Chapter 6

Similarity of Figures

GOALS

When you draw an image of something that is larger or smaller than the original, you create an image that is similar to the actual object. Architects, engineers, and construction workers use blueprints and plans that are smaller than the real buildings they create. Microbiologists and computer engineers use diagrams that are larger than the real-life items they study or make. Toy and furniture makers create scale drawings of toys and furniture to help customers assemble these items from parts.

In this chapter, you will learn what makes two figures similar to help you

- identify similar polygons;
- identify images that are *not* similar to the original diagrams;
- draw a polygon similar to another polygon; and
- understand what characteristics make triangles similar.

KEY TERMS

- congruent
- convex polygon
- corresponding angles
- corresponding sides
- proportional
- similar figures

START TO PLAN

PROJECT OVERVIEW

Have you ever been to a community games room? A community games room may be part of a youth centre, a friendship centre, or at a seniors' facility. Some common equipment in a community games room includes card tables, air hockey tables, shuffleboards, foosball tables, pinball machines, and chess or croquinole boards.

For this project, you will be part of a committee that uses scale drawings to plan and design a new community games room. The proposed room will have an area of about 70–80 square metres. Your sketch will be 50 times smaller than the actual room and will look like a floor plan. In your room, you must include at least four pieces of community game equipment. You can include any other items you would like in your community games room.

A community games room is a good place to relax, visit with friends, or work on your pool or dart skills.

GET STARTED

To begin your project, start thinking about how you will arrange the items in your community games room and how you will create the floor plan. When people build a new building, they think about the following questions.

- Who will use the room, and at what times of the day?
- What games will you include for the community games room?
- Will you need places for people to sit?
- Is there enough space for people to move around?
- What other elements can you put in your community games room to make it original and interesting?
- How will you present your design? Will it be on a poster, or perhaps in a virtual tour created with computer-assisted design software? Will you colour in the community games room to show your colour choices?

FINAL PRESENTATION CHECKLIST

Your final presentation will be displayed at a community meeting and members will decide on their favourite design. Your presentation should include:

- A sketch or computer-generated visual tour of your community games room.
- A separate sheet of paper with your calculations. Include the actual measurements of each piece of community game equipment or furniture in your room, as well as the sizes on your scale diagrams and an explanation of how you calculated those sizes.

6.1 Similar Polygons

MATH *ON THE JOB*

Paul Messier is a francophone cabinetmaker who lives in Calgary, Alberta. He designs, builds, and installs fine cabinets in kitchens, bedrooms, and living spaces as part of renovations that people do to improve their older homes. In his work, he creates scale diagrams to help the clients visualize what the new cabinets will look like in the old space. The plans show exactly what the new installation will look like, because he draws them to scale, or with the exact same proportions and angles. He is very good at drawing plan views of three-dimensional objects, and he is careful to always duplicate the angles on the diagram to be exactly the same as the angles that will be in the actual space.

Paul plans to create a bank of cabinets that will be 10 feet long and 3 feet high. He is creating a scale diagram on graph paper with measurements that are one-twelfth of the real-life measurements. What will the size of the cabinets be on the diagram? How can he be certain that they will look similar to the real cabinets?

Paul Messier uses scale diagrams in his work as a cabinetmaker.

EXPLORE THE MATH

Different-sized toolboxes help tradespeople organize their equipment.

When two figures are similar, one figure has dimensions (length and width) that are proportional to the other figure. A set of toolboxes, such as the one shown on the left, can be placed inside each other to save space (and therefore shipping costs) when shipping the toolboxes to retailers. These toolboxes may have dimensions that are proportional; each box has length and width dimensions in the same proportion, or ratio, as the next larger size.

If the largest toolbox has a length of 600 mm and a width of 300 mm, and each smaller box is $\frac{4}{5}$ the size of the box before it, what would be the length and width of the second and third toolboxes? What would be the length and width of the fourth toolbox in the series?

DISCUSS THE IDEAS

SIMILAR FIGURES

Similar figures have the same shape but are usually different sizes. Starting at the top left corner, label the four corners of your classroom board A, B, C, and D. Now take a piece of writing paper and hold it up. Is it the same shape as the board? Label its corners A′, B′, C′, and D′, again starting at the top left corner. Angle A and angle A′, both 90° angles, are **congruent**. They are also both at the top left corner of the shapes. In similar figures, **corresponding angles** refers to two congruent angles in separate similar shapes that occupy the same relative position.

similar figures: figures that have the same shape

corresponding angles: two angles that are congruent and occupy the same relative position in similar figures

congruent: the same, or equal, shape and size

corresponding sides: two sides that occupy the same relative position in similar figures

For the paper and the board to be truly similar figures, their **corresponding sides** would have to be proportional, or in the same ratio all around. Measure the top of the blackboard (side AB), and the top of your piece of paper (side A′B′). Now measure the left side of each figure. The figures are proportional if the following is true.

$$\frac{AB}{A'B'} = \frac{AD}{A'D'}$$

The symbol ~ can be used to mean similar. If rectangle ABCD is similar to rectangle EFGH, this is written as ABCD ~ EFGH.

HINT

A' is another way to name a point, and is pronounced "A prime."

Example 1

Debbie works for a magazine and is adjusting the dimensions of some images to fit into an article. She is not certain that she has changed the length and the width by the same amount and wants to find out if she has used the same ratio for each direction.

1. Look at the original photo on the left and the changed image. What is the ratio of the lengths (the horizontal measurement)?
2. What is the ratio of their widths (the vertical measurement)?
3. Has the same ratio been used in both cases?
4. Are the two photographs similar to each other? Explain why.

SOLUTION

1. Compare the length of the enlargement with the length of the original. The enlargement is 6 cm long and the original is 4 cm long. The ratio of the lengths is 6:4 or 3:2.

