Chapter 7

Trigonometry of Right Triangles

GOALS

In this final chapter, you will be looking at trigonometric ratios. You will be actively involved in creating tables of values that will help you understand what the different ratios stand for and how to apply them.

Trigonometric ratios are used in many different professions and trades. Carpenters, pipefitters, and even seamstresses use them on a regular basis. In this chapter, you will be applying prior knowledge about triangles and similar figures to

- determine the trigonometric ratios;
- determine lengths of sides of right triangles using the ratios;
- determine the sizes of angles if you know the ratios.

KEY TERMS

- angle of depression
- angle of elevation
- cosine
- hypotenuse
- leg
- Pythagorean theorem
- sine
- tangent

START TO PLAN

PROJECT OVERVIEW

There are many different structures in a community that enhance the visual appeal of its surroundings. One type of structure is a staircase, a necessary feature in architecture that adds beauty to an entranceway. For this chapter project, you will design a staircase to connect the main floor of a house to the second floor. The distance between the two floors is 9 feet 10 inches and the maximum floor area available for the staircase is 11 feet by 11 feet. You can design a straight staircase, one with a turn or landing in it, or a spiral staircase. You will need to consider the height, depth, and width of each stair and design your staircase so that it fits into the designated area.

This set of stairs is in an ice castle built for Yellowknife's annual Snowking Winter Festival.

GET STARTED

Most of you will have one or two staircases in your home. Take a look at them. Usually, each step is the same size, but sometimes there is a wider one, or a landing where the direction changes. Do you have a spiral staircase in your home? How do the steps on it differ from the ones on a straight staircase? Do you have a staircase on the outside of your house leading to a porch or sundeck? How is it different from the staircases inside your house?

To begin your project, look on the internet or talk to your woodworking teacher or a carpenter to learn about staircase design. Make a list of all the things you will need to consider. For example:

- What are the building code regulations for stairs? What are the allowable range of heights and depth of stairs?
- What types of lumber do you want to use?
- What types and thicknesses of finishing wood are available?
- What terms are used in stair construction?

Use your information to determine the number of stairs you will need and if you will have a turn or landing in your staircase. Begin by drawing top-down and side views of your ideas. Consider more than one possibility.

FINAL PRESENTATION CHECKLIST

You will present your final drawings and measurements as a poster or a scale model, along with instructions for a carpenter to build the stairs. The final presentation must include:

- a scale drawing of your staircase;
- accurate measurements of each part of the staircase;
- steps for the work involved, including tools and instruments that will be needed;
- at least one pair of similar figures in your diagram;
- the measurements calculated in imperial units; and
- a discussion as to why you designed your staircase as you did.

7.1 The Pythagorean Theorem

MATH *ON THE JOB*

Jim Jenner is a drywaller from Surrey, British Columbia. His passion is woodworking and he loves creating boxes with cedar, maple, and beech.

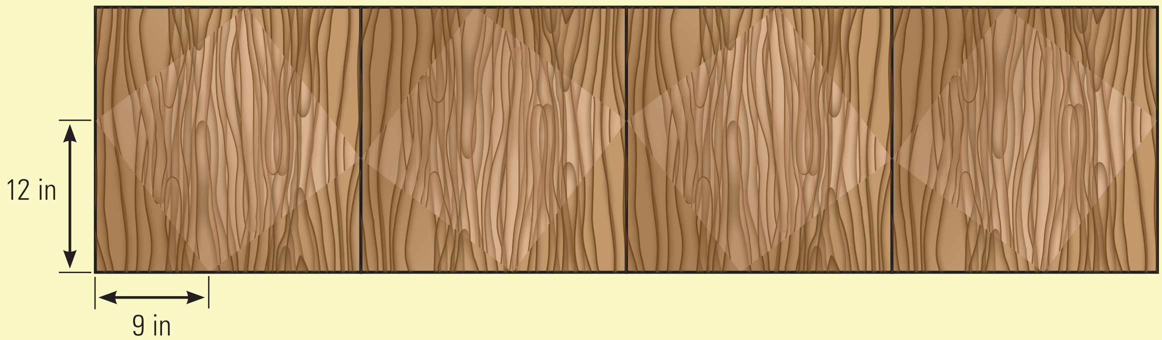
Jim often creates geometric designs to inlay on a box. These have to be cut perfectly. Why do you think this is? Also, the sides of a box are usually set at right angles to the base. Why do think this is important? What tools could Jim use to make sure that the bottom and sides are perpendicular?

Jim is designing a toybox for his grandson, Zack. He wants to decorate the top of it with large squares inlaid with smaller squares that are slightly rotated, as shown in the illustration.

Jim wants the inner squares to be rotated according to the measurements shown in the illustration. What is the length of the sides of the inlaid squares?

Should Jim then cut the inner squares to exactly those lengths? Why or why not?

Building a toy box is a hobby for Jim, who loves working with specialty wood



EXPLORE THE MATH

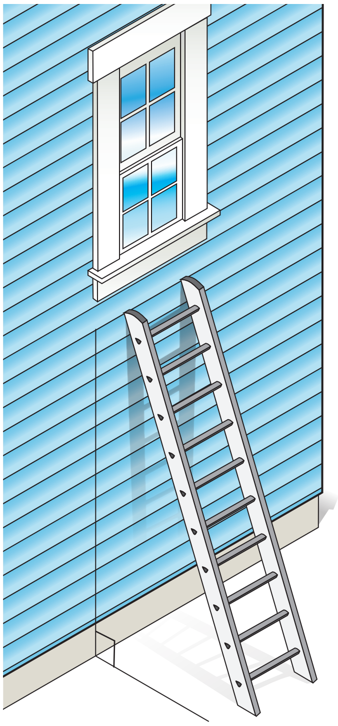
right triangle: a triangle with one right angle

Pythagorean theorem: in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse

In the example above, Jim used the **Pythagorean theorem**, often referred to as the Pythagorean relation, in order to determine the lengths of the sides of the **right triangles**. On a daily basis, carpenters, surveyors, bricklayers, and many other professionals use the formula below, which is based on the Pythagorean theorem. In the past you, too, have worked with this formula.

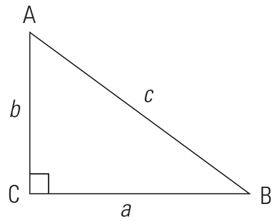
$$a^2 + b^2 = c^2$$

The longest side of a right triangle is always represented by the letter c. The two shorter sides are always represented by the letters a and b.



When a ladder is placed against a wall so that a painter can paint the house, the ladder, the wall, and the ground form a right triangle.

In this example, the wall and the ground form the right angle of the right triangle, and the ladder forms the **hypotenuse**. The ground and the wall are the **legs** (sometimes called the arms or sides).

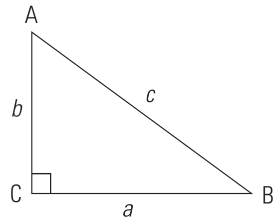


Triangle ABC represents the house and ladder. Leg BC of the triangle is said to be adjacent to angle B and opposite angle A. In general, a leg of a right triangle is adjacent to the angle if it, along with the hypotenuse, forms the acute angle. The other leg is said to be opposite that acute angle. In triangle ABC, leg AC is opposite angle B.

A lower case letter is used to label the side opposite an angle identified with a capital letter.

hypotenuse: the longest side of a right triangle, opposite the 90° angle

leg: in a right triangle, the two sides that intersect to form a right angle

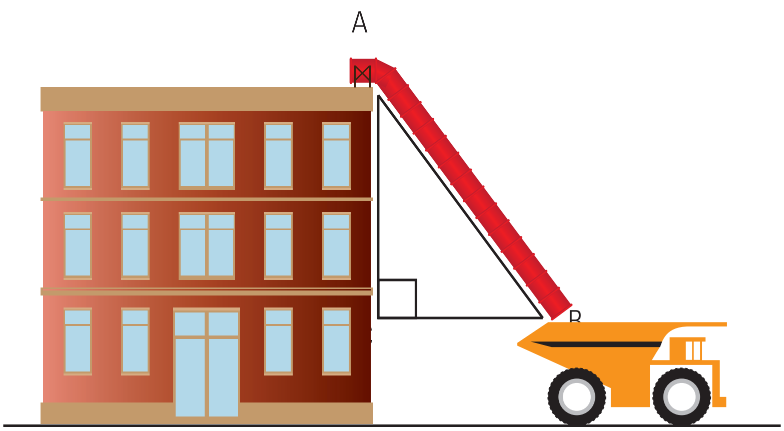


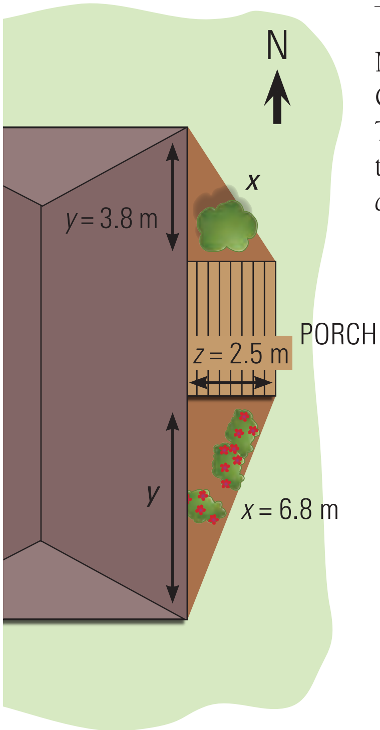
Leg AC, or *b*, is adjacent to angle A and opposite angle B

Leg BC, or *a*, is adjacent to angle B and opposite angle A

Mental Math and Estimation

In the diagram shown, if AB is 30 ft, and AC is 25 ft, approximately how far from the building is the back of the truck?





Example 1

Mary has submitted a plan to plant a herb garden in front of the proposed Centre Communitaire Beaumont Community Centre in Beaumont, Alberta. The garden will be made of two triangular pieces of earth on either side of the centre's porch. It will be used to grow traditional French herbs such as *cerfeuil* (chervil), *sarriette* (savory), *thym* (thyme), and *romarin* (rosemary).

a) Given the dimensions of the legs of the triangle as shown in the diagram, what will be the length of the hypotenuse of the plot to the north of the porch?

b) How far along the front of the house will the garden in the plot to the south of the porch reach?

SOLUTION

Use the Pythagorean theorem.

$x^2 = y^2 + z^2$

a) x represents the hypotenuse.

$x^2 = (3.8)^2 + (2.5)^2$ — Substitute known values into the Pythagorean theorem.

$x^2 = 14.44 + 6.25$

$x^2 = 20.69$

$x = \sqrt{20.69}$

$x \approx 4.5$

The hypotenuse will be about 4.5 m long.

b) The hypotenuse is 6.8 m.

$6.8^2 = (2.5)^2 + y^2$ — Substitute known values into the Pythagorean theorem.

$46.24 = 6.25 + y^2$

$y^2 = 46.24 - 6.25$

$y^2 = 39.99$

$y = \sqrt{39.99}$

$y \approx 6.3$

The garden will extend approximately 6.3 m along the house.

DISCUSS THE IDEAS

PYTHAGOREAN TRIPLES

The theorem you have been using is named after Pythagoras, who was born about 570 BCE. But ancient Egyptians were putting the theory into practice as early as 2500 BCE.

After the Nile flooded its banks, the ancient Egyptians had to lay out the boundaries of their fields again. They had to make sure the fields were laid out at right angles. To do this, they marked off 12 even lengths in a cord (by tying 11 knots in it, evenly spaced from the two ends of the cord).

1. Using a felt marker and a piece of string, make 11 equally-spaced marks that separate the string into 12 equal lengths. How do you think the ancient Egyptians would have used this string to ensure that they had a right angle?
2. What would have been the lengths of the sides of the triangles the Egyptians used? Check with your classmates to see if you arrived at the same answers.

The numbers you have found are consistent with the Pythagorean theorem. They are referred to as a **Pythagorean triple**.

Pythagorean triple: any set of three natural numbers that satisfy the Pythagorean theorem

3. Using your understanding of similar triangles from chapter 6, if you doubled the lengths of the sides of your triangle, would it still be a right triangle? What if you tripled them?
4. Excluding multiples of the Pythagorean triple that the ancient Egyptians used, find at least 3 other sets of Pythagorean triples that are distinct, meaning not multiples of each other.

Mental Math and Estimation

In a right triangle, if the hypotenuse is 20 in and one leg is 12 in, how long is the other leg?

Example 2

Marc is going to paint the exterior of his house. He has a 40-foot ladder and knows that for safety reasons the base of the ladder must be between 9 and 12 feet from the base of the wall. What are the maximum and the minimum heights the ladder will reach up the wall?

SOLUTION

The ladder will form the hypotenuse of the right triangle whose legs will be the ground and the height up the wall.

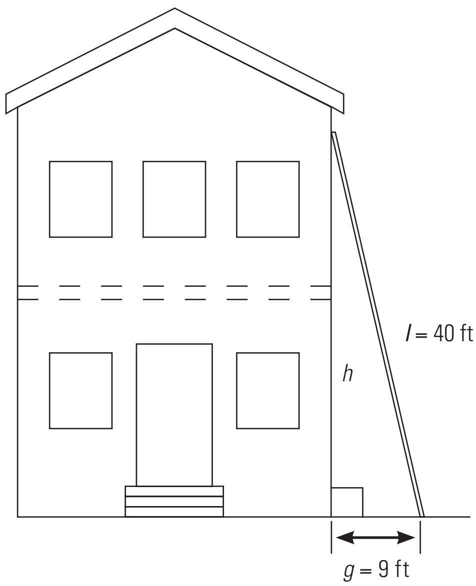
HINT

Choose letters that remind you of what they stand for. For example, use l for ladder, g for ground, h for height.

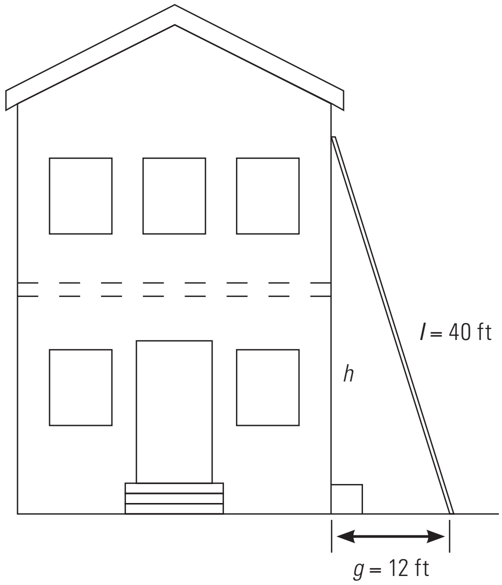
If the ladder is 9 feet from the base, calculate its height.

$$g^2 + h^2 = l^2$$
$$9^2 + h^2 = 40^2$$
$$81 + h^2 = 1600$$
$$h^2 = 1600 - 81$$
$$h^2 = 1519$$
$$h = \sqrt{1519}$$
$$h \approx 39$$

The ladder will reach approximately 39 feet up the wall.



