.2	948.0	827.8	1002.7	314.5	648.9
<b>Total:</b> 4458.1 mm					

Since November is in the fall and January, February, and most of March are in the winter, I can make a conjecture about which seasons have the most precipitation.

I checked the totals for the five-year period. I found that spring and summer had a total of 1435.6 mm of precipitation, and fall and winter had a total of 4458.1 mm of precipitation.

The data support my conjecture.

### Your Turn

Make a different conjecture based on patterns in the precipitation chart.

### EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

#### Jay's Solution

$$(+3)(+7) = (+21)$$

Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.

$$(-5)(-3) = (+15)$$

Next, I tried two negative odd integers. The product was again positive and odd.

$$(+3)(-3) = (-9)$$

Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.

My conjecture is that the product of two odd integers is an odd integer.

I noticed that each pair of integers I tried resulted in an odd product.

$$(-211)(-17) = (+3587)$$

I tried other integers to test my conjecture. The product was again odd.

### Your Turn

Do you find Jay's conjecture convincing? Why or why not?

**EXAMPLE 3****Using inductive reasoning to develop a conjecture about perfect squares**

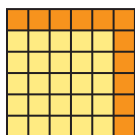
Make a conjecture about the difference between consecutive perfect squares.

**Steffan's Solution: Comparing the squares geometrically**

I represented the difference using unit tiles for each perfect square. First, I made a  $3 \times 3$  square in orange and placed a yellow  $2 \times 2$  square on top. When I subtracted the  $2 \times 2$  square, I had 5 orange unit tiles left.

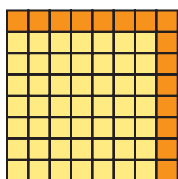


Next, I made  $3 \times 3$  and  $4 \times 4$  squares. When I subtracted the  $3 \times 3$  square, I was left with 7 orange unit tiles. I decided to try greater squares.



My conjecture is that the difference between consecutive squares is always an odd number.

I saw the same pattern in all my examples: an even number of orange unit tiles bordering the yellow square, with one orange unit tile in the top right corner. So, there would always be an odd number of orange unit tiles left, since an even number plus one is always an odd number.



I tested my conjecture with the perfect squares  $7 \times 7$  and  $8 \times 8$ . The difference was an odd number.

The example supports my conjecture.

**Francesca's Solution: Describing the difference numerically**

$$2^2 - 1^2 = 4 - 1$$

$$2^2 - 1^2 = 3$$

I started with the smallest possible perfect square and the next greater perfect square:  $1^2$  and  $2^2$ . The difference was 3.

$$4^2 - 3^2 = 7$$

$$9^2 - 8^2 = 17$$

Then I used the perfect squares  $3^2$  and  $4^2$ . The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

$$12^2 - 11^2 = 23$$

To test my conjecture, I tried the perfect squares  $11^2$  and  $12^2$ . The difference was a prime number.

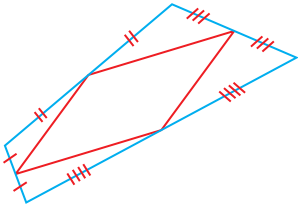
The example supports my conjecture.

**Your Turn**

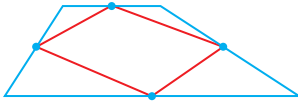
How is it possible to have two different conjectures about the same situation? Explain.

**EXAMPLE 4****Using inductive reasoning to develop a conjecture about quadrilaterals**

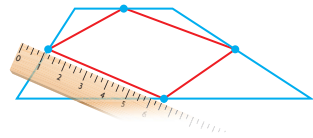
Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

**Marc's Solution: Using a protractor and ruler**

I drew an irregular quadrilateral on tracing paper. I used my ruler to determine the midpoints of each side. I joined the midpoints of adjacent sides to form a new quadrilateral. This quadrilateral looked like a parallelogram.



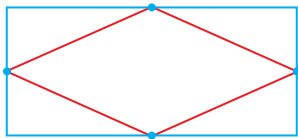
Next, I drew a trapezoid with sides that were four different lengths. I determined the midpoints of the sides. When the midpoints were joined, the new quadrilateral looked like a parallelogram.



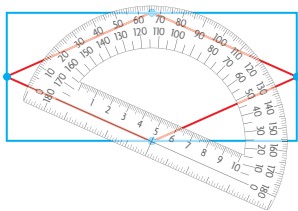
I used my ruler to confirm that the opposite sides were equal.

My conjecture is that joining the adjacent midpoints of any quadrilateral will create a parallelogram.

Each time I joined the midpoints, a parallelogram was formed.



To check my conjecture one more time, I drew a rectangle. I determined its midpoints and joined them. This quadrilateral also looked like a parallelogram.