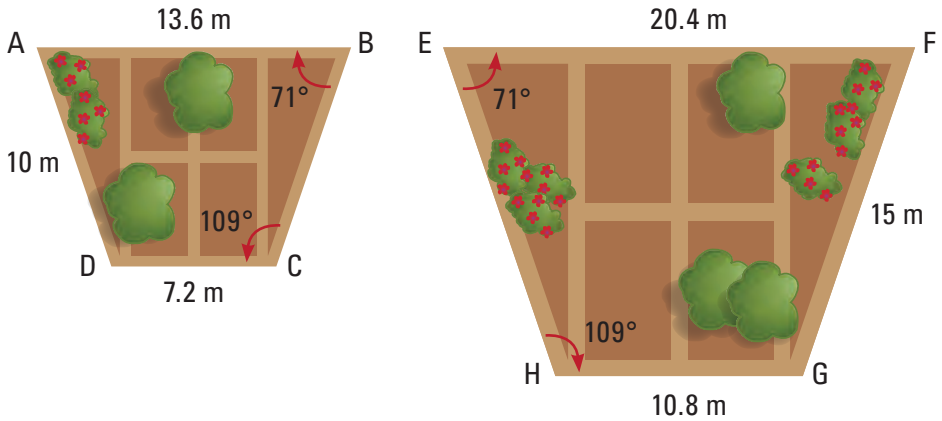
2. Compare the width of the enlargement with the width of the original. The ratio of the widths is 3:2.

3. Yes, a ratio of 1.5 has been used in both cases.

4. The two photographs are similar to each other because the angles are congruent and the ratios of the corresponding sides are equal.

Example 2

A family built a home in Regina, Saskatchewan and designed a backyard garden in the shape of a trapezoid. Over the years, they loved the design they had created, and when they moved to a larger property in Avonlea, Saskatchewan, they decided to use the same shape for their new backyard garden. They mapped what they did in the first garden, then created a map of a trapezoid that would fit in the new space.



a) If the family uses the plan they have sketched for the new garden, will the shape of their two backyards be similar to each other?

b) Do you think laying out the trees and flowers in an exactly similar way makes the most sense? Explain why or why not.

SOLUTION

a) For two figures to be similar, the measures of the corresponding angles need to be congruent and the measures of the corresponding sides need to be proportional.

The angles in the first figure are represented by the letters A, B, C, and D. The angles in the second figure are represented by the letters E, F, G, and H.

- Angle A in the first figure corresponds with angle E in the second figure because they are in the same positions. Angle A is congruent to angle E because they both have measures of 71°.
- Angle B corresponds to angle F and they are congruent with measures of 71°.
- Angle C corresponds to angle G and they are congruent with measures of 109°.
- Angle D corresponds to angle H and they are congruent with measures of 109°.

Determine the length ratios of the corresponding sides.

$$\frac{EH}{AD} = \frac{15}{10}$$
$$\frac{EH}{AB} = 1.5$$

$$\frac{EF}{AB} = \frac{20.4}{13.6}$$
$$\frac{EF}{AB} = 1.5$$

$$\frac{HG}{DC} = \frac{10.8}{7.2}$$
$$\frac{HG}{DC} = 1.5$$

There are many names for gardens designed in geometrical shapes. For example, a formal French garden, or jardin à la française, has shrubbery and plants cultivated to form triangles and diamonds, and arranges paths in geometric patterns.

Since the measures of the corresponding angles are congruent and the measures of the corresponding sides are proportional, we know that ABCD is similar to EFGH. We can write this as ABCD ~ EFGH.

b) Making the new garden in exactly the same proportions, but larger, may not be the most sensible design. This could result in some vegetables or flowers having too much area, and it could also make it harder to reach to the centre of the garden beds to do the weeding.

Mental Math and Estimation

A poster has a length of 3′1″ and a width of 2′4″. Kerri plans to create a reproduction of the poster by enlarging each dimension by a scale factor of 2.

Estimate the length of framing wood she will need to build a frame for the enlarged poster.

HINT

When you estimate how much material you will need for a project, you should always overestimate instead of underestimate.

Example 3

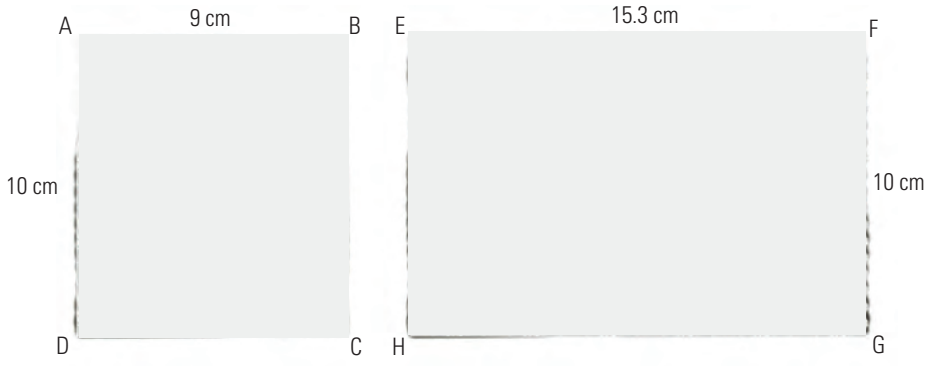
A tissue company creates tissues for their dispensers in the shape of rectangles. Each tissue has a length of 9 cm and a width of 10 cm. They want to increase the length of their tissues by a factor of 1.7 but keep the same width so they can still fit in their dispensers.

a) Draw a sketch of the original tissue with its dimensions and a sketch of the new tissue with its dimensions.

b) Do the figures in your sketches represent similar figures? Why or why not?

SOLUTION

a)



The tissues are drawn to scale because their actual dimensions restrict drawing these figures. The original tissue has dimensions of 10 cm by 9 cm. The new tissue will keep the same width, but will have a length that is 1.7 times longer.

$9 \text{ cm} \times 1.7 = 15.3 \text{ cm}$

So the new tissue will have dimensions of 10 cm by 15.3 cm.

b) All of the angles in ABCD that correspond with the angles in EFGH are congruent and are equal to 90°.

But since only the corresponding lengths of ABCD and EFGH were increased and the widths were not increased by the same amount, the ratios of corresponding sides are not equal. Therefore, these figures are not similar.