If the ladder is 12 feet from the base, how far up the wall will it reach?



$$g^2 + h^2 = l^2$$
$$12^2 + h^2 = 40^2$$
$$144 + h^2 = 1600$$
$$h^2 = 1600 - 144$$
$$h^2 = 1456$$
$$h = \sqrt{1456}$$
$$h \approx 38.2$$

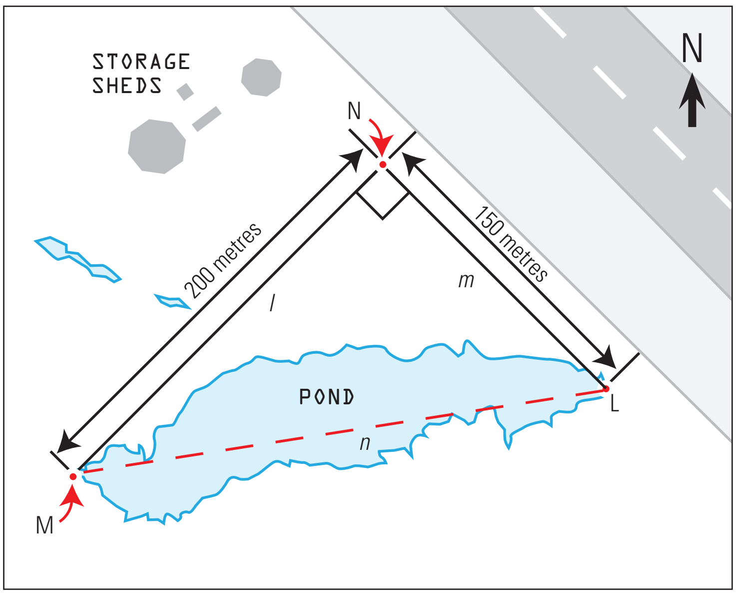
The ladder will reach approximately 38.2 feet above the ground.

Therefore the ladder, when placed safely, will reach between 38.2 feet and 39 feet above the ground.

ACTIVITY 7.1
INDIRECT MEASUREMENT

Cam is a surveyor working in Nunavut. He needs to estimate the length of a small pond between the Iqaluit Airport and Sylvia Grinnell Territorial Park. He decides to use a right triangle, as shown in the diagram, as an indirect method of measurement.

1. Why might a surveyor use an indirect method of measurement in the example above?
2. What is the length of the pond?
3. With a partner, find two objects in your neighbourhood for which you cannot take direct measurements and use right triangles to find their lengths or widths.



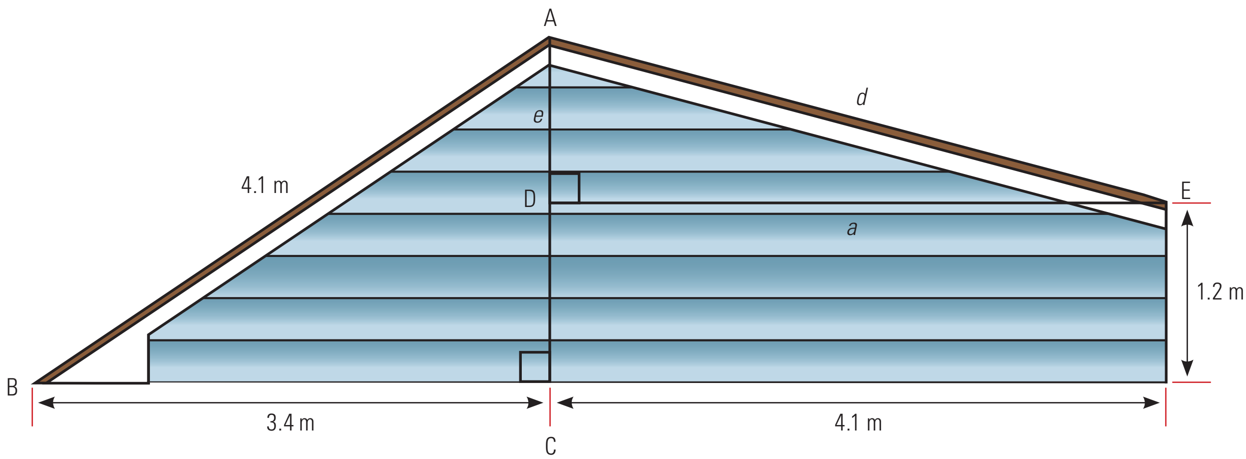
ACTIVITY 7.2
GENERALIZATIONS OF THE PYTHAGOREAN THEOREM

Euclid (born circa 300 BCE) is called the Father of Modern Geometry. In his famous book *The Elements*, he generalized the Pythagorean theorem by stating that if one erects similar figures on the sides of a right triangle, then the sum of the areas of the two smaller figures will equal the area of the larger figure.

Work with a partner. Use grid paper and a 6-8-10 right triangle.

1. Draw a diagram to illustrate Euclid's statement for squares. Use your knowledge about areas to prove that the statement is true for this situation.
2. Draw a right triangle. Then, draw 3 isosceles triangles, with each one having a side of the right triangle as its base. Make the height of each isosceles triangle equal the length of its base. Find the area of each of these triangles. Does Euclid's statement hold?

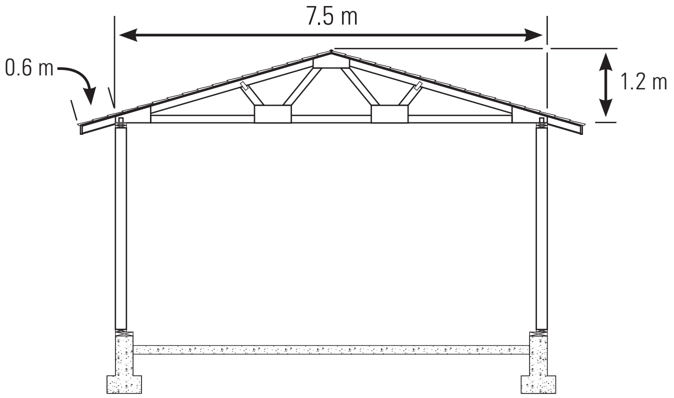
BUILD YOUR SKILLS



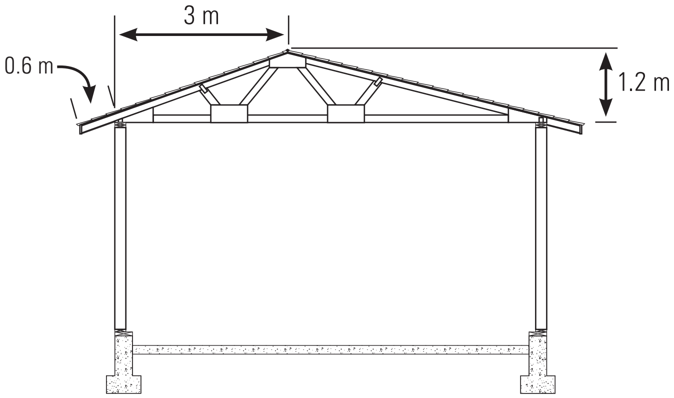
1. The roof of a shed is offset as in the diagram shown. Ben must determine its measurements so that he can order materials to repair it.
   a) How high is the peak (AC)?
   b) What is the length of the right-hand side (AE)?

2. Al has been contracted to build a garage in Prince Albert, Saskatchewan. The garage will be 7.5 m wide and the roof will have a 0.6 m overhang.

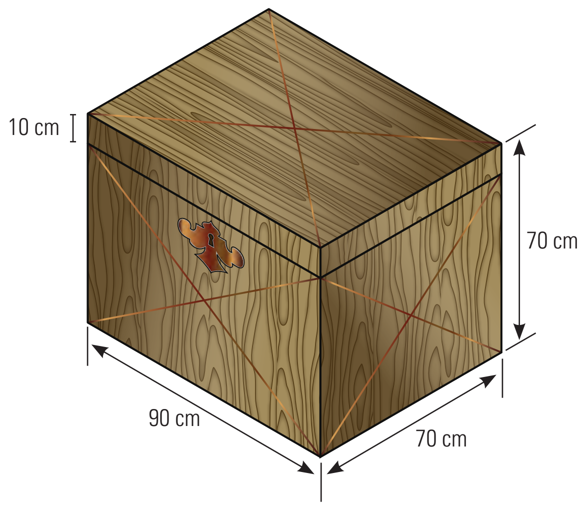
   a) If the peak of the garage is 1.2 m higher than the walls, how long does the rafter on each side have to be?



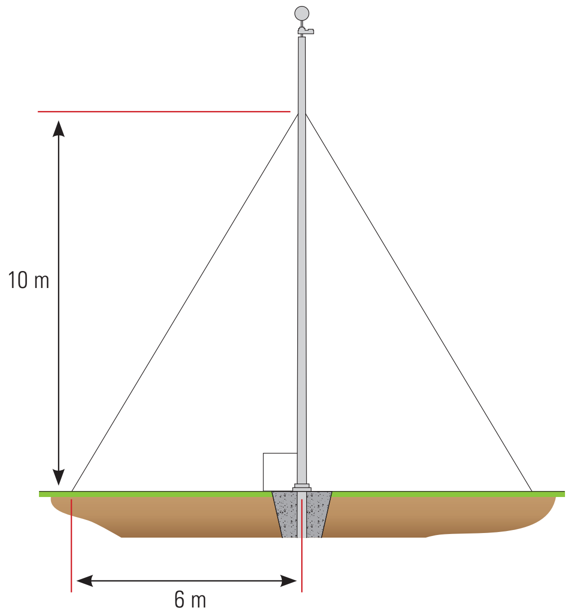
   b) The owner changes his mind and wants the peak to be off-centre. If it is 3 m from one side, how long will Al have to make each of the rafters? Note: There will still be a 0.6 m overhang.



3. Suzanne is designing a rectangular storage box. The box will be built of solid oak. The lid of the storage box will extend above the sides by 10 cm. Each face except the bottom will have an embedded X made of thick copper wire as shown in the diagram. How much copper wire must Suzanne buy if the storage box is 90 cm long, 70 cm deep, and 70 cm high? Do you think she should buy any extra wire? Why or why not?

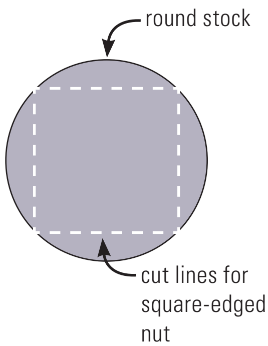


4. Brigitte and René are installing a new flagpole at a community centre in Portage la Prairie, Manitoba. The height of the flagpole is 12 m. Two guy wires must be attached 10 m above the ground and secured 6 m from the base.



   a) Allowing 153 cm total for fastening both guy wires, how much guy wire will they need? Answer to the nearest centimetre.

   b) Why are guy wires necessary?



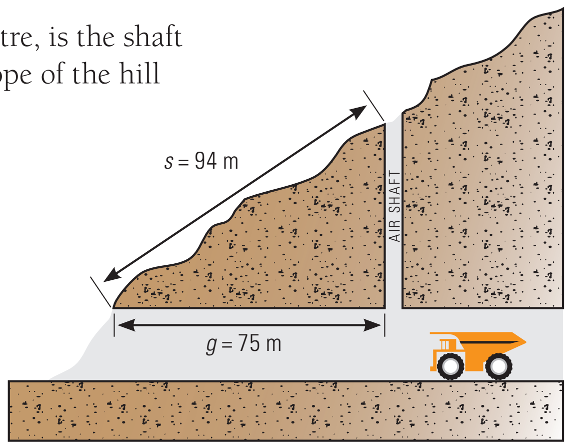
5. Bupinder must cut a square-edged nut from a piece of round stock in his machine technology class in Dawson City, Yukon.

   a) If metric round stock comes in diameters that are multiples of 5, what is the smallest diameter of stock needed if the nut must have a side of 30 mm?

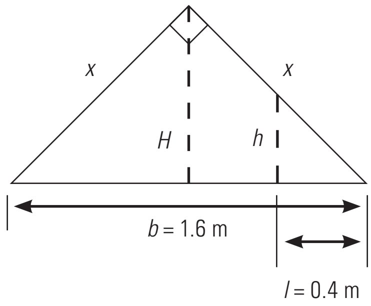
   b) If imperial unit round stock comes in diameters that are multiples of $\frac{1}{4}''$, what is the smallest diameter of stock needed if the nut must have a side of $\frac{3}{4}''$?

6. Mining is a major industry in Saskatchewan, and safety is a primary concern. An air shaft must be drilled from a mine tunnel to the surface of a hill at 75 m intervals, measured horizontally along the tunnel.

   How long, to the nearest metre, is the shaft if it emerges 94 m up the slope of the hill as indicated in the diagram?



7. Suki is building an A-frame doghouse in her backyard. There will be a 90° angle at the vertex, and the base of the front will be 1.6 m wide. Answer to the nearest tenth of a metre.



   a) What will the lengths of the sloping roof pieces be?

   b) How high will the doghouse be at its peak? (Hint: H = height.)

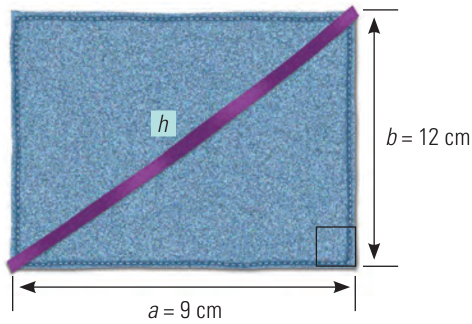
   c) At 0.4 m in from the base, how high will the doghouse be?

   d) Would this be a suitable doghouse for a large dog? Why or why not?

Extend your thinking

8. Marc works for a trucking company that makes regular trips between a stone quarry and a construction site. He usually follows a route that runs from the quarry north for one mile and then east for 2 miles to the construction site. He's now found a new route along a road that runs straight from the quarry to the site. How much shorter is the new route?

