6 Chapter Review

FREQUENTLY ASKED Questions

Study Aid
- See Lesson 6.4, Examples 1 and 2, and Lesson 6.6, Examples 1 and 2.
- Try Chapter Review Questions 7 and 8.

Q: What do each of these parts describe in the algebraic model of an optimization problem: a system of linear inequalities and an objective function?

A: The system of linear inequalities describes the constraints of the problem (each one represented by a linear inequality) and the restrictions on the variables (the set of numbers that the variables belong to and any limits on the values of the variables).

The objective function describes how the quantity that is being optimized relates to the variables.

Study Aid
- See Lesson 6.5 and Lesson 6.6, Examples 1 and 2.
- Try Chapter Review Questions 9, 10, and 11.

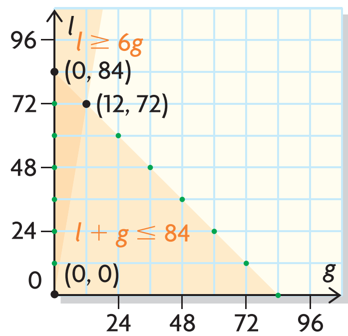
Q: Once you have modelled an optimization problem, how do you solve it?

A: The feasible region of a graph for a system of linear inequalities represents all the valid solutions to the system. The optimal solution is usually represented by a point at a vertex of the feasible region.

For example, consider this problem and its optimization model:

An office supply store sells no more than 84 packages of lined paper and graph paper, in total, in a day. The store also sells at least six times as much lined paper as graph paper. Lined paper sells for $2.75 and graph paper for $4.25. How many packages of each paper does the store need to sell to maximize revenue?

Optimization Model:
Let g represent the number of packages of graph paper.
Let l represent the number of packages of lined paper.
Let R represent the revenue.
Restrictions:
$l \in \mathrm{W}, g \in \mathrm{W}$
Constraints:
$l \geq 0$
$g \geq 0$
$l + g \leq 84$
$l \geq 6g$
Objective function:
$R = 2.75l + 4.25g$



If (g, l) is (0, 0), $R = 2.75(0) + 4.25(0)$ $R = \$0$	If (g, l) is (0, 84), $R = 2.75(84) + 4.25(0)$ $R = \$231$	If (g, l) is (12, 72), $R = 2.75(72) + 4.25(12)$ $R = \$249$

The solution that results in the greatest revenue, $249, is 72 packages of lined paper and 12 packages of graph paper.

PRACTISING

Lesson 6.1

1. Graph each linear inequality. Justify each boundary and the half plane you shaded.
   a) $\{(x, y) \mid 6x + y > 12, x \in \text{I}, y \in \text{I}\}$
   b) $\{(x, y) \mid 10 + 2y \leq 7x, x \in \text{W}, y \in \text{W}\}$
   c) $\{(x, y) \mid -7y \geq 14, x \in \text{R}, y \in \text{R}$
2. Selma and Claudia are working on a project. The project will take, at most, 50 h to complete. They want to know how many hours each will need to work.
   a) Define the variables and their domain and range. Write and graph a linear inequality to represent this problem.
   b) Choose three combinations of numbers of hours that Selma and Claudia could work. How could you verify each combination?

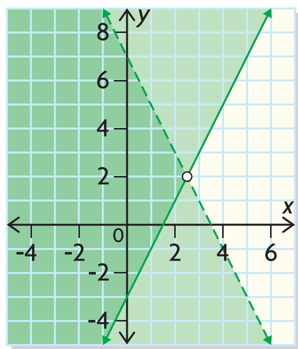
Lesson 6.2

3. Paul is making a mixture of raisins and peanuts.
   - He is making no more than 8 kg of the mixture.
   - He wants at least twice the amount of raisins as peanuts.
   a) Define the variables and their domain and range. Write a system of linear inequalities to represent this situation.
   b) Graph the system. Describe the solution region.
   c) Choose three points in the solution region, and explain what each point represents.

Lesson 6.3

4. a) Graph the solution set for the following system of linear inequalities:
   $2x + 1 > 3y$
   $2y - 5x \leq 10$
   b) How would the graph change if
      i) the domain and range were from the set of integers?
      ii) the domain and range were from the set of whole numbers?
      iii) the inequality signs were reversed?
5. George is replacing the halyards (ropes that lift the sails) and sheets (ropes that control side movement of the sails) on his boat.
   - He wants no more than 50 m of rope for the halyards.
   - He needs no more than 120 m of rope altogether.
   a) What are the restrictions on the variables? How do you know?
   b) Create a graphical model of this situation and use it to choose two possible combinations of lengths of rope.

6. Consider the following graph of a system of linear inequalities.



   a) Determine the linear equation that represents the boundary of each linear inequality. Explain how you determined each equation.
   b) Represent the system of linear inequalities algebraically.
   c) Verify that your system in part b) matches the graph.
   d) What are the restrictions on the variables? How do you know?

Lesson 6.4

7. A pet store specializes in birds.

- It sells at least three times more male birds than female birds of the same species. The males' colourful feathers make them more popular.
- Over the past two weeks, no more than 28 birds, in total, have been sold.
- Males were sold for \$115, and females were sold for \$90.

What combinations of sales of male and female birds would have maximized the pet store's revenue? Create a model of this problem.

8. A zoo has categorized its exhibits as herbivores and carnivores.

- There are no more than 50 exhibits altogether.
- No more than 50% of the 50 possible exhibits are herbivores, and no less than 30% are carnivores.
- A ticket to any herbivore exhibit costs \$15, and a ticket to any carnivore exhibit costs \$18.

What combinations of herbivore and carnivore exhibits would maximize the zoo's revenue? Create a model of this problem.

Lesson 6.5

9. The following model represents an optimization problem. What point in the feasible region would result in the minimum value for the objective function? What point would result in the maximum value? Explain how you know.

Optimization Model

Restrictions:

$x \in I, g \in I$

Constraints:

$x \geq 0$

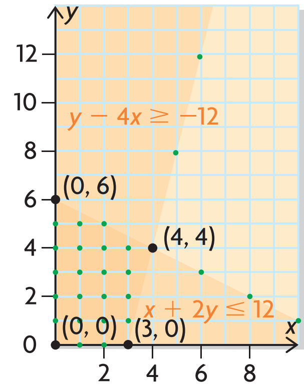
$y \geq 0$

$y - 4x \geq -12$

$x + 2y \leq 12$

Objective function:

$B = -2x + y$



Lesson 6.6

10. The following model represents an optimization problem. Determine the maximum solution.

Optimization Model

Restrictions:

$x \in R, y \in R$

Constraints:

$x \geq 0$

$y \geq 0$

$3y \geq 5x - 15$

$3y + x \leq 3$

Objective function:

$M = 1.5x + 3.1y$

11. The stylists in a hair salon cut hair for women and men.

- The salon books at least four women's appointments for every man's appointment.
- Usually there are 90 or more appointments, in total, during a week.
- The salon is trying to reduce the number of hours the stylists work.
- A woman's cut takes about 75 min, and a man's cut takes about 30 min.

What combination of women's and men's appointments would minimize the number of hours the stylists work? How many hours would this be?