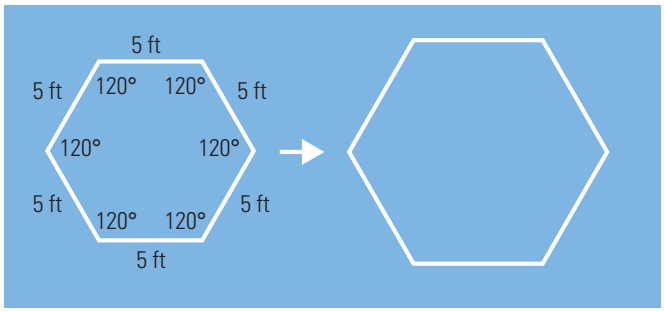
ALTERNATIVE SOLUTION

To determine if the tissues represent similar figures, you can create a similar version of the original tissue using the factor 1.7 to increase the dimensions. The length of the new tissue would be 9 times 1.7, which is 15.3 cm. The width of the new tissue would be 10 times 1.7, which is 17 cm.

Since the width of the new tissue is 10 cm and not 17 cm, the two figures will not be similar.

ACTIVITY 6.1
ENLARGING BLUEPRINTS

Three summer students were working together on a landscaping project in Winkler, Manitoba. Their boss asked them to enlarge the blueprint before they lay out the hexagonal gazebo on the lawn. The sides need to be doubled, while keeping the shape exactly the same.



After the students discussed the problem, they each suggested their own strategy for drawing out the larger hexagonal blueprint.

Student 1: Double the length of each side and double the measure of each angle.

Student 2: Keep all of the lengths the same and double the measure of each angle.

Student 3: Keep all of the angles the same and double the length of each side.

In a group of three people, have each person choose one of the strategies above. Show what shape each strategy produces and explain why the strategy works or does not work.

BUILD YOUR SKILLS

1. Suppose you create a similar polygon using each of the given scale factors below. How will the side lengths and angle measurements compare to those of the original?

   a) doubled

   b) tripled

   c) halved

2. Maxime has entered a contest to design a logo for a local basketball team. After she drew her logo below, she found out that for it to meet the contest criteria, she had to increase its dimensions.

An all-star basketball player needs good shooting, defense, and ball-handling skills.

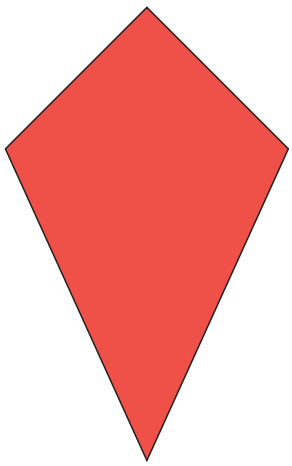
Which of the following rules could she use to create a similar figure that is an enlargement of the original?

   a) Use side lengths that are one-third the length of the original figure and keep the angle measurements the same.

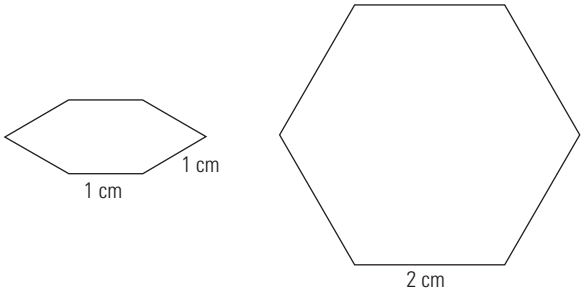
   b) Use angle measurements that are double the size of the original figure and keep the side lengths the same.

   c) Use side lengths that are triple the size of the original figure and keep the angle measurements the same.

   d) Use side lengths that are 8 cm longer and keep the angle measurements the same.

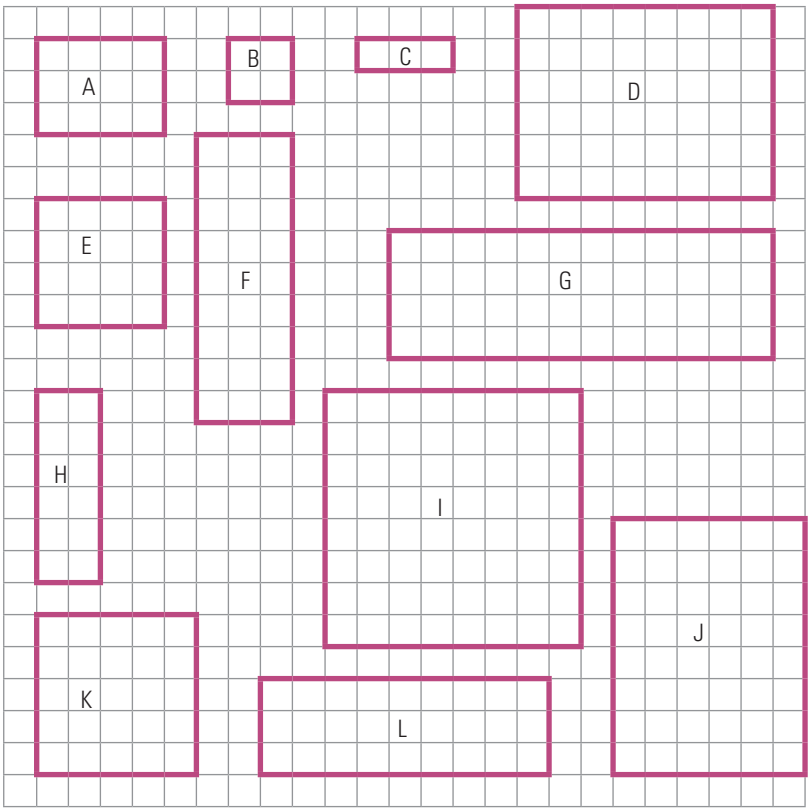


3. Duane drew the two figures shown.



Duane thinks the two shapes are similar because the side lengths of the larger figure are twice as long as those of the smaller figure. How would you explain to Duane why the two figures are not similar?

4. Talise created a blueprint of her garden with rectangular plots to plant different vegetables. She created 3 different plots, A, B, and C. She then created similar figures of each of the 3 plots. Identify the 3 sets of rectangular plots that create similar figures in the blueprint below.



Building simple raised beds allows gardeners to care for different plants in different ways. For example, carrots grow better in sandy soil than celery.

