Name: KEY **TA:**______

Math 11 Pre-Calculus LG 2 Ver B **/20**

1. Write the first 4 terms of the geometric sequence where $t_1 = 5$ and $r = -3$. *(2 marks)*

$t_1 = 5$
$t_2 = 5(-3) = -15$
$t_3 = (-15)(-3) = 45$
$t_4 = (45)(-3) = -135$

$\boxed{5, -15, 45, -135}$

2. If the geometric sequence has $t_1 = -4$ and $r = -3$, determine t_5. *(1 mark)*

$t_n = t_1 \cdot r^{n-1}$
$t_5 = (-4)(-3)^{5-1}$
$= (-4)(-3)^4$
$= (-4)(81) = \boxed{-324}$

3. The number of insects in a population increases 25% each day. If there are 1500 insects now, how many days will it take before there are at least 50 000? *(2 marks)*

$r = 1.25$
$t_1 = 1500$
$t_n = 50000$

$t_n = t_1 \cdot r^{n-1}$
$\frac{50000}{1500} = \frac{1500}{1500} \cdot 1.25^{n-1}$
$33.\overline{3} = 1.25^{n-1}$

By inspection $1.25^{15} = 28.42$ $(n=16)$
$1.25^{16} = 35.53$ $(n=17)$

If there are 1500 insects on the first day, on the 17th day there'll be over 50000 (ie. 16 days from now)

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4. The population of Cherryville was 1 500 in 2006. In 2015, the population was 4 200. Determine the value of the growth rate (as an annual percentage, to two decimal places) from 2006 to 2015. *(2 marks)*

2006 → 2015 ⟹ 10 terms

$t_1 = 1500$

$t_{10} = 4200$

$t_n = t_1 \cdot r^{n-1}$

$\frac{4200}{1500} = \frac{1500}{1500} \cdot r^{10-1}$

$2.8 = r^9$

$r = \sqrt[9]{2.8} \doteq 1.1212$

$\boxed{\text{Growth rate is } 12.12\%}$

5. Determine the sum of the first 12 terms of the geometric series: 30 – 15 + 7.5 + … *(2 marks)*

$n = 12$

$t_1 = 30$

$r = \frac{-1}{2}$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$= \frac{30\left(\left(-\frac{1}{2}\right)^{12} - 1\right)}{-\frac{1}{2} - 1}$

$= \frac{30\left(\frac{1}{4096} - 1\right)}{-1.5} \doteq 19.995 \quad \boxed{20}$

6. Find the sum of the following geometric series: 243 - 81 + 27 + … $-\frac{1}{9}$. *(2 marks)*

$t_1 = 243 = 3^5$

$r = \frac{-1}{3}$

$t_n = \frac{-1}{9} = 3^{-2}$

$S_n = \frac{rt_n - t_1}{r - 1}$

$= \frac{-\frac{1}{3}\left(\frac{-1}{9}\right) - 243}{-\frac{1}{3} - 1}$

$= \frac{\frac{1}{27} - 243}{-1.\overline{3}} = \boxed{182.\overline{2}}$

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7. A website experienced 40 unique visitors on its first day. Each day, the number of unique visitors increased by 25%. How many total unique visitors would have visited the site after 10 days? *(2 marks)*

$t_1 = 40$
$r = 1.25$
$n = 10$

$$S_n = \frac{t_1(r^{10}-1)}{r-1}$$
$$= \frac{40(1.25^{10}-1)}{1.25-1}$$
$$= 1330.1$$

$\boxed{1330 \text{ visitors}}$

8. Determine the sum of each infinite geometric series, if it exists. *(1 mark each)*

a) $t_1 = 8$ and $r = \frac{1}{4}$

$$S_\infty = \frac{t_1}{1-r} = \frac{8}{1-\frac{1}{4}} = \frac{8}{\frac{3}{4}} = \boxed{10.\overline{6}}$$

b) 12 – 18 + 27 + ……

$r = \frac{-18}{12} < -1$ $\boxed{\text{DIVERGES}}$

9. The first term of an infinite geometric series is 12 and the sum is 60. Determine the common ratio. *(2 marks)*

$t_1 = 12$
$S_\infty = 60$

$$S_\infty = \frac{t_1}{1-r}$$
$$60 = \frac{12}{1-r}$$
$$1-r = \frac{12}{60} = \frac{1}{5}$$
$$1-r = \frac{1}{5}$$
$$\boxed{r = \frac{4}{5}}$$

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10. An oil well produces 100 000 barrels of oil per year. If production drops by 8% per year, estimate the total production before the well runs dry. *(3 marks)*

$t_1 = 100\,000$

$r = 1 - 0.08 = 0.92$

$$S_\infty = \frac{t_1}{1-r}$$

$$= \frac{100\,000}{1-0.92} = \frac{100\,000}{0.08}$$

$$= \boxed{1\,250\,000 \text{ barrels}}$$

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