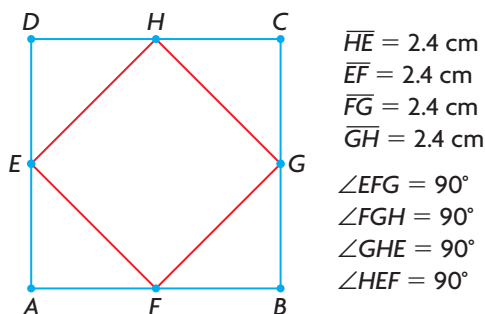


I checked the measures of the angles in the new quadrilateral. The opposite angles were equal. The new quadrilateral was a parallelogram, just like the others were.

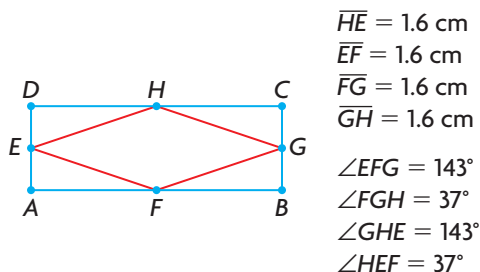
The rectangle example supports my conjecture.



## Tracey's Solution: Using dynamic geometry software



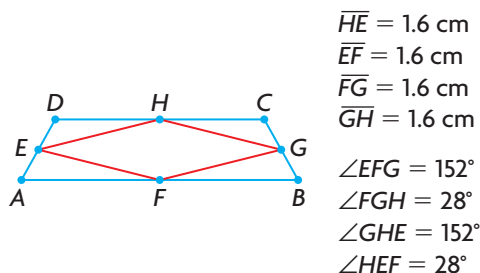
I constructed a square and the midpoints of the sides. Then I joined the adjacent midpoints.  $EFGH$  looked like a square. I checked its side lengths and angle measures to confirm that it was a square.



Next, I constructed a rectangle and joined the adjacent midpoints to create a new quadrilateral,  $EFGH$ . The side lengths and angle measures of  $EFGH$  showed that  $EFGH$  was a rhombus but not a square.

My conjecture is that the quadrilateral formed by joining the adjacent midpoints of any quadrilateral is a rhombus.

Since a square is a rhombus with right angles, both of my examples resulted in a rhombus.



To check my conjecture, I tried an isosceles trapezoid. The new quadrilateral,  $EFGH$ , was a rhombus.

The isosceles trapezoid example supports my conjecture.

### Your Turn

- Why did the students draw different conjectures?
- Do you think that both conjectures are valid? Explain.

## In Summary

### Key Idea

- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

### Need to Know

- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.

## CHECK Your Understanding



Canada's most popular ski destination is Whistler/Blackcomb in British Columbia. This area draws more than 2 million visitors each year.

1. Troy works at a ski shop in Whistler, British Columbia, where three types of downhill skis are available: parabolic, twin tip, and powder. The manager of the store has ordered 100 pairs of each type, in various lengths, for the upcoming ski season. What conjecture did the manager make? Explain.

2. Tomas gathered the following evidence and noticed a pattern.

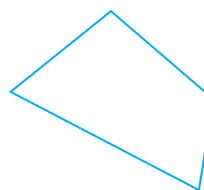
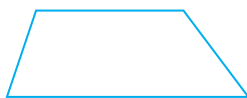
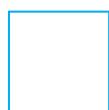
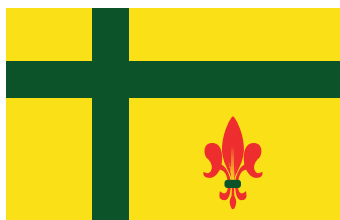
$$17(11) = 187 \quad 23(11) = 253$$

$$41(11) = 451 \quad 62(11) = 682$$

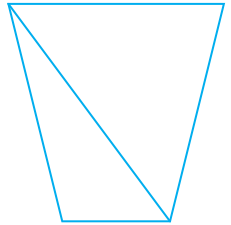
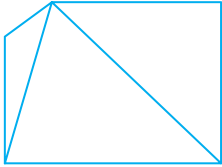

Tomas made this conjecture: When you multiply a two-digit number by 11, the first and last digits of the product are the digits of the original number. Is Tomas's conjecture reasonable? Develop evidence to test his conjecture and determine whether it is reasonable.

## PRACTISING

3. Make a conjecture about the sum of two even integers. Develop evidence to test your conjecture.
4. Make a conjecture about the meaning of the symbols and colours in the Fransaskois flag, at the left. Consider French-Canadian history and the province of Saskatchewan.
5. Marie studied the sum of the angles in quadrilaterals and made a conjecture. What conjecture could she have made?



6. Use the evidence given in the chart below to make a conjecture. Provide more evidence to support your conjecture.

Polygon	quadrilateral	pentagon	hexagon
Fewest Number of Triangles	 2	 3	 4

7. Sonia noticed a pattern when dividing the square of an odd number by 4. Determine the pattern and make a conjecture.
8. Dan noticed a pattern in the digits of the multiples of 3. He created the following table to show the pattern.