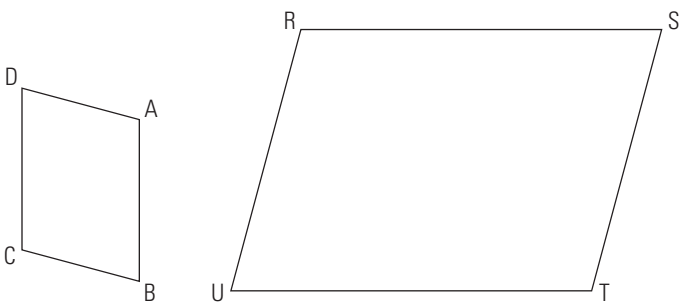
5. Aaliyah works in a custom framing shop. She is given two paintings to frame. The customer wants a 20-centimetre wide mat (a decorative cardboard border surrounding the painting inside the frame) around each painting. For each painting, compare the dimensions of that painting with the dimensions of the framed painting. Are the dimensions similar? Justify your response.

A.

B.

Swimming is a great way to strengthen muscles and improve stamina.

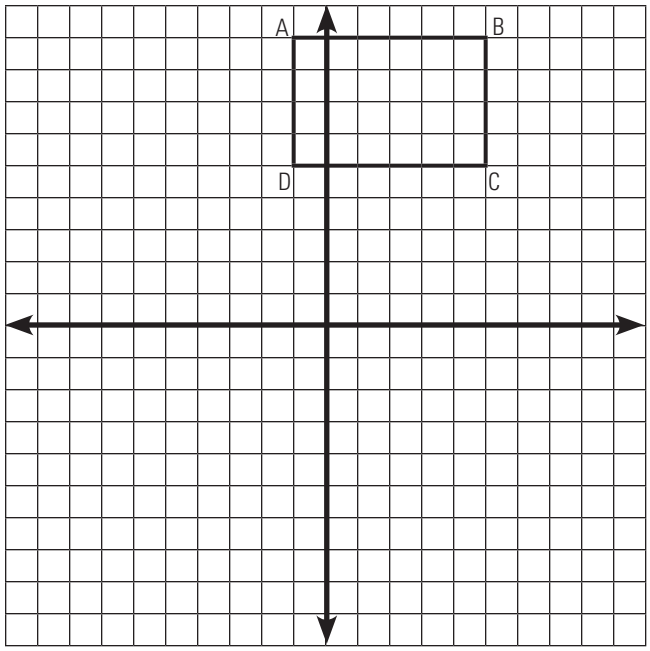
6. Renée is designing a pool for her client's backyard. She drew a sketch of the pool, which she labelled ABCD below. If the actual pool is represented by the similar figure RSTU, help Renée determine the corresponding angles and corresponding sides to make sure that she adds the right finishing details to each side.



Extend your thinking

7. As a builder and contractor, George must determine and map out the placement of the buildings he will construct in relation to each other. He does this before he begins each new job.

   George has been hired to build a greenhouse and a retail building where the plants can be sold. George draws a rectangle on the grid below to represent the retail building. The greenhouse will be double the size of the retail space. He starts drawing the greenhouse so that point E corresponds with point A. If he plots point E at (-7, 2), what would be the other ordered pairs that represent the greenhouse (points F, G, and H)?



6.2 Determining if Two Polygons Are Similar

MATH *ON THE JOB*

Quinn Keast-Wiatrowski is a junior designer working for Relish Design Studio in Winnipeg, Manitoba.

Quinn is currently completing a graphic design diploma from Red River College. "I primarily use math for ratios and measurements," says Quinn. He cites the "golden section," a mathematical ratio that is often incorporated into design work like websites and print advertisements. Quinn also uses grids as a design tool. "By using a grid as a base structure to place content on, I'm able to create something that feels organized, balanced, and is easy to read and understand," he says.

Quinn uses a computer to create his designs in any size he requires.

As part of a branding strategy, Quinn designs a logo for an electronics store. He produces a diamond-shaped logo that will appear on the store's business cards, fliers, and website. When the logo appears on the business card, the two line segments composing the top of the diamond measure 21 mm and the line segments composing the bottom measure 26 mm. The logo must be made one third larger when it appears on the website and two thirds larger when it appears on fliers.

Quinn must calculate how big the logos will be, so that he can design the website page and flier in relation to it. Approximately what size will the logos on the website and the fliers be? Do your calculations to two decimal points and round your final answer up to a whole number.

EXPLORE THE MATH

A map is an example of a drawing that is a reduced image of the original. Most maps indicate a scale, which is the factor used to reduce the size of the original. Therefore, a correct map is an example of a perfect scale drawing.

A scale factor is written in this form:

new measurement:old measurement.

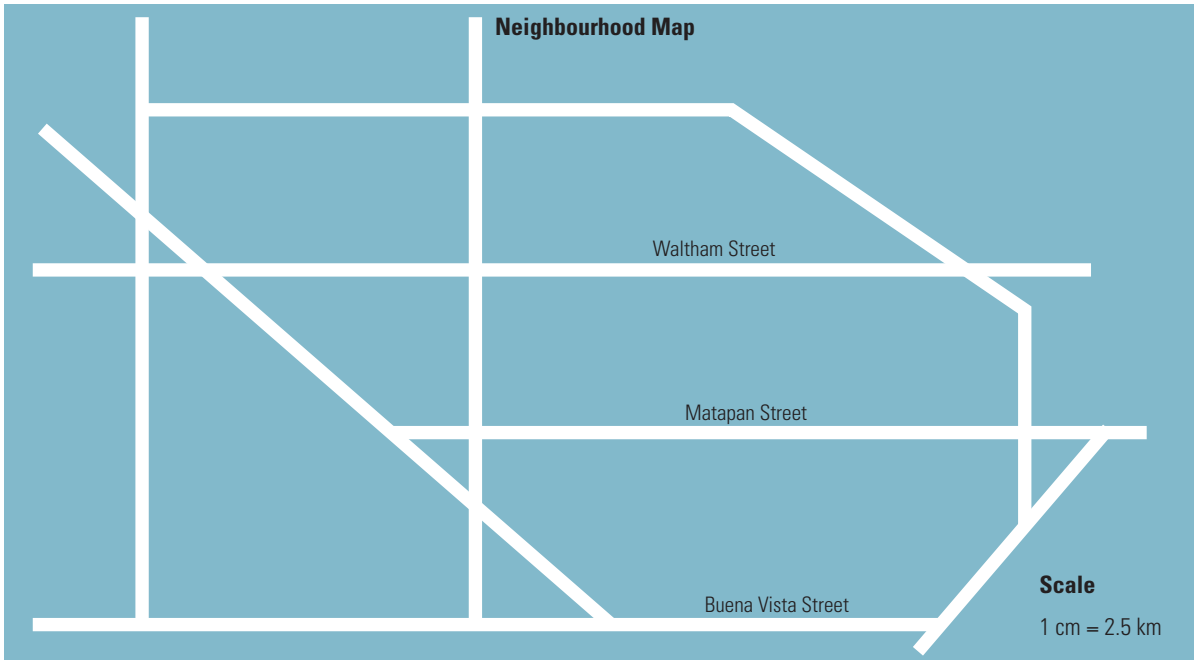
This means that when you are asked to determine what scale factor was used to create a second figure, you need to compare a measurement from the new figure to the corresponding measurement in the old figure.