9. Sara is taking a course on quilt design so that she can make and sell her quilts at craft fairs. Her first quilt design has 100 rectangles on it, with a ribbon running across each rectangle in a single diagonal line, as in the diagram shown. Sara is trying to figure out how much ribbon to buy. She calculates:



$$h^2 = a^2 + b^2$$

$$h^2 = 81 \text{ cm} + 144 \text{ cm}$$

$$h = 9 \text{ cm} + 12 \text{ cm}$$

$$h = 21 \text{ cm}$$

$$21 \text{ cm} \times 100 \text{ rectangles} = 2100 \text{ cm}$$

   Sara buys 2100 cm of ribbon.

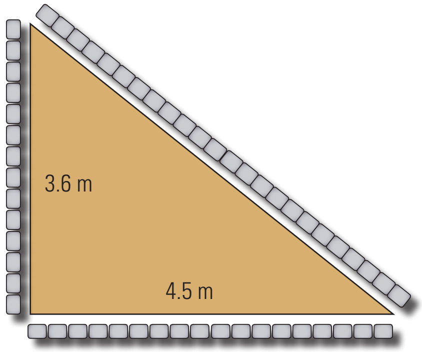
   a) Sara has ribbon left over when she is finished the quilt. Why?

   b) How much ribbon should Sara have bought?

10. In the Math on the Job at the beginning of this unit, Jim could have built a box in which the sides were not perpendicular to the base, as shown in the image here. If so, what would Jim have had to consider?

Sea chests were traditionally used by sailors to store their belongings in. They were commonly made of pine, with rope handles, and brass detailing.

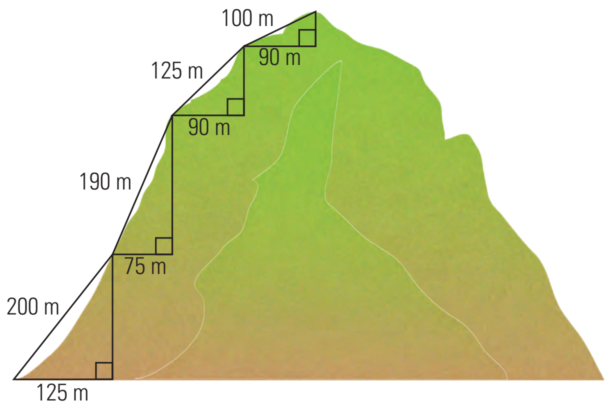
11. Harpreet is designing a triangular garden plot in Saskatoon, Saskatchewan and will surround it by paving blocks that are 30 cm long. If the legs of the plot are 4.5 m and 3.6 m respectively, how many paving blocks will he need? (Ignore the corner overlap.)



12. John is a surveyor who is asked to measure the height of a hill in Hinton, Alberta. He is unable to do so directly, so reverts to measuring the slant distance and the horizontal distance of shorter segments as he climbs up.

    a) How high is the hill?

    b) Why would John measure the hill in this manner?



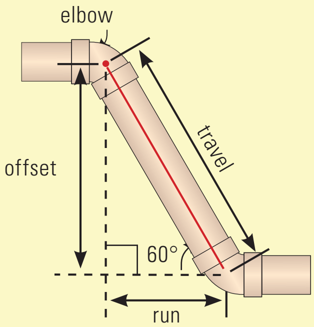
13. Can you find any other value of n so that

    $a^n + b^n = c^n$

    where a, b, and c are positive integers? Explain your reasoning.

The Sine Ratio 7.2

A pipefitter must know how to cut, install, maintain, and repair pipes.



MATH *ON THE JOB*

Alexia is a plumber/pipefitter who works in the Northwest Territories installing and repairing pipes. Often the pipes have to be fitted around obstructions, and Alexia must take careful measurements, then cut and attach the pipes using elbows of different angles. An elbow is a curved fitting that is used to form a corner joining two pieces of straight pipe. Three terms Alexia uses in her work are *offset*, which is the vertical displacement between the centres of the pipes; *run*, which is the horizontal displacement between the end of one pipe and the beginning of the next; and *travel*, the diagonal distance between the centres of the two pipes.

In construction projects, professionals often simplify a 3-D object by drawing a 2-D sketch, as shown. Also, their professional installation manuals often provide them with a formula that helps them determine what they need to know. When Alexia consults her manual, it gives her a formula that indicates that, in joining a lower horizontal pipe to a travel pipe with a 60° elbow, she must use the following formula.

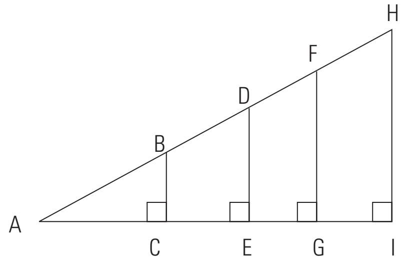
$$\frac{\text{offset}}{\text{travel}} = 0.866$$

If the offset in a particular project is 75 inches, what must the length of the travel pipe be?

EXPLORE THE MATH

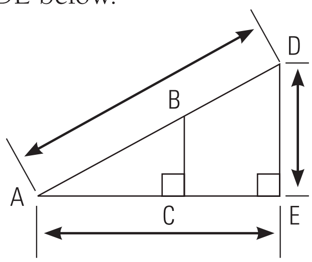
Many trades use terms for particular situations. In Math on the Job, above, Alexia was working with a right triangle whose “travel” was the hypotenuse. The side opposite the base was the “offset.” Alexia uses a formula that mathematicians refer to as the **sine** ratio. In this section we will explore what is meant by sine ratio by considering the ratio of sides *within* triangles.

sine: in a right triangle, the ratio of the length of the side opposite a given angle to the length of the hypotenuse (abbreviated as sin)



In the diagram above, you will notice that you have four similar right triangles, ΔABC, ΔADE, ΔAFG and ΔAHI. Explain how you know the four triangles are similar.

Consider ΔABC and ΔADE below.



1. Because the triangles are similar, you know that $\frac{BC}{DE} = \frac{AB}{AD}$. Use your understanding of equations to rearrange the letters in this equation so that each side represents a ratio of sides from the same triangle.
2. How would you describe sides BC and DE with respect to ∠A?
3. What is the name given to sides AD and AB in their respective triangles?
4. Write the ratios from question 1 in words relating to the sides of the triangles.
5. Consider ΔAFG and ΔAHI. What would the corresponding ratios be?

DISCUSS THE IDEAS

THE SINE RATIO

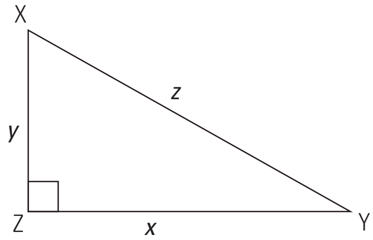
In the previous section, you looked at the ratios of sides within given right triangles. You discovered that when the triangles are similar (have the same acute angle measures), the ratio of the length of the side opposite a given angle to the length of the hypotenuse of the triangle remains the same, regardless of the overall size of the triangle. This ratio is the sine ratio that Alexia was given to use in the Math on the Job. She was told that, anytime she had a 60° angle, the ratio of the opposite side (offset) to the hypotenuse (travel) will be approximately 0.866.

$$\text{sine A} = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{or, simply, } \sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{or, } \sin A = \frac{\text{opp}}{\text{hyp}}$$

In the diagram shown here, look at angle Y.



$$\text{sine } Y = \frac{y}{z}$$

$$\text{or, } \sin Y = \frac{y}{z}$$

Angle X is also an acute angle.

$$\sin X = \frac{x}{z}$$

ACTIVITY 7.3
THE SINE OF AN ANGLE

In the preceding Discuss the Ideas, you learned that the sine of an acute angle of a right triangle is calculated by finding the ratio of the side opposite the angle and the hypotenuse. Because of similarity, in a right triangle, the ratio of the opposite side to the hypotenuse will be the same if the angle is the same. You therefore need to calculate this ratio only once. In this activity, you will develop a table of values for the sine ratio. You will then use it to draw a graph and solve related problems.

Part A: Taking Measurements

1. Work with a partner. Your teacher will provide you with a set of three similar right triangles with a specified acute angle. Your assigned angles will be labelled as $\angle A_1$.
2. Carefully measure, to the nearest millimetre, the length of the side opposite each ∠A and the hypotenuse of each triangle. Record the results on the table provided by the teacher.

THE SINE OF ANGLES

size of ∠A	length (o) of side opposite $\angle A_i$			length (h) of hypotenuse			ratio $\left(\frac{o}{h}\right)$: $\frac{\text{opposite}}{\text{hypotenuse}}$			sine ratio [average of ratio $\left(\frac{o}{h}\right)$]	sin A values
	Δ_1	Δ_2	Δ_3	Δ_1	Δ_2	Δ_3	Δ_1	Δ_2	Δ_3		
10°											

3. Calculate the ratio of the opposite side to the hypotenuse for each of your triangles to the nearest hundredth.
4. If your ratios are not all the same, find the average of the three ratios and record this in the column labelled sine ratio.
5. Record your sine ratio on an overhead chart provided by your teacher. The last column will remain empty for now.

Part B: Discussion

1. When the class has filled in the overhead chart, compare the sine ratios. What do you notice about the values of sin ∠A as A increases from 10° to 80°?
2. Do the data appear to form a linear relationship between an angle and its sine? Discuss why or why not.
3. Between what two values do you think sin 45° would fall?
4. Approximately what do you think sin 5° would be?
5. About what value do you think sin 85° would be?
6. What is the smallest value the sine of an acute angle of a right triangle can have? How about the largest? Use diagrams of right triangles with very small acute angles and acute angles that are near 90° to discuss and explain your reasoning.

Part C: Drawing a Graph

1. Using the table supplied by your teacher, fill in the data from the table that was generated by the class. Leave the last column empty for now.
2. Using the data from your table, sketch a graph of y equals sin ∠A.
3. Extrapolate to extend the graph to values of A near 0° and 90°.
4. Do the values correspond to your predicted values above?

Many mathematical operations, including finding the sine, cosine, or tangent of an angle, can be performed on a scientific calculator.

DISCUSS THE IDEAS

REPAIRING A TRUSS BRIDGE

Benson is a structural engineer working in Sainte-Anne, Manitoba. He is repairing a truss bridge on which the angle of one of the beams is 60° compared to the horizontal. He knows that the height of the bridge is 2.8 m. Using the sine table you developed in Activity 7.3, determine approximately how long the beam that he must replace will be.

In designing and construction, accuracy is important. Engineers cannot rely on approximated values such as we determined for the sine above because slight errors can lead to serious flaws in a design. Thus, they generally rely on a calculator to determine the values.

T Using your scientific calculator, find sin 60.

1. Once you have the answer of approximately 0.8660, what does this tell you?
2. How does this compare with the value you found in Activity 7.3 for the sine of a 60° angle?
3. Use your calculator to find the values of sin A for the values in your table. Put these values in the last column of the table and compare them to your measurement values. Were your measurements accurate? Discuss possible sources of error.

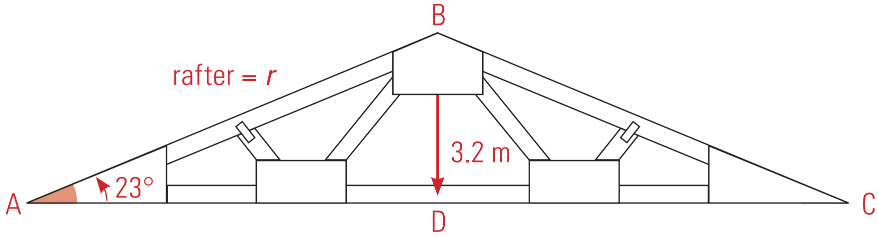
HINT

Make sure your calulator is in degree mode.

Example 1

In this and the next example, we will use the terms angle of elevation and angle of depression. Based on your understanding of the terms "elevation" and "depression," what do you think is meant by these expressions?

Hélène is building a garage on her farm near Stavely, Alberta. She knows that the angle of elevation of the roof must be 23° for the peak of the roof to be 3.2 metres above the ends of the rafters, as shown in the diagram. How long is each rafter?



SOLUTION

In the diagram, BD is opposite the 23° angle, and the length of the rafter is the hypotenuse of the right triangle ABD.

$$\sin A = \frac{BD}{AB}$$

$$\sin 23° = \frac{3.2}{r}$$

$$\frac{r \sin 23°}{\sin 23°} = \frac{3.2}{\sin 23°}$$

$$r = \frac{3.2}{\sin 23°}$$

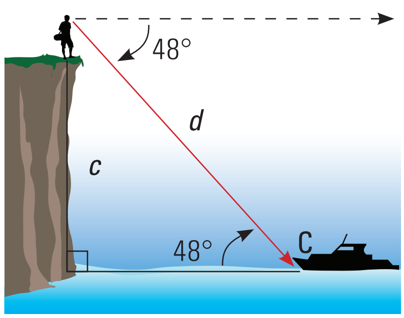
$$r \approx 8.2$$

Each rafter is 8.2 m long.

Example 2

angle of depression: the angle formed between the horizontal and the line of sight while looking downwards

From the top of a cliff by the ocean, Cedric sights a boat at an **angle of depression** of 48°. If the top of the cliff is 73 m above the surface of the water, and Cedric is 2 m tall, how far is Cedric from the boat?

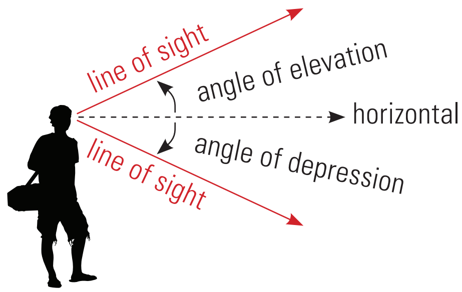


SOLUTION

Sketch the scene described.

Since the angle of depression from Cedric to the boat is 48° and the angle of depression is measured from the horizontal, the **angle of elevation** from the boat to him is also 48°. Thus, using the definition, we solve the distance as follows.

angle of elevation: the angle formed between the horizontal and the line of sight while looking upwards; sometimes referred to as the angle of inclination



$$\sin C = \frac{c}{d}$$

$$\sin 48° = \frac{75}{d}$$ Substitute the known values.

$$d \sin 48° = 75$$ Multiply both sides by d.

$$d = \frac{75}{\sin 48°}$$ Divide both sides by sin 48°.

$$d \approx 101$$

Cedric is approximately 101 m from the boat.