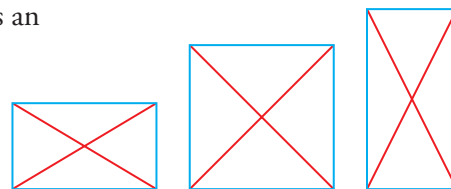
<b>Multiples of 3</b>	12	15	18	21	24	27	30
<b>Sum of the Digits</b>	3	6	9	3	6	9	3

- a) Make a conjecture based on the pattern in the table.
- b) Find a classmate who made a different conjecture. Discuss the reasonableness of both conjectures.
- c) Test one of the conjectures.
9. Make a conjecture about the sum of one odd integer and one even integer. Test your conjecture with at least three examples.
10. Make a conjecture about the temperature on November 1 in Hay River, Northwest Territories, based on the information in the chart below. Summarize the evidence that supports your conjecture.

<b>Year</b>	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
<b>Maximum Temperature (°C)</b>	+3.1	-2.2	-1.1	-10.1	-1.6	-3.9	-3.2	+2.9	+1.8	-3.0

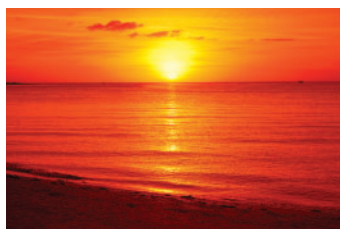
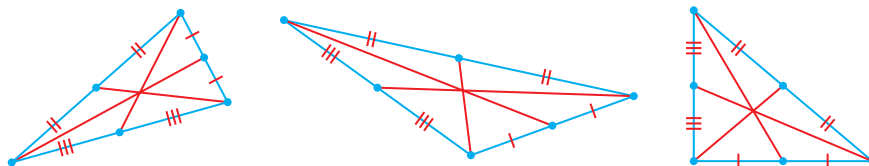
11. Paula claims that whenever you square an odd integer, the result is an odd number. Is her conjecture reasonable? Justify your decision.

12. Ursula studied the diagonals of these rectangles to look for patterns. Make a conjecture about the diagonals of rectangles. What evidence supports your conjecture?



13. Text messages often include cryptic abbreviations, such as L2G (love to go), 2MI (too much information), LOL (laugh out loud), and MTF (more to follow). Make a conjecture about the cryptic abbreviations used in text messages, and provide evidence to support your conjecture.

14. Nick made a conjecture about the medians of a triangle. He used triangles of different sizes and types to gather evidence. The evidence always supported his conjecture. What might his conjecture have been? Provide additional evidence to support the conjecture.



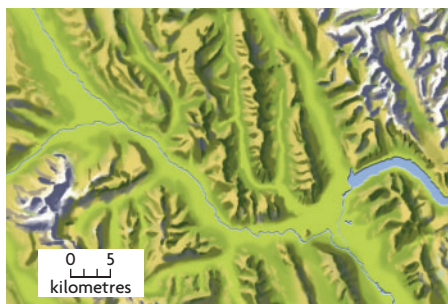
Red sky at sunset may predict calm weather. Based on the photo, how else could you predict that a storm isn't coming?

15. Farmers, travellers, and hunters depend on their observations of weather and storm systems to make quick decisions and to survive in different weather conditions. Weather predictions, passed on through oral tradition or cited in almanacs, are often based on long-term observations. Two predictive statements about weather are given below.

- If cows are lying down, then it is going to rain.
- Red sky at night; sailor's delight.

Find another such predictive statement from oral tradition, an Elder, a family member, an Internet source, or a text. Explain how and why this prediction may have been reached.

16. The map below does not have any roads or urban areas marked. However, there is a town of 7000 people somewhere in this area. Make a conjecture about where the town is. Justify your decision.



17. Suppose that social networking sites were the only method for passing information among people, and that everyone in Canada was a member of one of these sites. Make a conjecture about the amount of time it would take for the entire population of Canada to get information first shared at 8 a.m. Central Standard Time. With a partner, discuss the reasonableness of each of your conjectures and decide how you could gather evidence to support your conjectures.
18. Thérèse held up a piece of notebook paper in one hand and a pair of scissors in the other hand, and made the conjecture that she could walk through the piece of paper. With a partner, explore how Thérèse's conjecture could be possible.

## Closing

19. Lou says that conjectures are like inferences in literature and hypotheses in science. Sasha says that conjectures are related only to reasoning. With a partner, discuss these two opinions. Explain how both may be valid.

## Extending

20. Photographs lead to conjectures about what was happening around the time that the images were captured. With a partner, develop at least three different conjectures about what could have led to the situation in this photograph.
21. In advertising, we often see statements such as “four out of five dentists recommend it.” Discuss this statement with a partner, and decide whether it is a conjecture. Justify your decision.
22. From a news source, retrieve evidence about an athlete’s current performance or a team’s current performance. Study the evidence, and make a conjecture about the athlete’s or team’s performance over the next month. Justify your conjecture, and discuss the complexity of conjectures about sports.



## Math in Action

### Oops! What Happened?

- Identify the pieces of given evidence that are conjectures.
- Make a conjecture about what caused the accident.
- What evidence supports your conjecture?
- If you could ask three questions of the drivers or the witness, what would they be?
- Can the cause of an accident such as this be proved? Why or why not?