Like the scale on a map, a scale factor can be used to reduce or enlarge any figure. This is the number you multiply each side length by to create a similar figure.

DISCUSS THE IDEAS

SCALE FACTORS AND MAPS

The scale on the neighbourhood map on the next page shows that 1 cm on the map represents an actual distance of 2.5 km.



1. On the map, Waltham Street has a length of 14 cm. Using the scale, what would be the length of the actual street?

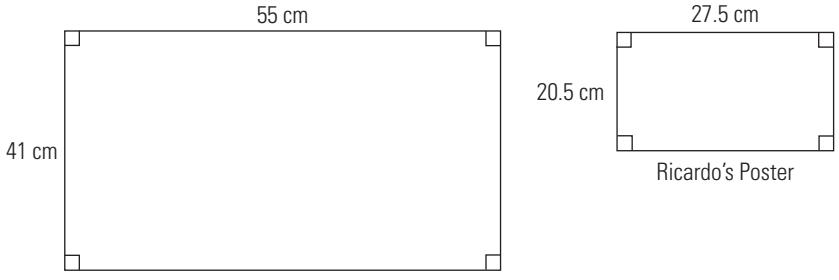
2. Matapan Street has an actual length of 25 km. Show your work to find the right length of the street on the map. Use a ruler to see if your calculation is correct.

3. You have probably seen maps that are not proper scale drawings. What was the purpose of these maps? What advantages did the mapmakers get by distorting the scale?

DISCUSS THE IDEAS

SCALE FACTORS AND POSTERS

Ricardo was given the poster on the left and was asked by his employer to reduce the poster by a factor of $\frac{1}{2}$. Ricardo figured out the size of the new poster and drew the rectangle on the right.



1. Do you think Ricardo correctly interpreted what his employer asked him to do?
2. What do you notice about the surface area with this reduction?
3. Do you think there could be more interpretations of what his employer asked? Draw a figure for each interpretation you can think of.

Example 1

This scale model shows the renovations of Edmonton's Cité Francophone. The building is a gathering place for those wishing to learn about and celebrate the francophone culture and language.

The Cité Francophone, Edmonton's francophone centre, is undergoing renovations. To display what the renovations will look like when completed, a scale model of the future centre was built.

a) If a building is reduced to 10% of its original size in order to build a scale model, what scale factor was used?

b) If a room measuring 16 feet by 12 feet in real life is reduced to 10% of its original size for its reproduction in the scale model, what would the room's new dimensions be?

c) If a rectangular hallway in the Cité Francophone were reduced in size by 60%, what would be the angle measurements of the new hallway?

SOLUTION

a) The percentage 10% can also be written as 0.10, so the scale factor used was 0.10.

b) The percentage 10% can also be written as 0.10 or $\frac{1}{10}$. You can multiply each side length by 0.10 or $\frac{1}{10}$ to get the new dimensions.

$0.10 \times 16 = 1.6$ ft
$0.10 \times 12 = 1.2$ ft

The new dimensions of the reduced room would be 1.6 ft by 1.2 ft.

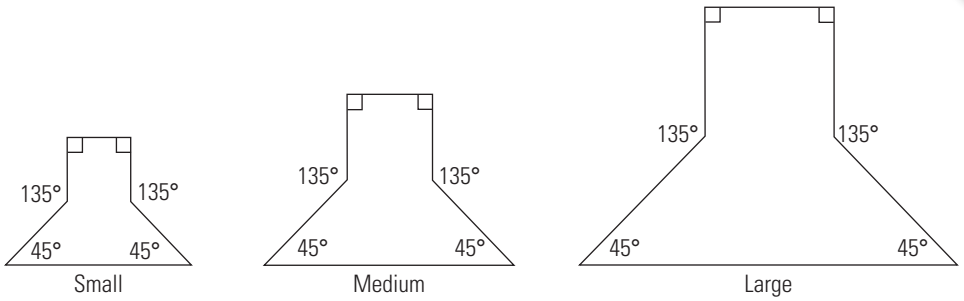
c) Enlarging or reducing the size of a figure only affects the measures of the side lengths, not the angles. If the angles were changed, the shape would no longer be the same. So, the angle measurements of the new rectangle will still be 90°.

Example 2

Althea is designing cedar hats to sell at her shop at Skidegate on Haida Gwaii. She needs to make three sizes of the same hat. She sketches out the designs for each hat using their angle measurements.

a) How do you determine if the three sketches are the same shape?

b) Does this mean they are similar figures? Do you need to know the side length measurements? Explain your answer.



First Nations people on B.C.'s coast, such as the Haida, weave traditional hats from split spruce roots and cedar bark. The hats were used as trade items with other First Nations people. Hat weavers often incorporate images of animals such as frogs and ravens into their work.