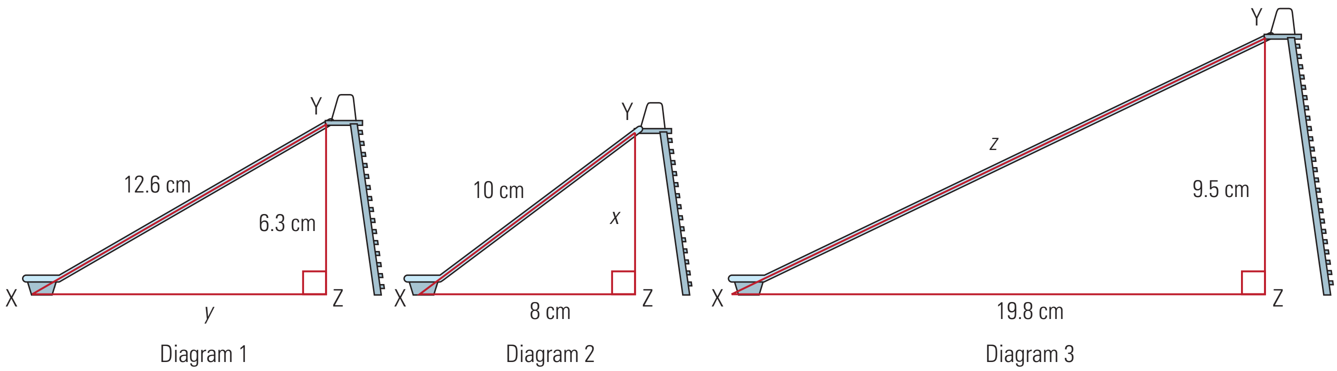
BUILD YOUR SKILLS

1. Use your calculator to find sin 16°, sin 28°, sin 51°, and sin 83°, to four decimal places.

2. Joanne is designing a children's slide for a playground in the community of Carcross, Yukon. She has submitted three scaled designs, as shown here.

   a) For each design, determine the sine ratio of angle X.

   b) Find the length of each slide if the actual height is to be 2.6 m.



3. Downstream from the confluence of the North and South Saskatchewan Rivers, archaeologists are excavating the site of a former Cree settlement. They find a stone circle that represents where a tipi once stood. (Stones were used to weigh down the hides that covered the tipi poles.) The average tipi had about 17 poles, which ranged from 10 to 24 feet long. An archaeologist determines that the tipi was 15 feet high at its peak and 12 feet wide. What would the length of the tipi poles be?

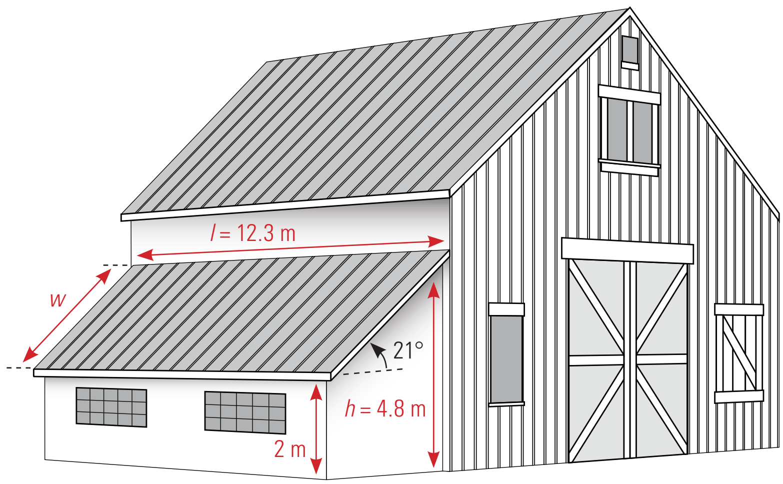
4. Laiwan, who lives in Grand Forks, BC, must have a wheelchair ramp built to her front porch. The porch is 1.9 m above ground level and the steepest angle of elevation allowed by the building code is 6°.

   a) What is the shortest ramp that Laiwan can have installed?

   b) About how many metres (to one decimal) from the base of the porch must the ramp start?

   c) Why do you think regulations state that the ramp cannot be any steeper?

5. Johan's barn is 12.3 m long. He is constructing a lean-to against the side of it. The angle of elevation of the roof of the lean-to is 21° and it meets the side of the barn at a point 4.8 m above the ground. How much roofing will he need to cover the roof of the lean-to? Give your answers in square metres, to one decimal.



Theodolites are small mounted telescopes that can be moved horizontally and vertically. Navigators and surveyors use them to measure angles and bearings.

6. Darren works on a road construction crew in Wha Ti, Northwest Territories. He is able to measure the angle of elevation using an instrument called a theodolite. The angle of elevation from one point to another is 9°. The slope distance between the two points is 250 m.

   a) How much does the road rise over that distance?

   b) Do you think this would be considered a steep road? Explain your thinking.

7. Sally is flying a kite in Cochrane, Alberta. She has let out 210 m of string. Ignore Sally's height for the following calculations.

   a) If the angle of elevation is 50°, how high above the ground is the kite to the nearest metre?

   b) If an updraft catches the kite so that the angle of elevation changes to 65°, how high is the kite now?

8. The Leaning Tower of Pisa leans at an angle of approximately 84.5° to the ground. If the guardrail at the top of the tower is 55.86 m on the lowest side and 56.70 m on the highest side, determine the length of the two sides.

The Leaning Tower of Pisa in Italy was built over a span of 177 years. It began to sink, and lean, while its third floor was being built.

Extend your thinking

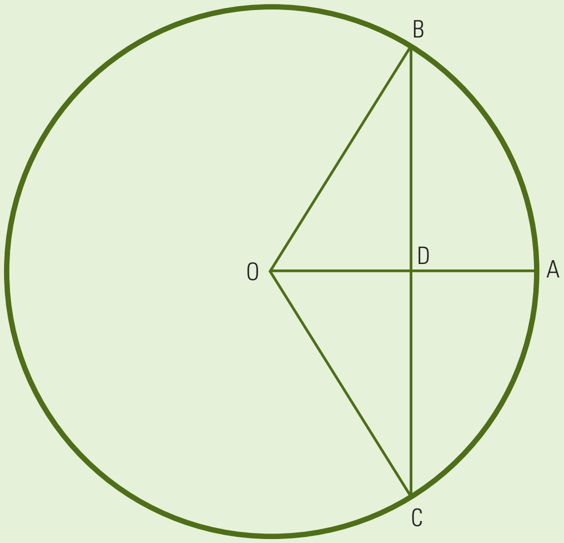
9. Refer to Example 1 on p. 288. Hélène wants her roof to be steeper but keep the width the same. How will this affect the length of the rafter? What happens to the distance between the peak and the line of the ends of the rafters? Explain and create a scenario with a diagram and actual measurements to illustrate your answer.

The roof of a garage gains strength and stability from its triangular form. By supporting a roof with triangular trusses, the roof is made even stronger.

10. Bridge builders use trigonometry to complete their work. They must be familiar with the sine ratio and its values, since right triangles are frequently used to construct bridges. Imagine you are a bridge builder using trigonometry at work. Given any right triangle, between what two values must the sine of an acute angle fall? That is, what are the maximum and minimum values for sin x when x is greater than 0° and less than 90°? Explain.

THE ROOTS OF MATH

TRIGONOMETRY IN HISTORY



The Rhind papyrus, kept in the British Museum, contains problems and tables that show how ancient Egyptians used mathematics.

The term trigonometry is derived from the Greek words trigon (triangle) and metria (measure). It was first used in 1595 by Bartholomaeus Pitiscus, in his influential work *Trigonometria: sive de solutione triangulorum tractatus brevis et perspicuus*. When this book was translated into English and French in 1614 and 1619 respectively, the term *trigonometry* became used in these languages.

However, the development of the mathematics of triangles began long before that in many cultures. Ancient cultures dating back to as early as 4500 BCE in Britain may have used some Pythagorean triples. Old Sanskrit texts from India dating back to about 3100 BCE discuss the concepts of angles and measurement. The Rhind Papyrus, copied from an older document in about 1650 BCE by the Egyptian scribe Ahmes, displays some use of what we now call trigonometry in discussions about building pyramids. Much later (about 150 BCE), Hypsicles of Alexandria used chord functions of angles in a circle to make triangular computations.

In about 140 BCE, a Greek astronomer named Hipparchus is believed to have been the first to make systematic use of trigonometry. He computed a table of chords roughly equivalent to trigonometrical sines. However, it was not until the fifth century CE that trigonometry was introduced in its present form.

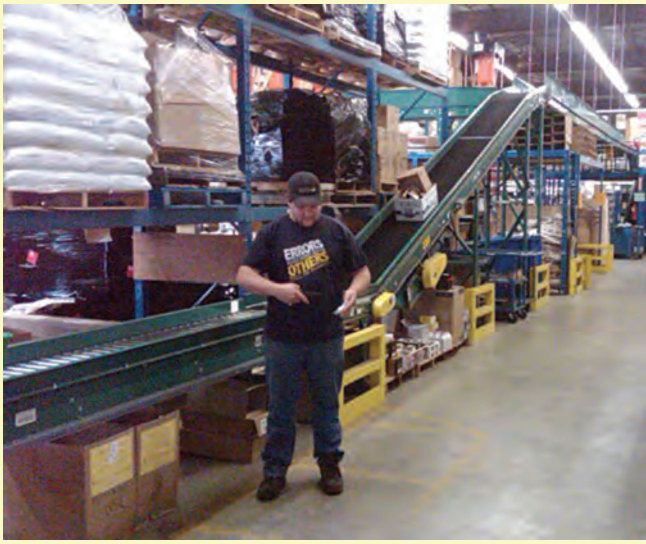
BC is a chord, a straight line segment joining any two points on a circle. If the radius (OA) of the circle is 1 unit, then the sine of the angle AOB is the half-chord DB.

For thousands of years, people used tables of calculated trigonometric values, like the one you calculated for sine. These tables were replaced by calculators only in the latter part of the twentieth century.

1. Using your understanding of circles, the sine function, and the image provided, explain how ancient scholars could have used this information to create a table of half-chords to represent the sine function.
2. Conduct research on the internet to find ways to determine the sine function other than by taking measurements of the sides of triangles.

The Cosine Ratio

7.3



Richard uses a conveyer belt in his warehouse job.

MATH *ON THE JOB*

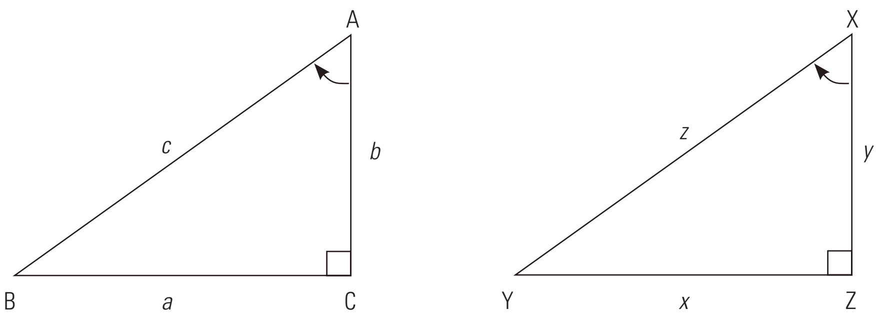
Richard McCaffrey works at an auto parts warehouse in Burnaby, BC. His work involves keeping inventory of what parts are in stock, ordering new parts when stock is low, and delivering parts to various dealers in the Lower Mainland. Often stock arrives and is stored on one level, but is then transported to a different level for loading into a truck for delivery. Since the parts are often too heavy to lift, a conveyor belt is used to move them.

The conveyor belt needs replacing, and Richard must determine approximately what length of belt to order. He knows that the angle of depression from the upper floor along the conveyor belt is 38°. The belt reaches the lower level at a point 6.1 m further along the floor. How can Richard use similar triangles with a scale diagram to determine the length of belt he must order? Remember that a conveyor belt is a continuous loop with the belt returning on the underside of the conveyor. Drawing a scale diagram of the situation can help you understand it.

EXPLORE THE MATH

Using similar triangles is one way that Richard could determine the length of the conveyor belt. However, he could also use a second trigonometric ratio, the **cosine** ratio, to find the length. While the sine ratio considers the ratio between the side opposite an acute angle in a right triangle to the hypotenuse, the cosine ratio considers the ratio of the side adjacent the acute angle to the hypotenuse.

cosine: in a right triangle, the ratio of the length of the side adjacent to a given angle to the length of the hypotenuse (abbreviated as cos)



Begin by comparing the two similar triangles, $\Delta ABC \sim \Delta XYZ$.

1. Because the triangles are similar, you know that $\frac{x}{a} = \frac{z}{c}$. Use your understanding of equations to rearrange the letters in this equation so that each side represents a ratio of sides from the same triangle.

2. How would you describe side a with respect to angle B? How would you describe side x with respect to angle Y?

3. What is the name given to sides c and z in their respective triangles?

4. Write the ratio a/c with respect to angle B in words. Write the ratio x/z with respect to angle Y in words.

The ratio of the adjacent side and the hypotenuse is equivalent in both triangles. It is defined as follows

$$\text{cosine X} = \frac{\text{length of the side adjacent to } \angle X}{\text{length of the hypotenuse}}$$

$$\cos X = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos X = \frac{\text{adj}}{\text{hyp}}$$

Example 1

Now that you know how to use the cosine function, use it to determine the length of conveyor belt that Richard needs to order in the Math on the Job on p. 293.

SOLUTION

Consider ΔPQR. The distance from P to Q represents the distance from the loading edge to the place where the conveyor belt meets the floor.

Therefore:

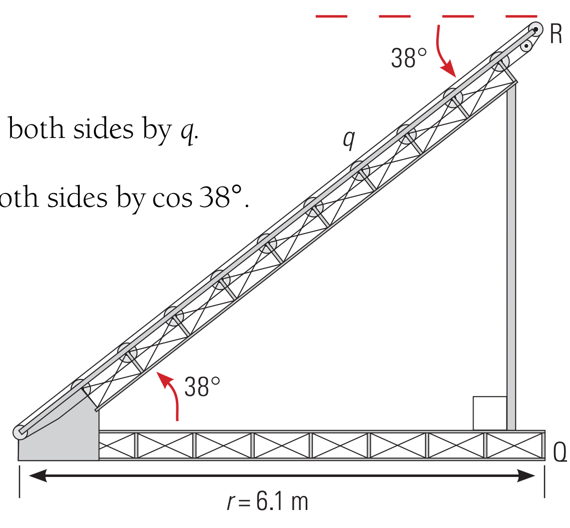
$$\cos P = \frac{r}{q}$$

$$\cos 38° = \frac{6.1}{q}$$

$$q \cos 38° = 6.1 \quad \text{Multiply both sides by } q.$$

$$q = \frac{6.1}{\cos 38°} \quad \text{Divide both sides by } \cos 38°.$$

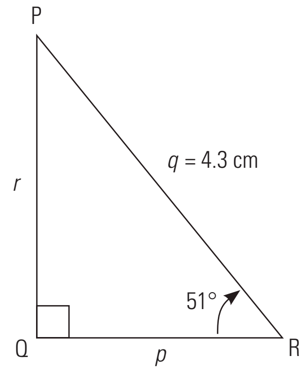
$$q \approx 7.74$$



The distance along the conveyor belt is approximately 7.74 m. Richard therefore has to order at least twice this amount, at least 15.5 m.

