7 Chapter Review

Q: How can I solve a quadratic equation algebraically?

A: Write the equation in standard form:

$$ax^2 + bx + c = 0$$

Then determine the roots of the equation by factoring or by using the quadratic formula.

Study Aid

- See Lessons 7.5, Examples 1, 4, and 5, and 7.7, Examples 1, 3, and 4.
- Try Chapter Review Questions 8 and 13.

Factoring

If the expression $ax^2 + bx + c$ is factorable, then the equation $ax^2 + bx + c = 0$ is true when either of the factors is equal to 0.

For example:

$$2x^2 + 2x = 5x + 20$$
$$2x^2 - 3x - 20 = 0$$
$$(2x + 5)(x - 4) = 0$$

$2x + 5 = 0$ or $x - 4 = 0$

$2x = -5$ $\quad x = 4$

$$x = -\frac{5}{2}$$

The roots are

$$-\frac{5}{2} \text{ and } 4$$

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For example:

$3x^2 - 4x - 5 = 0$

$a = 3$, $b = -4$, and $c = -5$

Substitute these values into the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{76}}{6}$$

The radicand is positive, so the equation has a solution.

$$x = \frac{4 + \sqrt{76}}{6} \text{ or } x = \frac{4 - \sqrt{76}}{6}$$

The roots are

$$\frac{2 + \sqrt{19}}{3} \text{ and } \frac{2 - \sqrt{19}}{3}$$

Study Aid

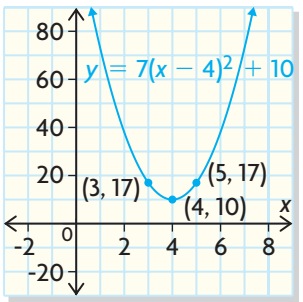
- See Lesson 7.6, Examples 1 and 2.
- Try Chapter Review Question 9.

Q: How can you graph a quadratic function in vertex form, $y = a(x - h)^2 + k$?

A: Use the information provided by the form of the quadratic equation.

For example: Sketch the graph of the following quadratic function:

$$y = 7(x - 4)^2 + 10$$

The vertex is at (4, 10).	Determine the coordinates of the vertex, (h, k).
$y = 7[(5) - 4]^2 + 10$ $y = 7(1)^2 + 10$ $y = 17$ One other point on the graph is (5, 17).	Locate one other point on the function by substituting a value for x into the equation. In this example, substitute 5 for x because the calculation is easy to check.
Another point on the graph is (3, 17), because 3 is the same distance from 4 as 5 is. (Another way of looking at this is that $(x - 4)^2 = 1$ when x is 5 and when x is 3.)	Apply symmetry to the first located point. In this example, the vertical line of symmetry is $x = 4$.
	Connect the three points with a smooth curve.

Study Aid

- See Lesson 7.8, Examples 1 to 4.
- Try Chapter Review Questions 14 to 17.

Q. When using a quadratic model, how do you decide whether you should determine the vertex of the parabola or solve a quadratic equation?

A. If you want to determine a maximum or minimum value, then you should locate the vertex of the function. If you are given a specific value for the dependent variable (any number, including 0), then you should solve the corresponding quadratic equation by graphing, factoring, or using the quadratic formula.

PRACTISING

Lesson 7.1

1. Graph the following quadratic functions without using technology.

a) $f(x) = x^2 - 6x + 8$

b) $g(x) = -2(x + 1)(x - 3)$

c) $h(x) = 0.5(x + 4)^2 - 2$

Lesson 7.2

2. The points $(-2, -41)$ and $(6, -41)$ are on the following quadratic function:

$$f(x) = -3x^2 + 12x - 5$$

Determine the vertex of the function.

3. In the photograph, the fisherman is holding his fishing rod 0.5 m above the water. The fishing rod reaches its maximum height 1.5 m above and 1 m to the left of his hand.

a) Determine the quadratic function that describes the arc of the fishing rod. Assume that the y-axis passes through the fisherman's hand and the x-axis is at water level.

b) State the domain and range for the function that models the fishing rod.