SOLUTION

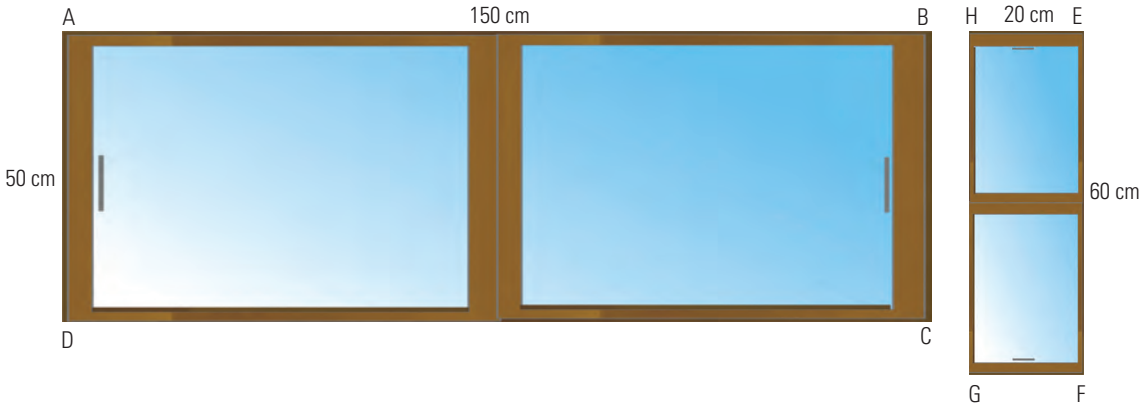
a) Since the sketches all have the same corresponding angle measurements, each sketch is the same shape as the other two.

b) It is impossible to determine if the shapes represent similar figures without knowing the side lengths. It is possible that the enlargements are relatively wider or narrower than the original.

DISCUSS THE IDEAS

SCALE FACTORS IN SIMILAR FIGURES

Drew is a building contractor who sketched a window, labelled ABCD for a house. He wants to install a smaller but similar window at his next job. He drew the similar figure EFGH to represent the new window.



If you are not told which angles are corresponding, you should follow the letters alphabetically. That means that A corresponds to E, B corresponds to F, C corresponds to G, and D corresponds to H. That also means that the corresponding sides are AB corresponds to EF, BC corresponds to FG, CD corresponds to GH, and AD corresponds to EH.

What is the scale factor Drew used to draw the smaller figure? In a group, discuss how to solve this question.

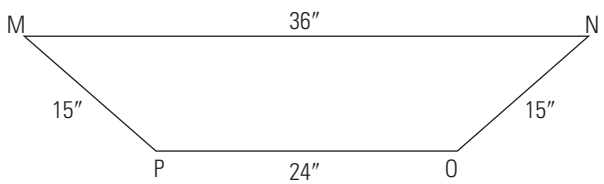
Mental Math and Estimation

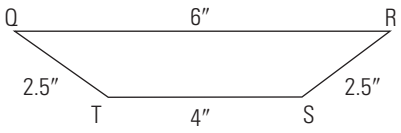
Determine the value of x in each proportion without the use of a calculator.

a) $\frac{x}{10} = \frac{30}{100}$ b) $\frac{50}{x} = \frac{45}{9}$ c) $\frac{72}{x} = \frac{108}{12}$

Example 3

Kirsten is a boat builder in Comox, BC. Figure MNOP is the pattern for the transom, or back end, of a simply plywood rowboat that she makes. To make a model of the rowboat as a gift for her brother, Kirsten sketches pattern QRST.





Is the model similar to the full-sized boat? If so, what scale factor is used to build the model?

SOLUTION

To determine if the figures are similar, the ratios of the corresponding side lengths of the two figures should be equal to each other. Determine the corresponding ratios of the side lengths.

$$\frac{MN}{QR} = \frac{36}{6}$$
$$\frac{MN}{QR} = 6$$

$$\frac{NO}{RS} = \frac{15}{2.5}$$
$$\frac{NO}{RS} = 6$$

$$\frac{OP}{ST} = \frac{24}{4}$$
$$\frac{OP}{ST} = 6$$

$$\frac{MP}{QT} = \frac{15}{2.5}$$
$$\frac{MP}{QT} = 6$$

A boat, such as these punt boats, can be built with basic tools such as a jigsaw, hand plane, and hammer.

Since the ratios of the corresponding sides are all equal to 6, the figure is similar.

The scale factor for the model is

$$\frac{\text{New measurement}}{\text{Old measurement}} = \frac{\text{model}}{\text{full-size}}$$

Since the figures are similar, pick any pair of corresponding sides, for example MN and QR.

$$\text{scale factor} = \frac{QR}{MN}$$
$$\text{scale factor} = \frac{6}{36}$$
$$\text{scale factor} = \frac{1}{6}$$

ACTIVITY 6.2
SCALING MÉTIS SASHES

Métis people use the sash as a symbol of cultural identity. It is also a practical item that can be used as a bag strap or rope.

Métis sashes vary in colour and length. Different colours on the sash represent different information. Green can represent prosperity, while blue and white represent the Métis flag. Colours can also represent different information about the individual wearing the sash and his or her family. Métis sashes are about 3 metres long, but men's sashes tend to be longer, sometimes as long as 12 feet, and wider than women's sashes.

In pairs or groups, discuss how to solve the following questions.

1. A man's sash measures 3.5 m long and a woman's sash measures 2.8 m long. In comparison to the man's sash, by what scale factor has the woman's sash been reduced?

2. Métis sashes are made using a technique called finger weaving. Elizabeth starts weaving a man's sash that is 10 cm wide. She realizes that the sash is too narrow and starts again, this time weaving the sash so that it is 16 cm wide. By what scale factor did she enlarge the width of the sash?

ACTIVITY 6.3
APPLYING RATIOS TO DISTANCES

Shania lives in Inuvik, NT and wants to travel to Dawson, YT. She had been planning a picnic break at the Arctic Circle, but her brother Roland says they should stop in Eagle Plains, YT, which is closer to the halfway point of the trip.

Shania looks at a roadmap and determines the distance from Inuvik to Eagle Plains, and the distance from Eagle Plains to Dawson. She calculates that the ratio of these is 0.88.

Then she calculates the ratio of distance from Inuvik to the Arctic Circle over distance from the Arctic Circle to Dawson, and the ratio is 0.74.