Example 2

Given, ΔPQR where $\angle Q$ equals 90°, q equals 4.3 cm, and $\angle R$ equals 51°, solve the triangle.



SOLUTION

First, draw a diagram.

Since the sum of the two acute angles of a right triangle is 90°, you can find the third angle.

$\angle P = 90° - 51°$

$\angle P = 39°$

Find r.

$$\begin{aligned}\sin R &= \frac{r}{q}\\ \sin 51° &= \frac{r}{4.3}\\ 4.3\sin 51° &= r\\ r &\approx 3.3\end{aligned}$$

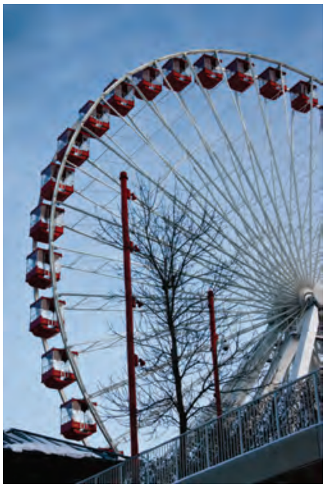
Therefore r is approximately 3.3 cm.

Find p.

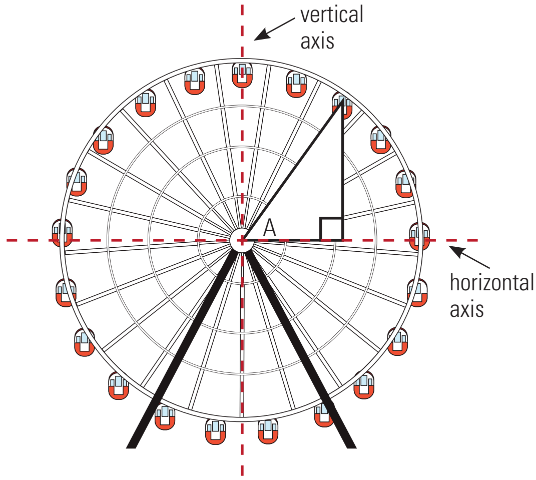
$$\begin{aligned}\cos R &= \frac{p}{q}\\ \cos 51° &= \frac{p}{4.3}\\ 4.3\cos 51° &= p\\ p &\approx 2.7\end{aligned}$$

Therefore p is approximately 2.7 cm.

ACTIVITY 7.4
MOVEMENT OF A FERRIS WHEEL



Ferris wheels are constructed using many right angle triangles.



You and your friends are on a Ferris wheel at the local summer fair. As your chair moves, you will always be the same distance from the centre as this is the radius of the wheel, but you will be closer or further away from the vertical diameter and the horizontal diameter depending on the movement. Consider the angle formed by the horizontal diameter, the axis to your chair (the radius of the Ferris wheel) and the vertical line from your chair to the horizontal axis. If you let the radius of the Ferris wheel be represented by 1 unit, the cosine of the angle formed will be the length of the horizontal leg of this triangle. Using angle measurements that are multiples of 10°, find the lengths of these horizontal segments by filling in a chart like the one below. You may use your calculator to find the values. Use this information to sketch the graph of y = cos A.

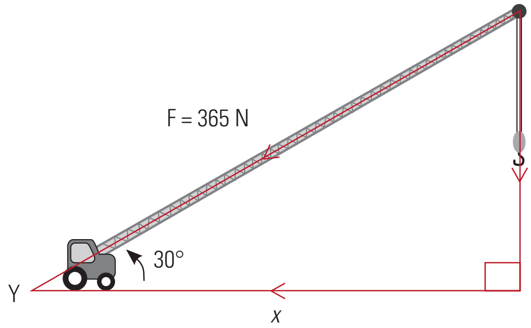
COSINE RATIOS

A	10°	20°	30°	40°	50°	60°	70°	80°
y = cos A								

1. What do you notice about the value of y as A increases from 10° to 80°?
2. Does this appear to be a linear graph? Explain how you know.
3. How does this graph compare to the graph of y = sin A from the previous section? Explain.
4. Extend your graph to values of A = 0° and A = 90°.
5. As the value of A approaches 0° (gets very small), what happens to the value of cos A? Using diagrams of right triangles, discuss why this is so.
6. What happens to the value of cos A as A gets close to 90°? Explain using diagrams.
7. What does this graph tell you about the horizontal distance of the chair from a vertical line through the centre of the Ferris wheel?

Example 3

In construction, Marie knows that a force acting at an angle can be broken up into a vertical force and a horizontal force. If a force of 365 Newtons is exerted diagonally downward at an angle of 30° to the horizontal, what force will be applied horizontally?



SOLUTION

The horizontal force x is adjacent to the 30° angle, so you use the cosine function.

$$\cos Y = \frac{x}{F}$$

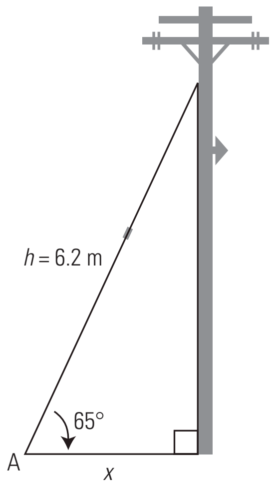
$$\cos 30° = \frac{x}{365}$$

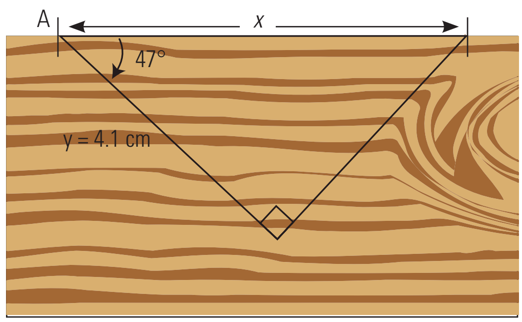
$$365 \cos 30° = \frac{x}{\cancel{365}} \times \cancel{365} \qquad \text{Multiply both sides by 365.}$$

$$316 \approx x$$

The horizontal force will be about 316 N.

BUILD YOUR SKILLS





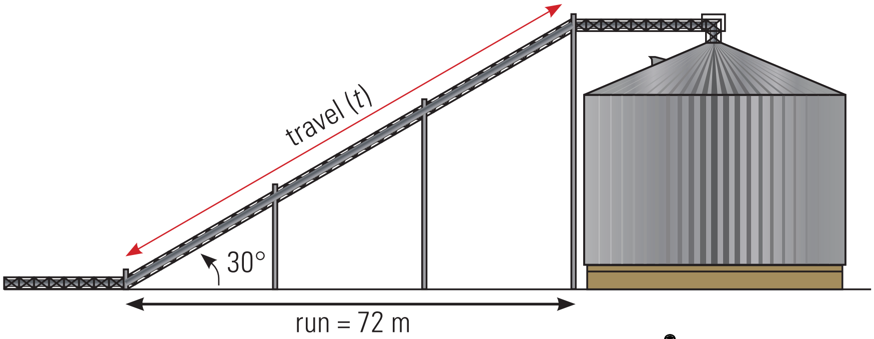
Guy wires add stability to poles carrying electrical lines.

1. Refer to the diagrams above.

   a) How far from the base of a pole must a 6.2 m long guy wire be attached if the angle of elevation is 65°?

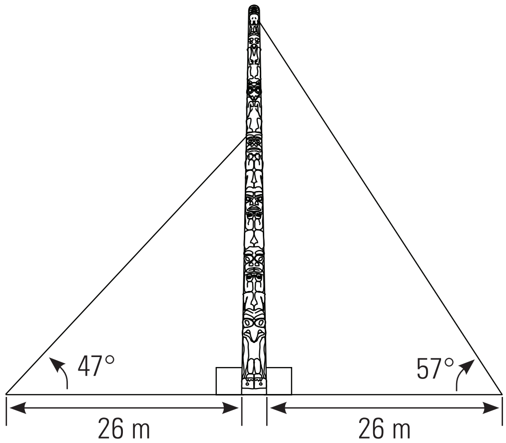
   b) A notch is cut from a block of wood as indicated. What is the width of the opening of the cut-out portion?

2. The angle of elevation between a grain auger and the grainery to which it is to be connected is 30°. If the run is 72 m, how long must the travel pipe be?

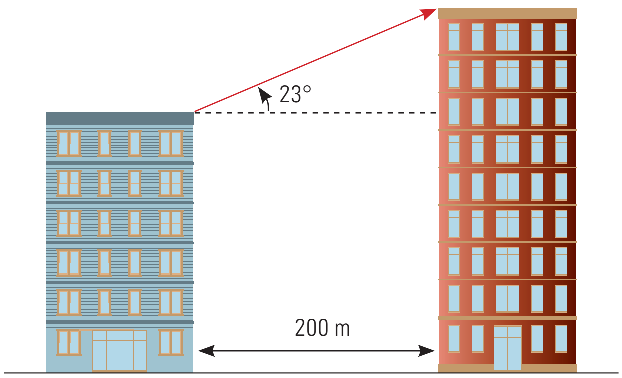


This freestanding totem pole is in Victoria, BC's Beacon Hill Park. Standing 38.9 m tall, it was carved by Henry Hunt, David Martin, and Mungo Martin.

3. Totem poles are almost always erected by being pulled upright with ropes into a wooden scaffold support until they are stable. Suppose that two of the ropes attached to a pole are at angles of elevation of 47° and 57° respectively. If the base of the ropes is approximately 26 m from the base of the totem pole, how long is each rope?

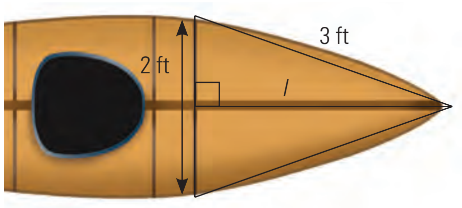


4. A surveyor standing at the edge of one building notes that the angle of elevation to the top of another building is 23°. If the buildings are 200 m apart at the base, how far is the surveyor from the top of the second building?



5. A telephone pole is diagonally braced by a piece of timber 6.8 feet long. The angle between the pole and the timber is 34°. How high up the pole does the timber reach?

6. Roger is a craftsperson and boat builder who lives and works in Iqaluit, Nunavut. One of Roger's skills is building kayaks. While traditional kayaks are covered in sealskin and use sinew as fastenings, Roger's boats are covered in canvas and use epoxy as a fastening. Roger must know the length of the tapered part at the front of the kayak to ensure he builds a boat with enough leg room. Each tapered side that forms the kayak's nose is 3 ft long. The kayak is 2 ft wide. What is the length (l) of the tapered section of the kayak?

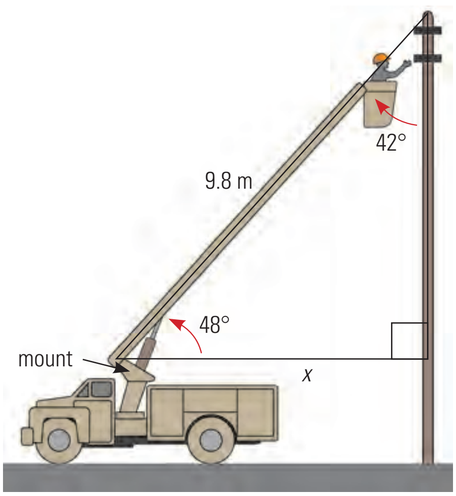


The kayak was originally used by Inuit people for hunting and transportation. The frame was made of driftwood or whalebone and the kayak's surface was made waterproof with whale fat. Today, kayaks can be made of fiberglass or molded plastic.

Extend your thinking

7. Laurie has been hired to design ski chalets in Jasper, Alberta. In her blueprint, she draws a right triangle with acute angle x, for which $\sin x = \cos x$, to represent the roof of the building. Draw the same figure Laurie drew. What type of triangle is it?

8. Frank uses a truck with a long arm called a "cherry picker" in his job repairing telephone lines. The arm has a maximum length of 9.8 m. If the angle of elevation from the mount to the top of the telephone pole is 48°, how far from the pole is the mount?



9. Two parallel chords of a circle are 4 cm apart and subtend angles of 120° and 90° at the centre. Find the radius of the circle and the length of each chord.

MAKING A SCALE DIAGRAM OR A SCALE MODEL

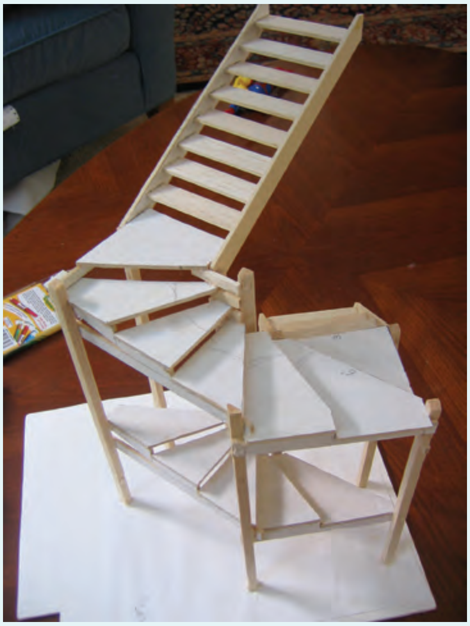
Now that you have decided what type of staircase you will build, you can draw a scale diagram using similar triangles. Remember:

- the stringer is a timber that supports the treads and risers in a staircase
- the riser is the vertical distance between two stairs
- the run is the distance from the front to the back of each stair
- the tread is a separate piece of wood that covers a stair.

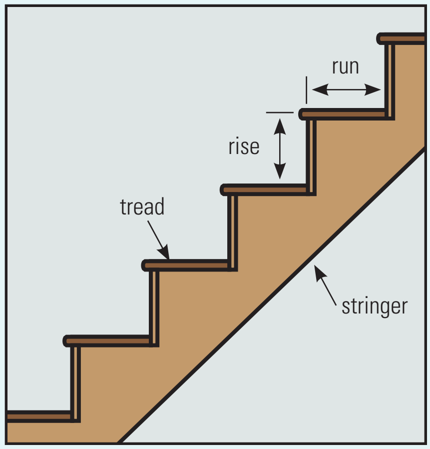
Determine the length and width of the stringer and the amount of wood needed for the risers and treads.

