Name: KEY **TA:**______

Math 11 Pre-Calculus LG 2 Ver A **/20**

1. Write the first 4 terms of the geometric sequence where $t_1 = 4$ and $r = -3$. *(2 marks)*

$t_1 = 4$

$t_2 = 4(-3) = -12$

$t_3 = (-12)(-3) = 36$

$t_4 = (36)(-3) = -108$

$\boxed{4, -12, 36, -108}$

2. If the geometric sequence has $t_1 = 3$ and $r = 2$, determine t_5. *(1 mark)*

$t_n = t_1 \cdot r^{n-1}$

$t_5 = 3 \cdot (2)^{5-1}$

$= 3 \cdot 2^4$

$= 3 \cdot 16 = \boxed{48}$

3. A battery loses 6% of its charge each day. If the battery is 100% charged at the end of the first day, on which day does the battery's charge drop below 25%? *(2 marks)*

$r = 1 - 0.06 = 0.94$

$t_1 = 100$ (%)

$t_n = 25$ (%)

$t_n = t_1 \cdot r^{n-1}$

$\frac{25}{100} = \frac{100}{100} \cdot (0.94)^{n-1}$

$0.25 = 0.94^{n-1}$

By inspection,

$0.256 = 0.94^{22}$ $(n = 23)$

$0.241 = 0.94^{23}$ $(n = 24)$

$\boxed{\text{The charge drops below 25\% on day 24}}$

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4. The population of Murrayville was 2 000 in 2003. In 2015, the population was 3 400. Determine the value of the growth rate (as an annual percentage, to two decimal places) from 2003 to 2015. *(2 marks)*

2003 → 2015 ⟹ 13 terms

$t_1 = 2000$

$t_{13} = 3400$

$n = 13$

$t_n = t_1 \cdot r^{n-1}$

$\frac{3400}{2000} = \frac{\cancel{2000}}{\cancel{2000}} \cdot r^{13-1}$

$1.7 = r^{12}$

$r = \sqrt[12]{1.7} \doteq 1.04521$

$\boxed{\text{Growth rate is } 4.52\%}$

5. Determine the sum of the first 15 terms of the geometric series: 12 – 6 + 3 + ... *(2 marks)*

$t_1 = 12$

$n = 15$

$r = -0.5$

$S_n = \frac{t_1(r^n - 1)}{r-1} = \frac{12((-0.5)^{15} - 1)}{-0.5-1}$

$= \frac{12\left(\frac{-1}{32768} - 1\right)}{-1.5} \doteq \frac{-12}{-1.5}$

$\boxed{\doteq 8.0}$

6. Find the sum of the following geometric series: 32 + 16 + 8 + ... + $\frac{1}{16}$. *(2 marks)*

$t_1 = 32$

$t_n = \frac{1}{16}$

$r = \frac{1}{2}$

$t_n = t_1 \cdot r^{n-1}$

$\frac{1}{16} = 32 \cdot \left(\frac{1}{2}\right)^{n-1}$

$2^{-4} = 2^5 \cdot (2^{-1})^{n-1}$

$2^{-4} = 2^{5 + -1(n-1)}$

$\Downarrow$

$-4 = 5 - n + 1$

$-4 = 6 - n$

$n = 10$

not necessary.

$S_n = \frac{r \cdot t_n - t_1}{r-1}$

$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{16}\right) - 32}{\frac{1}{2} - 1}$

$= \frac{\frac{1}{32} - 32}{\frac{-1}{2}} \doteq \boxed{63.94}$

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7. A website experienced 500 unique visitors on its first day. Each day, the number of unique visitors increased by 20%. How many total unique visitors would have visited the site by the end of the week (7 days)? *(2 marks)*

$t_1 = 500$

$r = 1.2$

$n = 7$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$= \frac{500(1.2^7 - 1)}{1.2 - 1}$$

$$= 6457.95$$

$\boxed{\text{6457 or 6458 visitors}}$

8. Determine the sum of each infinite geometric series, if it exists. *(1 mark each)*

a) $t_1 = 3$ and $r = \frac{4}{3}$ $\quad r = \frac{4}{3} \quad r > 1$ so DIVERGES

b) $24 - 12 + 6 + \ldots\ldots$ $\quad r = \frac{-1}{2}$

$$S_\infty = \frac{t_1}{1-r} = \frac{24}{1-\left(\frac{-1}{2}\right)} = \frac{24}{1.5}$$

$$= \boxed{16}$$

9. The first term of an infinite geometric series is -15 and the sum is -20. Determine the common ratio. *(2 marks)*

$t_1 = -15$

$S_\infty = -20$

$$S_\infty = \frac{t_1}{1-r}$$

$$-20 = \frac{-15}{1-r}$$

$$1 - r = \frac{-15}{-20}$$

$$1 - r = 0.75$$

$$\boxed{r = 0.25}$$

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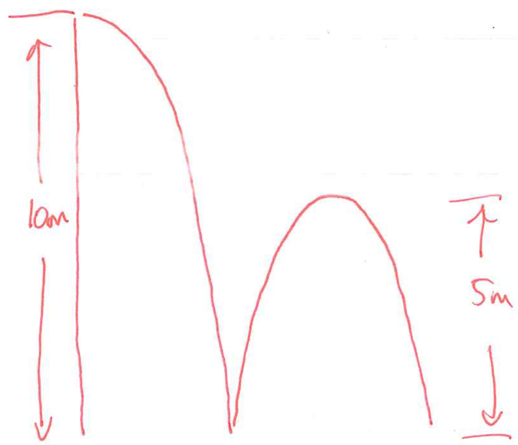
10. A ball is dropped from a height of 10m. The ball rebounds to one half of its previous height each time it bounces. If the ball keeps bouncing, what is the total vertical distance the ball travels? *(3 marks)*



Downwards : $10 + 5 + \cdots$ $\quad S_\infty = \frac{t_1}{1-r} = \frac{10}{1-\frac{1}{2}} = \frac{10}{0.5} = 20\,m$

Upwards : $5 + 2.5 + \cdots$ $\quad S_\infty = \frac{t_1}{1-r} = \frac{5}{1-\frac{1}{2}} = \frac{5}{0.5} = 10m$

TOTAL VERTICAL $= 10m + 20m$

$= \boxed{30m}$

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