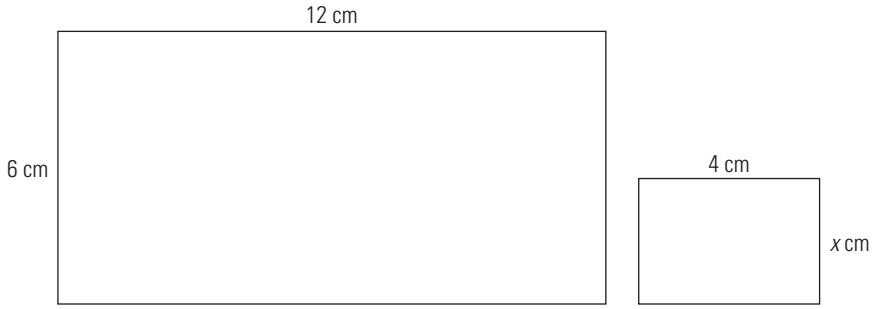
1. Roland asks, "What are your units? Does 0.74 represent 0.74 centimetres, 0.74 inches, or something else?"

   Discuss how you think Shania should respond to Roland.

2. After Roland understands her explanation to his last question, he says the ratio proves that Eagle Plains is closer to the midpoint of the trip than the Arctic Circle. Do you agree? Why?

BUILD YOUR SKILLS

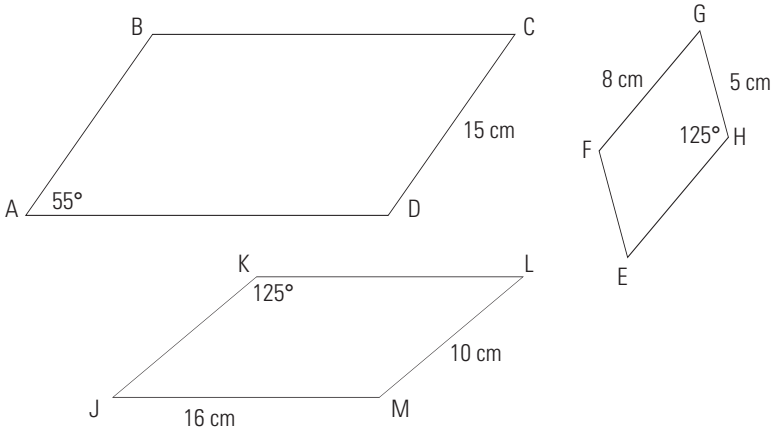
1. Mary is scaling down a pattern for a skating outfit to make a smaller size for her niece. One piece of the pattern is a rectangle, and the original and new piece that she cut are shown below.



   a) What scale factor did Mary use on her pattern to create the pattern for her niece's outfit?

   b) What is the value of x?

2. Rhoniel is an interior decorator who is creating a wall pattern with similar parallelogram stencils. She created the three similar parallelograms shown. She wants to make the corresponding sides of each stencil have the same four colours: brown, yellow, blue, and orange.



   a) Help Rhoniel list the pairs of corresponding sides to figure out her paint colours.

   b) What scale factor was used on ABCD to create EFGH?

   c) What is the measure of side BC?

   d) What is the measure of angle D?

   e) What is the angle measure of L? How do you know? Explain your answer.

3. Fiona drew two triangles that have all corresponding angles congruent. The first triangle has side lengths of 4, 6, and 8 centimetres. The second triangle has side lengths of 9, 6, and 12 centimetres. Fiona says that because $\frac{9}{4}$ equals 2.25, $\frac{6}{6}$ equals 1, and $\frac{12}{8}$ equals 1.5, the triangles are not scaled copies of each other. Do you agree?

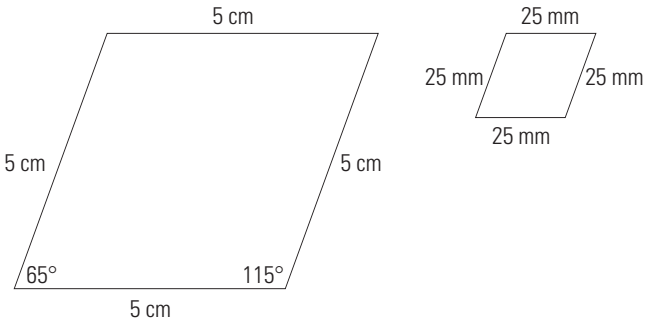
4. Sudi noticed that she has two similar rectangular bread pans in her bakery kitchen. The ratio of their lengths is 1.8. The smaller pan has a width of 3 inches. What is the width of the larger pan?

A doghouse is a comfortable place for this puppy to sleep.

5. Marco's boss has given him two sets of blueprints for trusses for similar structures. One structure will be a garage and the other is for a matching doghouse that their company will display at a home show. The trusses are in the shape of triangles. The doghouse truss has side lengths of 18 inches, 24 inches, and 30 inches. The garage truss has corresponding side lengths of 12 feet, 16 feet, and 20 feet. Marco's boss wants to make sure the designs are similar before they start construction. Are the trusses similar or not? How can Marco prove the answer for his boss?

These embroidered moccasins are made of smoke tanned moose hide and trimmed with rabbit fur.

6. Dene craftspeople of the Northwest Territories produce colourful, intricate beadwork patterns that are stitched onto jackets, hair clips, and moccasins. Some of these patterns use similar figures. Lise is designing a diamond-shaped beadwork pattern for two pairs of moccasins, one for a man and one for a small child. The larger rhombus below represents a shape Lise will stitch in blue beads for the man's moccasins. The smaller shape will be stitched in red on the child's moccasins. She will stitch smaller rhombuses inside each shape.



a) Are the two shapes similar? How do you know?

b) If they are similar, what scale factor should Lise apply to the rhombus in order to scale it from the larger moccasin to the smaller moccasin?

c) If Lise started by stitching the smaller rhombus and had to scale it up to the larger moccasin, what scale factor would she use?

7. Lance and Max are working together on a project about similarity. Lance writes, "All squares are similar." He states that since a square always has

four 90-degree angles, every square is similar to every other square. As a result, Max writes, "All rectangles are similar." He thinks that since every rectangle also has four 90-degree angles, all rectangles are also similar to each other.

a) Is Lance correct? Explain.

b) Is Max correct? Explain.