Be sure to keep track of all your calculations, because you will need them for the final presentation. Because many measurements in the construction trade are given in imperial, you may want to work in those units.

Discuss how tools such as dividers, stair gauges, and set squares will be used in helping you construct your stairs. How will the application of the tread boards affect the height of the steps?



A scale model of a staircase can help you to ensure that all the proportions are accurate.



7.4 The Tangent Ratio

Chris Haika applies mathematical skills on the job as an airline pilot.

MATH *ON THE JOB*

When Chris Haika, of Calgary, Alberta, was in elementary school, he knew he wanted to fly airplanes. He got his pilot's licence in his last semester of high school and went on to Mount Royal College to get his aviation diploma. He worked part time washing airplanes, which helped him make contacts in the airline industry.

After getting his diploma, Chris worked as a customer agent for an airline in Grande Prairie, Alberta. After seven months, he was promoted to pilot.

Chris later moved back to Calgary. He's now a pilot in a small airline company that does charter flights as well as regular routes in Alberta and British Columbia.

When flying into Calgary, Chris has been at an elevation of 28 000 ft. His descent to the airport is at an angle of depression of 3°. At what horizontal distance from the airport must he begin his descent?

EXPLORE THE MATH

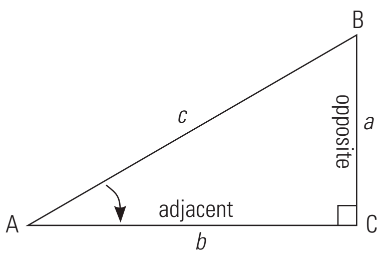
In previous sections, you worked with the sine and the cosine ratios, and in the example above you used them to determine the horizontal distance from the airport at which Chris must begin his descent. However, there is a more direct method for doing this calculation. You can use the **tangent** function. In the diagram shown, the tangent of angle A is $\frac{a}{b}$.

tangent: the ratio of the sides opposite and adjacent to an angle in a right triangle; abbreviated as tan

$$\text{tangent } \angle A = \tan A$$

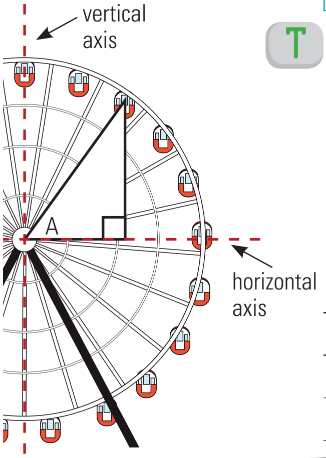
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}}$$



1. Use this definition to calculate the distance from the airport where Chris must begin his airplane's descent.
2. Did you arrive at the same answer as before? Explain any discrepancy.

ACTIVITY 7.5
DRAWING A TANGENT GRAPH



In Activity 7.4 you were asked to consider the horizontal distance of your Ferris wheel chair from a vertical line through the centre as you moved up the circle. Consider, now, the ratio of the vertical distance to the horizontal distance. This ratio would be the tangent of the angle. Using a table like the one provided, and determining the values using your calculator, fill in the values for y equals tan A for values of A that are multiples of 10° as before. Use these values to sketch the graph.

SOLVING TANGENTS

A	*10°*	*20°*	*30°*	[illegible]	*50°*	*60°*	*70°*	*80°*
y = tan A								

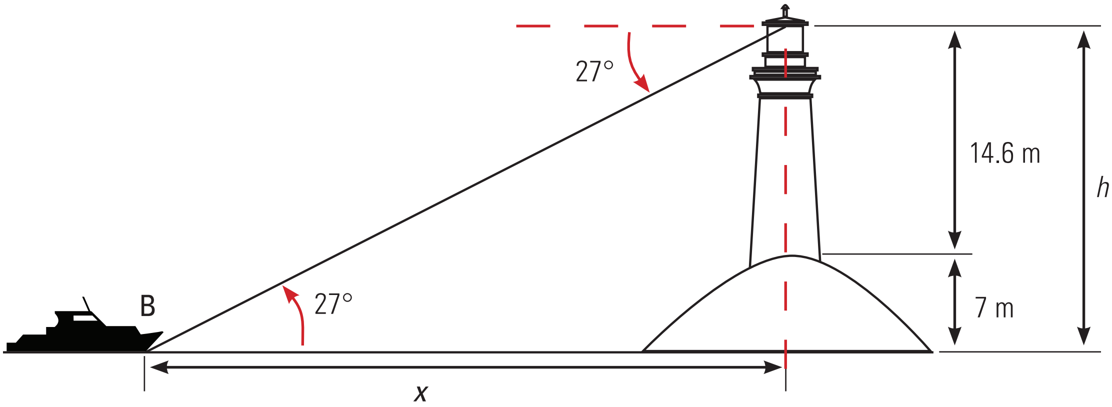
1. You will notice that tan A can be greater than one, whereas sine and cosine cannot be greater than one. Explain why this is so.
2. What do you notice about the value of y or tan A as A increases from 10° to 80°?
3. Does this appear to be a linear graph? Explain how you know.
4. What do you think will happen to the value of tan A as A approaches zero (becomes very small)? Explain.
5. What do you think will happen to the value of tan A as A gets close to 90°. Explain.
6. Extend the graph to verify your prediction.

Example 1

Gull Harbour Lighthouse is located on Manitoba's Lake Winnipeg. Assume the lighthouse is 14.6 m tall and stands 7 m above the surface of the lake. If the angle of depression to a boat on Lake Winnipeg is measured at 27°, approximately how far away from the base of the lighthouse is the boat?

The Gull Harbour lighthouse is located on Lake Winnipeg in Hecla Provincial Park. It was built in 1898.

SOLUTION



$$\tan B = \frac{h}{x}$$

$$\tan 27° = \frac{h}{x}$$

$$\tan 27° = \frac{21.6}{x}$$ Substitute known values into tan formula.

$$x \tan 27° = 21.6$$ Multiply both sides by x.

$$x = \frac{21.6}{\tan 27°}$$ Divide both sides by tan 27°.

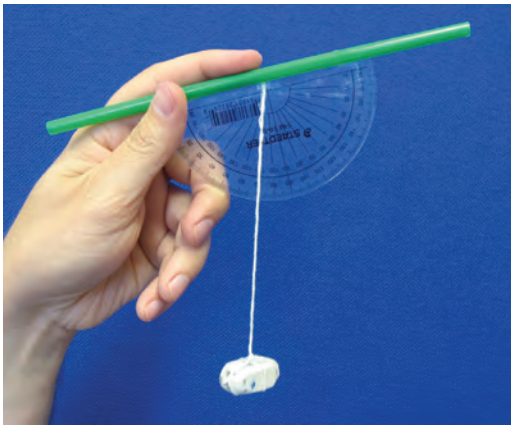
$$x = 42.4 \text{ m}$$

The boat is approximately 42.4 m from the island.

ACTIVITY 7.6
MAKING AND USING A CLINOMETER

To measure the angle of elevation or depression, several different instruments can be used. A clinometer can be improvised using a protractor, a straw, a string with a weight attached, and some tape for attaching them together.

1. Working with a partner, refer to the image shown here and use the necessary materials to help you construct a clinometer.
2. Explain how you will use your clinometer to find the angle of elevation.



You can assemble a basic clinometer using a few common objects.

3. Using your clinometer, with a partner, determine the angle of elevation to the top of at least five buildings in or around your school or other objects whose height you cannot measure directly. Record the angle as well as your distance from the base on a chart similar to the one shown. Use this information to determine the height of each object.

DETERMINING HEIGHTS

object	*angle of elevation*	*tangent of angle*	*distance from base*	*height of object*

SAMPLE

Example 2

A dockworker pulls a light crate (measuring 2 m × 2 m × 2 m) up to the dock using a pulley system. The angle of elevation of the rope is 50°. The man is 2 m from the edge of the pier and the bundle clears the pier by 0.5 m. How close are the pulleys to each other when the bottom pulley is at hand level?

SOLUTION

Draw a simplified diagram.

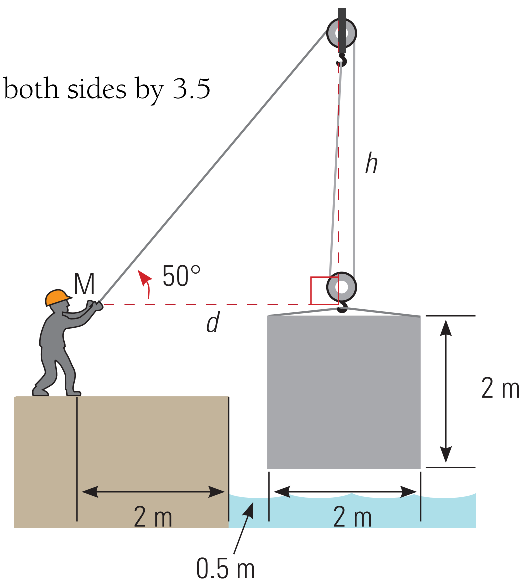
$$\tan M = \frac{h}{d}$$

$$\tan 50° = \frac{h}{3.5}$$

$$3.5 \tan 50° = h \qquad \text{multiply both sides by 3.5}$$

$$4.2 \approx h$$

The distance between the pulleys is approximately 4.2 m.



Mental Math and Estimation

The angle of depression from the top of a cliff to a boat in the water is 52°. What is the angle between the cliff and the line of sight?

BUILD YOUR SKILLS

1. The Lethbridge Viaduct, often referred to as the High Level Bridge, is the longest railway structure in Canada, at a length of 1624 m. It was built in 1908–09, at a cost of $1.3 million. The horizontal distance between the two sides of the coulees is 1620 m. An observer standing at one end notes that the angle of depression to the opposite side of the coulee over which it passes is 4°. What is the difference in elevation between the two sides of the coulee?

Napi, whose name means "old man," is a mythical being in the Siksika Nation's culture. Napi is a respected figure who sometimes personifies the sun. He is also portrayed as a powerful trickster. Alberta's Oldman River, over which the High Level Bridge (pictured here) passes, is named after Napi.

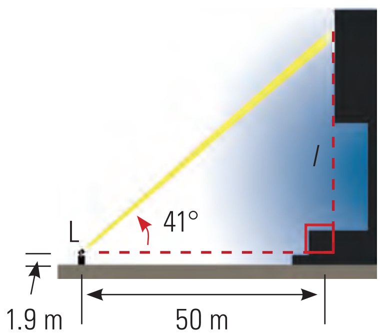
2. Near Estevan, Saskatchewan, Mary and James like to lie on their backs in an open field to watch planes landing. One day as they watch, a helicopter approaches and hovers over a building 1 km away from them. If the angle of elevation is 25°, how high above the ground is the helicopter?

3. Una observed a boat from the top of a 70 m cliff near Atlin, BC. She noted that the angle of depression to the boat was 25°.

   a) The height of the cliff is what fraction of the distance to the boat?

   b) How far is the boat from the base of the cliff?

4. Mike and Lianne are lighting technicians who work for a special events company. Their current job is to set up the spotlights for an outdoor music festival. The performers want to suspend a banner with the name of their troupe directly above the stage, as high as possible, with a spotlight shining on it. Mike and Lianne have only one spotlight left. It is 50 m away from the stage and mounted on a stand 1.9 m high. It has a maximum angle of elevation of 41°. How high will the performers be able to suspend the banner?



5. Johnny is in a hot air balloon 400 m above the ground. He observes his house at an angle of depression of 30°, his school at an angle of depression of 45°, and the soccer field house at an angle of depression of 60°.

    a) Which building is farthest away from Johnny?

    b) How far is the farthest building from a point on the ground directly under Johnny?

    c) How far is the closest building to the point on the ground directly under Johnny?

A hot air balloon can float above the ground because the hot air contained within its fabric is less dense than the cold air outside it. This makes the balloon buoyant.

6. A crime scene investigator (CSI) is investigating a bullet hole in the side of a building. The hole is 2.4 m above the floor and entered the wall at an angle of 83°.

    a) In order to determine how far from the wall the gun was fired, what other information does the CSI need to know?

    b) If the suspect was lying on the ground when he took the shot, about how far from the wall was he?

    c) If his target was 1.7 m tall and 4 m from the wall, would he have been hit?

7. If the angle of elevation to the top of a tree is 30° from a point 12 m from its base, how tall is the tree to the nearest metre?

Extend your thinking

8. For more than 25 years, Whitehorse's *Association franco-yukonnaise* (AFY) has served as a cultural association for the 3550 francophones living in Yukon Territory. Manjula finds that the angle of elevation to the top of the *Centre de la francophonie*, the AFY's meeting place, is 55°. She walks back 100 feet in a straight line from her initial observation point and finds that the angle of elevation is now 42°. Find the height of the centre, to two decimals.

7.5 Finding Angles and Solving Right Triangles

MATH *ON THE JOB*

Betsy is a highway engineer who works in Saskatchewan. A highway engineer is a civil engineer who specializes in the design and constructions of roads. Training for this job involves a great deal of higher mathematics, but at times it is the simple mathematics that proves most useful. For example, when designing a road, Betsy must consider the steepness or grade. Cars and trucks will have to brake excessively while going down a hill that is too steep, or they will slow down too much trying to drive up a hill that is too sleep. There are several different ways in which one can talk about the steepness or grade of a road.

Betsy has been told that the grade of a particular road is 6.6%. This means that the road rises 6.6 vertical units for every 100 horizontal units. For calculation purposes, however, Betsy finds it more convenient to use the angle of elevation.

Using the table of tangents you developed in Activity 7.5, determine the approximate angle of elevation of this road.

When highway engineers design and build highways, they must know the grade, or slope of the road.

EXPLORE THE MATH

T

In the previous sections, you determined the length of the sides of right triangles when you knew the size of an acute angle. However, many times in industry, it is the size of the angle that needs to be determined. Fortunately you can usually do this using your calculator. You will notice that above each of the trigonometric functions the same word appears with what looks like an exponent of negative 1. It is not really an exponent, but indicates something referred to as "the inverse" of the function. This means it "undoes" the function, or that it will give you the angle if you know the value of that particular trigonometric function. To do this, you will need to use the "second function" or "shift" button on your calculator.