Lesson 7.3

4. Solve by graphing.

a) $6x^2 - 13x + 6 = 0$

b) $-5x^2 - 8x + 3 = 0$

c) $4n^2 + 1 = n + 3$

d) $c^2 - 38c + 340 = 3c^2 - 96c + 740$

Lesson 7.4

5. **a)** Rewrite the following quadratic function in factored form:

$$f(x) = 2x^2 - 12x + 10$$

b) Identify the zeros of the function, and determine the equation of the axis of symmetry of the parabola it defines.

c) State the domain and range of the function.

d) Graph the function.

6. Determine the x-intercepts of the graph of this quadratic function:

$$f(x) = 2x^2 - 5x - 12$$

7. Determine the vertex of the parabola that is defined by each quadratic function. Explain your process.

a) $f(x) = 3x^2 - 6x + 5$

b) $g(x) = -1(x + 2)(x + 3)$

Lesson 7.5

8. Solve by factoring. Verify each solution.

a) $s^2 - 7s - 60 = 0$

b) $2a^2 + 10a + 12 = 0$

c) $16d^2 - 169 = 0$

d) $3x^2 - 2x = 81 - 2x - x^2$

Lesson 7.6

9. **a)** State the direction of opening of the parabola that is defined by the following quadratic function:

$$y = 2(x - 3)^2 - 7$$

b) Provide the equation of the axis of symmetry and the coordinates of the vertex of the parabola.

c) State the domain and range of the function.

d) Sketch the parabola.

10. Determine the quadratic function with zeros of -4 and -2, if the point $(-1, -9)$ is also on the graph of this function.

11. Determine the quadratic function that defines the parabola that has a vertex at $(3, -5)$ and passes through $(-1, -9)$.

12. The High Level Bridge in Edmonton is the source of the Great Divide Waterfall, which is open to the public on holiday weekends in the summer. The water falls a vertical distance of 45 m from the bridge and reaches the North Saskatchewan River 10 m horizontally from the base of the bridge. Determine a quadratic function that models the path of the water.

Lesson 7.7

13. Solve by using the quadratic formula.

a) $117x^2 - 307x + 176 = 0$

b) $f^2 + 2f - 2 = 0$

c) $7h^2 + 6h = 5$

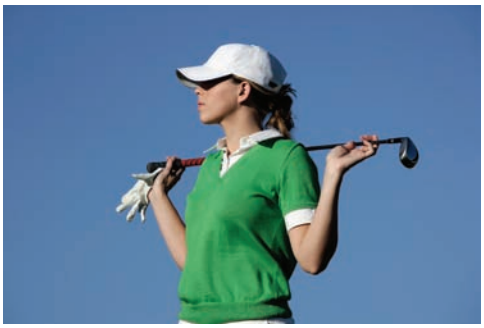
d) $6x^2 + 8x + 4 = 0$

Lesson 7.8

14. Determine three consecutive positive odd integers, if the sum of the squares of the first two integers is 15 less than the square of the third integer.

15. A right triangle has a perimeter of 120 cm. One side of the triangle is 24 cm long. Determine the length of the other side and the length of the hypotenuse.

16. On the 13th hole of a golf course, Saraya hits her tee shot to the right of the fairway. Saraya estimates that she now has 130 yd to reach the front of the green. However, she needs to clear some pine trees that are 40 yd from the green. The trees are about 10 yd high. Determine two different quadratic equations that model the flight of a golf ball over the trees and onto the green. Write one of your functions in factored form and the other in standard form.



17. A fishing boat leaves a dock at noon and travels due west at 40 km/h. A second boat leaves the same dock 20 min later and travels due south at 51 km/h. At what time, to the nearest minute, will the two boats be 116 km apart?