Make a table similar to the one shown and use your calculator to fill in the second row, accurate to 4 decimal places.

To fill in the last row, press the 2nd function or shift button on your calculator then the same trig function as in the first row. Enter the value from the second row. Put this answer in the third row.

HINT

Make sure your calulator is in degree mode.

TRIGONOMETRIC RATIOS

trig function	sin 20°	cos 43°	tan 71°	sin 47°	cos 82°	tan 47°	sin 35°	cos 75°	tan 12°
value									
inverse trig function									

T 1. What do you notice about the first and third rows?

2. Can you think of any other times when you "undid" an operation?

Now that you can find an angle given the trigonometric ratio, you can "solve" right triangles. This simply means that you can find all unknown parts of the triangle given two sides or one side and one acute angle of the triangle.

Example 1

Determine the angle indicated in each of the following.

a) A guy wire 8.5 m long is attached 5.7 m from the base of a pole.

b) The angle of depression from a point 10.1 m down a hill if the horizontal distance is 6.9 m.

c) The angle between the side of a house and the glass roof of a small bay window, if the bay window is 75 inches deep and the vertical displacement of the roof is 42 inches.

SOLUTION

a) $\cos B = \frac{\text{adj}}{\text{hyp}}$

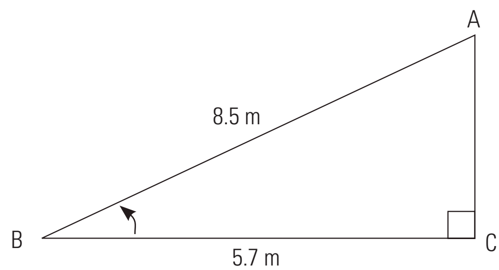
$\cos B = \frac{a}{c}$

$\cos B = \frac{5.7}{8.5}$

$\cos B = 0.6706$

$\cos B = \cos^{-1}(0.6706)$

$\cos B = 48°$



b) $\cos Y = \frac{\text{adj}}{\text{hyp}}$

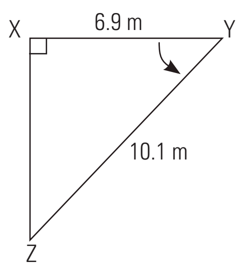
$\cos Y = \frac{z}{x}$

$\cos Y = \frac{6.9}{10.1}$

$\cos Y = 0.6832$

$\cos Y = \cos^{-1}(0.6832)$

$\cos Y = 47°$



c) Find the hypotenuse using the Pythagorean theorem.

$n^2 = m^2 + l^2$

$n^2 = (42)^2 + (75)^2$

$n^2 = 1764 + 5625$

$n^2 = 7389$

$n = \sqrt{7389}$

$n \approx 85.96''$

Next, find the cosine of L.

$\cos L = \frac{adj}{hyp}$

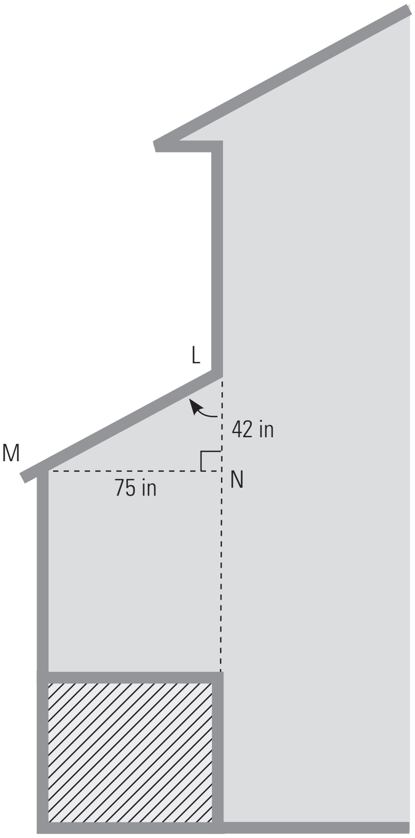
$\cos L = \frac{m}{n}$

$\cos L = \frac{42}{85.96}$

$\cos L = 0.4886$

$\cos L = \cos^{-1}(0.4886)$

$\cos L = 61°$



DISCUSS THE IDEAS

ALTERNATIVE APPROACHES

In Example 1a, *b* could be determined using the Pythagorean theorem, or by using trigonometric ratios and angles. Calculate *b* using both methods. Which method would be more accurate in this case? Why?

Example 2

The Pulaarvik Kablu Friendship Centre in Rankin Inlet, Nunavut, is a place where elders share their skills and knowledge with young people. Tagak is one of the maintenance people who cares for the centre. Her current job is to replace the centre's front steps. She knows that the distance between the ground and the landing is 0.86 m and that the stairs end at a point 1.2 m from the edge of the landing.

a) What will be the angle of elevation from the bottom to the landing?

b) What is the distance between the bottom of the stairs and the landing?

A staircase that is easy to use has wide steps and a gentle grade.

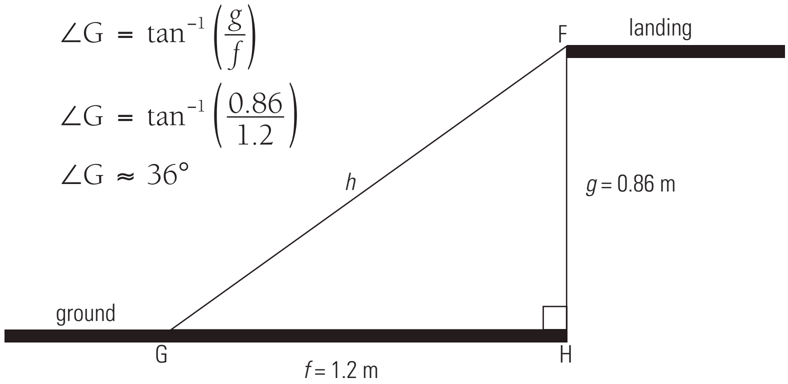
SOLUTION

a) First, find the tangent ratio. Remember that it is standard to keep four decimals in a trigonometric ratio.

$$\angle G = \tan^{-1}\left(\frac{g}{f}\right)$$

$$\angle G = \tan^{-1}\left(\frac{0.86}{1.2}\right)$$

$$\angle G \approx 36°$$



Therefore, the angle of elevation is approximately 36°.

b) Distance h is the hypotenuse of the right triangle. Use the Pythagorean theorem.

$$f^2 + g^2 = h^2$$

$$(1.2)^2 + (0.86)^2 = h^2$$

$$h^2 = 1.44 + 0.7396$$

$$h^2 = 2.1796$$

$$h = \sqrt{2.1796}$$

$$h \approx 1.5\text{m}$$

Therefore the distance from the bottom of the stairs to the landing is approximately 1.5 m.

ALTERNATIVE SOLUTION

$$\angle G = 36°$$

$$\sin G = \frac{0.86}{h}$$

$$\sin 36° = \frac{0.86}{h}$$

$$h \sin 36° = 0.86$$ Multiply both sides by h.

$$h = \frac{0.86}{\sin 36°}$$ Divide both sides by sin 36°.

$$h \approx 1.5 \text{ m}$$

ACTIVITY 7.7
ROCK BAND LIGHTING

You and your partner are working as lighting technicians for a rock band, L & N. The band has asked that you position lights off the floor 10 m from the lead singer. The red, blue, and green lights are to be placed at heights of 10 m, 9 m, and 7 m respectively.

Determine the angle of elevation at which to set each light so it lights up the lead singer.

Lighting technicians find work on television and film sets, or in theatres.

HINT

Draw a diagram to help you visualize the lighting arrangement.

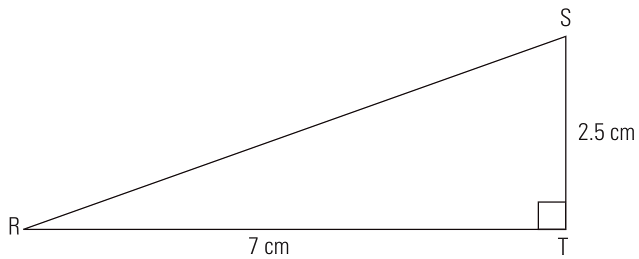
Mental Math and Estimation

What is the tangent of a 45° angle?

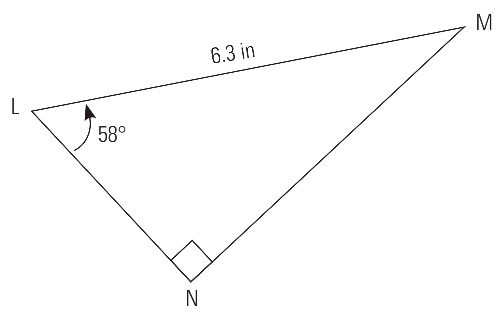
BUILD YOUR SKILLS

1. Emile is cutting pieces of stained glass to replace a window in his local church. He's been given rough diagrams of the pieces he needs to cut, but some of the measurements are missing. Provide the missing measurements for a) and b).

a)



b)



Hiking is a great activity to do with friends.

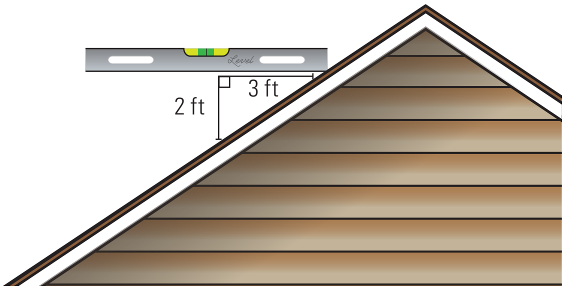
2. Heather is working on a brochure about hiking trails in Prince Albert National Park. One of the hills on a trail has angle of elevation of 15°, with a viewpoint 100 m from its base. Imagine that your walk along the trail from the base to the viewpoint.

   a) How much altitude would you gain?

   b) What horizontal distance would you cover?

   c) What is the grade of the trail, written as a percent?

3. David is a carpenter who is planning to renovate his own house. He is now working on the specification for the roof.



   a) The roof rises 2 feet for every 3 horizontal feet. What should David write in the specification as its angle of elevation?

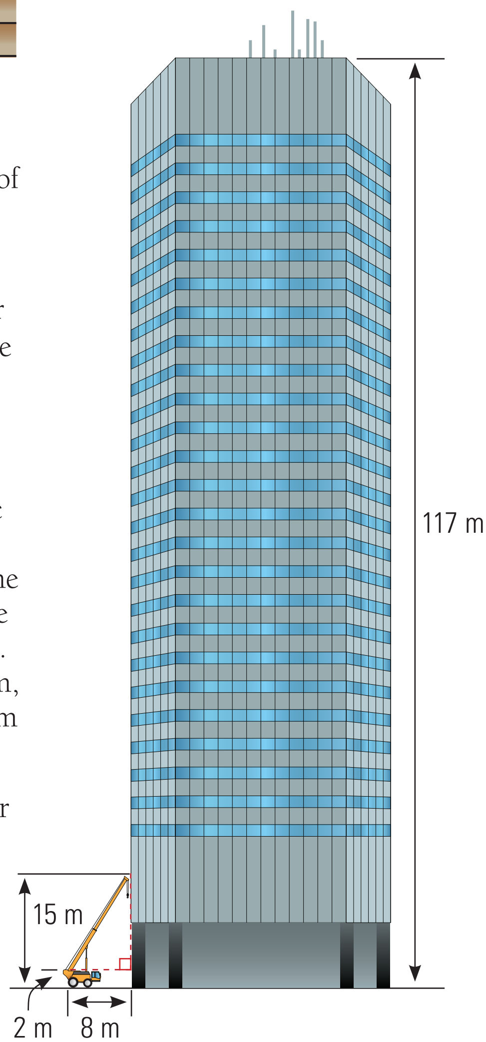
   b) The roof is going to be 20 feet long and rise 12 feet. David wants to cover it with shingles. What should he write in the specification as the roof's total area?

Wheat, barley and canola are some of the commodities that are traded at Winnipeg's Commodity Exchange Tower. The tower opened in 1979.

4. The Commodity Exchange Building in Winnipeg is 117 m tall. Pia is a telescopic crane operator who has been hired to do some repairs to the building's exterior. The base of her crane is 2 m tall and her crane is positioned 8 m away from the building. Pia extends the extension cylinder, or arm, of the crane until its tip is positioned 15 m up the side of the building.

   a) How long must the extension cylinder be to reach the side of the building?

   b) What is the angle of elevation of the crane's extension cylinder?



5. A 15 m ladder is placed against the side of an apartment and reaches a windowsill that is 12 m above the ground.

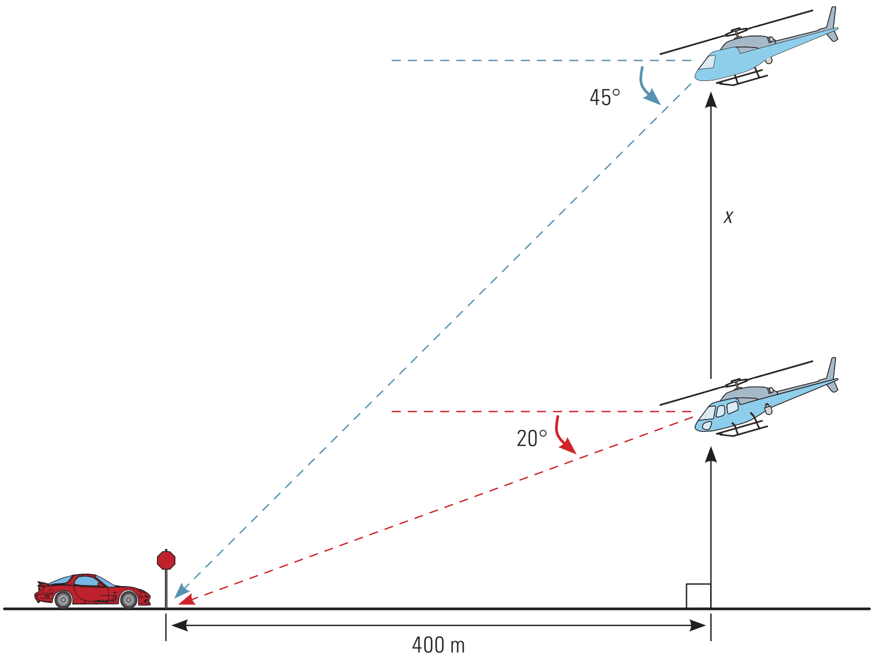
   a) What is the angle of elevation of the ladder?

   b) How far from the base of the apartment is the ladder?

6. Georges is a surveyor working near North Battleford, Saskatchewan. He looks through his theodolite, which is 1.8 m high, and sights the top of a small rock bluff from 15.9 m away. Georges determines that the bluff is 7.2 m tall.

   What is the angle of depression from the top of the bluff to George's theodolite?

7. A traffic helicopter is hovering over the evening rush hour in Vancouver, BC. The traffic reporter observes a traffic accident at an intersection. Using her GPS she determines that the horizontal distance from the helicopter to the intersection is 400 m. She also estimates that the angle of depression from the helicopter to the intersection is 20°. The helicopter begins to rise vertically, and 3 minutes later the reporter estimates that the angle of depression to the intersection is 45°.



   a) How far did the helicopter rise in 3 minutes?

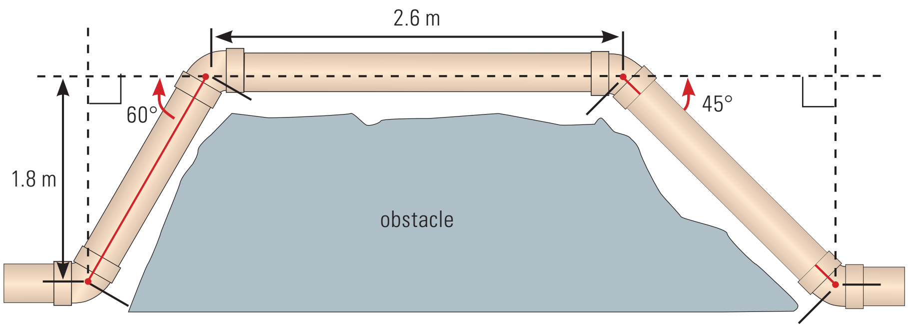
   b) At what speed did the helicopter rise, assuming constant speed?

8. Railway engineers must consider the steepness or grade of the terrain when constructing tracks. Early train tracks in England had very gentle gradients such as 0.05% because locomotive engines were weak. Now, locomotives are much more powerful. One of the steepest non-track railway lines is the Lisbon tram in Portugal. It has a grade of 13.5%. What is the angle of elevation of this track?

Electric streetcars, which run along tram tracks, came to Lisbon, Portugal in 1901. These bright yellow cars connect Lisbon's many neighbourhoods.

Extend your thinking

9. A pipe fitter must install pipes around an obstruction as in the diagram. He uses a 60° elbow at the left top, and a 45° elbow at the right top. The offset in each case is 1.8 m and the horizontal pipe is 2.6 m.



   a) How far is it from the end of one lower pipe to the other? (Round to the nearest tenth.)

   b) How much pipe will he need in total to get around the obstruction? (Round to the nearest tenth.)

   c) Do you think the pipe fitter would have to be more accurate than the nearest tenth of a metre in his measurements? Why or why not?

PUZZLE IT OUT

16 SQUARES

T

In this game, you will work with the Pythagorean theorem and trigonometric functions.

You will be given 16 squares with either an expression that needs solving or a solution on each side.

To play the game, calculate the value of a question on one side of a square and find the corresponding answer or an equivalent expression on the side of another square.

Match corresponding sides. The aim is to have all the pieces form a 4 by 4 square, with all the sides corresponding.

HINT

Some answers match more than one place and some do not match any side at all.

COMPILING YOUR WORK AND PREPARING YOUR PRESENTATION

You now have a package that you could present to a carpenter so that she or he could easily build the staircase that you designed. Your package includes:

- a staircase design drawn to scale, with all measurements shown;
- a list of all materials needed for construction (in imperial units) and the steps involved;
- a written discussion of how you would use the various tools to help you construct your staircase; and
- an organized set of calculations used in designing your staircase.

Good tradespeople often ask questions to clarify the requirements of a job. Your teacher and classmates may also ask you some questions about your staircase design. Be prepared to answer questions about your work, including:

- What is the angle of elevation of your staircase?
- Where are there pairs of similar triangles?
- Where did you use the Pythagorean theorem and the sine, cosine, or tangent functions in your calculations?
- Why did you select this design?

Determining the angle of elevation of a staircase is an important step in its design and construction.

REFLECT ON YOUR LEARNING

TRIGONOMETRY OF RIGHT TRIANGLES

Now that you have finished this chapter, you should be able to

- ❒ use the Pythagorean theorem to find the missing side of a right triangle;
- ❒ determine which of the three basic trigonometric functions applies to a given situation;
- ❒ apply the three basic trigonometric functions to find a missing side or angle of a right triangle;
- ❒ determine workplace applications of trigonometry.

You will also have finished a chapter project that allowed you to apply these skills in a practical way to a real-world task.

PRACTISE YOUR NEW SKILLS

1. Are the triangles with the following lengths of sides right triangles? Show how you know.

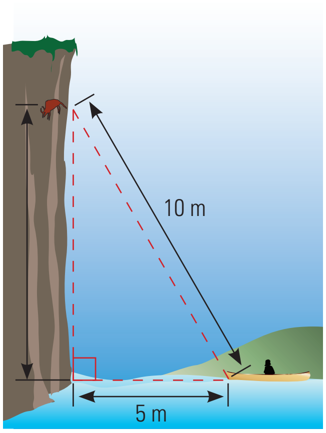
   a) 6 cm, 12 cm, 18 cm

   b) 4 ft, 5 ft, 9 ft

   c) 16 cm, 30 cm, 34 cm

   d) 25″, 60″, 65″

   e) 0.5 m, 0.12 m, 0.13 cm



2. Renée is on a canoe trip on Tramping Lake, Manitoba. She chose to visit the area so that she could see the pictographs of snakes, birds, and other animals that First Nations people drew on the lake's cliffs. They are estimated to be between 1500 and 3000 years old. Renée is in her canoe, 5 m away from the base of the cliff on which some pictographs are drawn. The distance from the tip of her canoe to the pictographs measures 10 m. How high up the cliff are the pictographs?

3. Sarbjit can walk to his school in Prince Rupert, BC, on the road or he can cut diagonally across the field. The field is 150 m by 90 m.

   a) How much distance does he save if he takes the shortcut?

   b) Why do you think he might not want to take the shortcut?

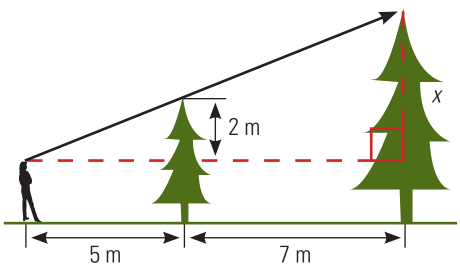
A road's steepness is measured by grade. For example, from Skagway to White Pass, the grade of the South Klondike Highway is 11%.

4. The grade of a road averages 5° to the horizontal.

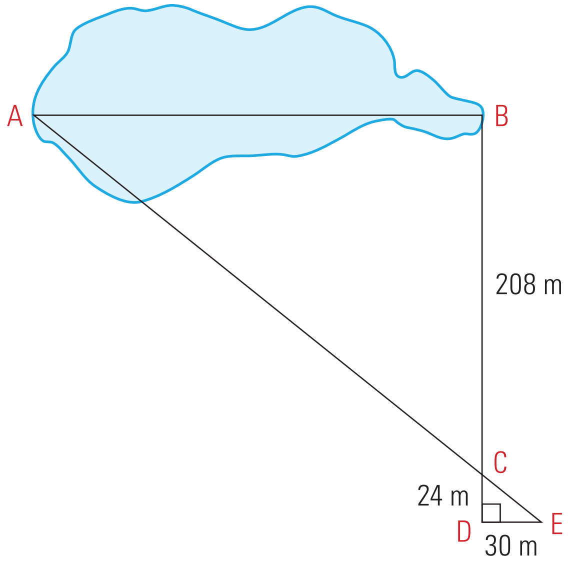
   a) If you move 2 km along the road, what is your change in altitude?

   b) If the grade were decreased, would the change in altitude be more or less? Explain your answer.

5. Marcy, who is 1.5 m tall, looks up towards a tree directly in her line of vision. Beyond the tree she sees the top of another tree. If the first tree is 2 m taller than Marcy and 5 m away from her and the second tree is 7 m beyond the first, how tall is the second tree?

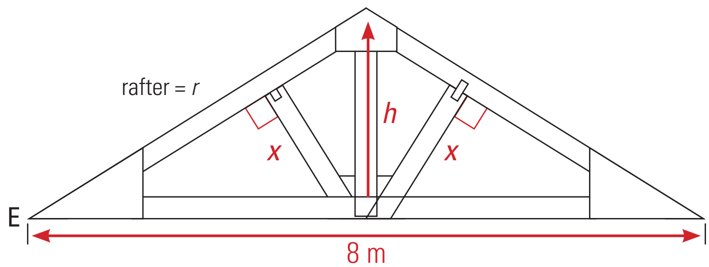


6. Wei Lee is trying to determine the length of the lake in the diagram shown. How long is it?




This view of Bennett Lake shows a bridge in Carcross, Yukon. Bennett Lake is well-known by boaters as a lake that can quickly become wavy and difficult to navigate when winds are strong.

7. The angle of elevation of a rafter is 32°. The width of the structure is 8 m.



a) Find the vertical height (h).

b) How long are the support pieces (x)?

c) What is the length of the rafter (r)?

8. To determine how much gravel has been removed from a gravel pit, a surveyor must determine the depth of the pit. Using a theodolite and electronic distance measurement equipment (EDM), he determines that the distance down the slope to the bottom of the pit is 300 m and the angle of depression of the slope is 46°.

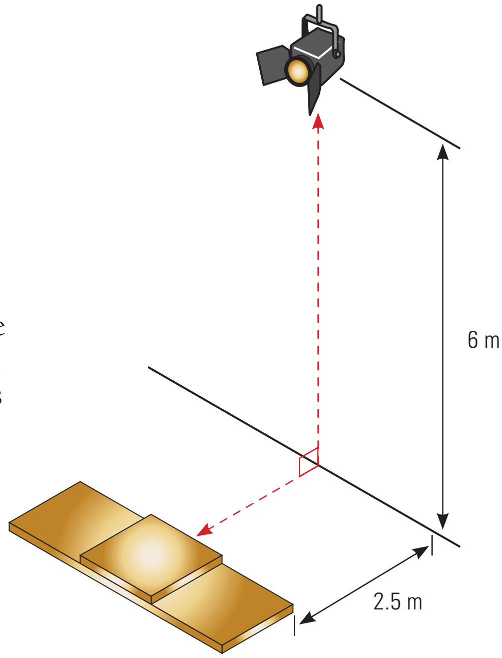
a) How deep is the gravel pit?

b) How would knowing the depth help the surveyor determine the amount of gravel removed? What else would he have to know?


Saskatchewan has several active gravel pits. The extracted gravel is used for road construction and maintenance.

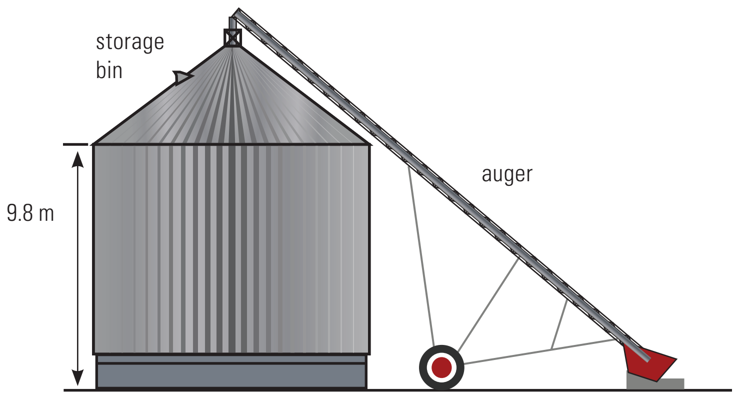
At the Paralympic Games, the fastest athletes in wheelchair racing have reached speeds of more than 30 kilometres an hour.

9. Nicolas is working as a lighting technician at the Paralympic Games. One of his jobs is to position spotlights on a podium during the medals ceremony for wheelchair racing. The lights are fixed on a backdrop that is 6 m tall. They must shine on the spot where the medalists are located, 2.5 m from the base of the backdrop. By how many degrees must the light directly behind the medalists be depressed?



Extend your thinking

10. The cylindrical part of a grain storage bin is 9.8 m high and 7.3 m in diameter. The grain must enter through a hole on the centre of the top.

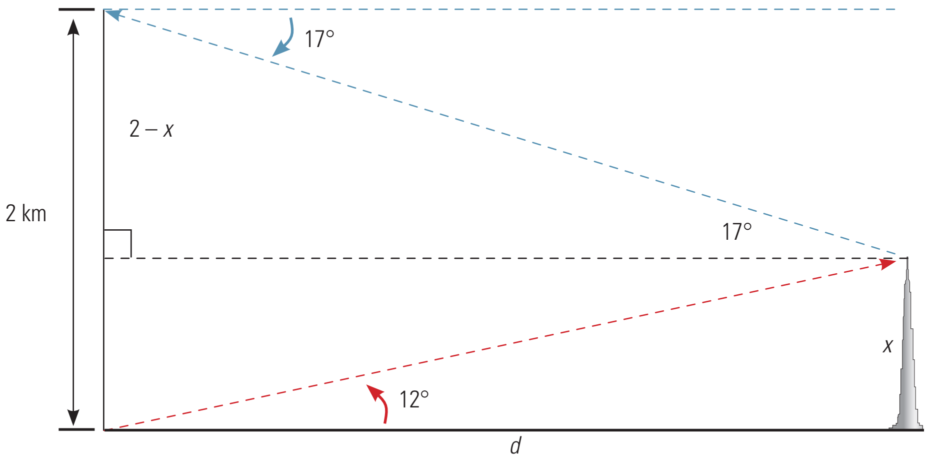


a) Sketch the storage bin and the auger and label the dimensions and the angle of elevation.

b) What is the shortest auger that can be used to fill the bin if the angle of elevation must be no more than 40°?

11. What is the grade of a flat road (one that does not have a hill)?

12. The world's tallest building, the Burj Dubai, was scheduled for completion in 2009. From a point 2 km above the ground, the angle of depression to the top of the Burj is approximately 17°. The angle of elevation from a point on the ground directly below this and to the top of the building is approximately 12°.



Determine the approximate height of the Burj.

13. Katarina says she does not need a calculator or table of trigonometric functions to solve right triangles that have acute angles of 45° or 60°. Explain (using diagrams) how she might do this.

The world's tallest building, the Burj Dubai, also contains the world's tallest service elevator. It has the capacity to carry 5500 kg.