8. A chef creates three casseroles. He makes them so that the shape of casserole A is similar to the shape of casserole B. Casserole B is also similar in shape to the third casserole, casserole C. Can you conclude that casserole A is similar to casserole C? Explain your thinking.

Extend your thinking

9. Jonas drives a forklift in a warehouse. He uses a forklift pallet size of 48 inches wide by 40 inches long, a common size in North America.

   To make the best use of storage space in the warehouse, the product boxes should fit exactly onto the pallet, so the company uses two box sizes.

   Box 1: 24″ long × 20″ wide
   Box 2: 20″ long × 16″ wide

   a) Draw two diagrams showing how the two box sizes make the best use of warehouse space.

   b) Are either of the two box shapes similar to the shape of the pallet? Which one?

   c) How does the area of the similar box compare to the area of the pallet? How does the ratio of the areas compare to the ratio of edge dimensions? Why do you think this is?

Forklift drivers are expected to have good driving skills and know about vehicle maintenance. They are often required to use data entry and computer skills.

EUCLIDEAN GEOMETRY

One of the greatest mathematicians of ancient Greece was Euclid, who lived around 300 BCE. His scholarly work in mathematics influenced the growth of many branches of science, including mechanics, astronomy, and reasoning.

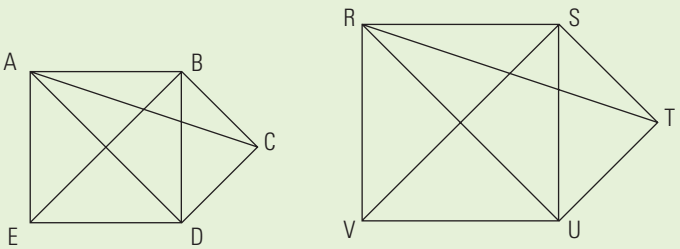
Euclid lived in Greece, but was sometimes known as Euclid of Alexandria. Alexandria had the best library in the ancient world and many great scholars gathered there to study and share ideas.

Euclid contributed a great deal to our understanding of geometry. One theorem he developed describes the relationship between the sides in similar triangles ABC and A'B'C'.

Euclid was the first person to put many of the axioms and theorems that were known at the time into a book called *Elements*. His 13 volumes contain hundreds of definitions, propositions, common notations, and postulates. A postulate is something assumed without proof because it is self-evident or generally accepted. In Book 6, he uses similar polygons to build one of his propositions: He states:

"Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the first polygon has to the second polygon the same ratio as the corresponding sides."

This proposition states that any two similar polygons can be divided into similar triangles. In the diagrams below, ABCDE is similar to RSTUV. This means BC corresponds to ST. This also means that CD corresponds to TU and has the same ratio to TU as BC does to ST.



In the diagrams shown, can you prove that ΔBCD will always be similar to ΔSTU?

Drawing Similar Polygons

6.3

Scale models can include many detailed features for which all the dimensions must be calculated.

MATH *ON THE JOB*

Ryan is a gas serviceperson trainee with Manitoba Hydro in Winnipeg, Manitoba. Before training with Manitoba Hydro, he completed courses at Red River College. Ryan's gas fitter training will take him four years to complete. To become a gas fitter at Manitoba Hydro, you need to have completed high school courses in mathematics and physics.

When at work, Ryan can be expected to install gas lines to appliances such as a natural gas burning fireplace. Before beginning this job, Ryan might read blueprints to familiarize himself with the layout of his work area. After assembling the materials needed to do the job, Ryan could be expected to cut openings in the walls to put gas pipes through, bend the pipes so that they will link the gas meter to the fireplace, and install valves, flues, vents, or burners.

Ryan is assembling the materials needed to help install two gas burning fireplaces. One fireplace is a small domestic model and the other is a large model for a hotel lobby. The face of the vent for the small fireplace is rectangular and measures 11 inches by 8 inches. Ryan's supervisor tells him the vent for the large fireplace is two-and-a-half times bigger and asks him to retrieve it from the warehouse where parts are stored. What is the scale used here? What are the measurements of the large vent?

EXPLORE THE MATH

Some museums have displays that are recreations of important times in history or displays of animals that may be extinct. When creating these displays, the museum may not have enough space to create each model using the full-size measurements. The museum needs to use the original measurements and apply a scale factor to each measurement to build a model that is similar to the original.

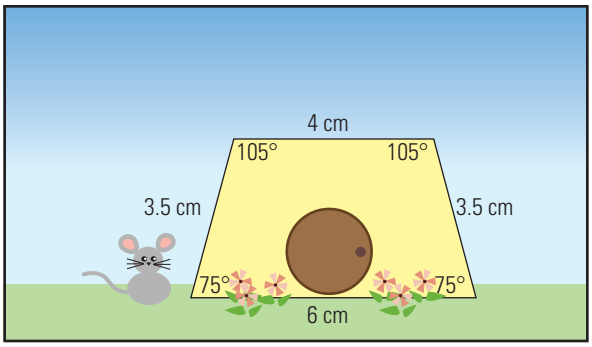
When creating a similar figure, there are two important characteristics to remember. The measures of their corresponding angles are equal, or congruent, and the measures of their corresponding sides are proportional because they have been increased or decreased by the same factor.

Mental Math and Estimation

A museum is building a dinosaur exhibit and wants to include a Tyrannosaurus Rex model. The original dinosaur was 13 metres long. The museum decides to use a scale factor of one-third to create the model. What would be the approximate length of the model?

ACTIVITY 6.4
ENLARGING A TRAPEZOID

Rochelle is an illustrator for a children's book. She drew the figure below, called a trapezoid, as the first step for drawing a cartoon mouse's house.

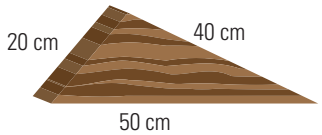


Rochelle decided she liked the shape, so she wanted to make a larger version for the story's cartoon raccoon. If Rochelle enlarges the trapezoid by a scale factor of 1.5, what would be the dimensions of the new trapezoid? Include the side lengths and angle measurements she would need to use.

Example 1

Lauren illustrates "how-to" manuals that show customers how to assemble furniture. One of her co-workers went home sick, and she was given the following diagram of a triangular shelf and told to redesign it. The triangular face of the new shelf has one side length of 60 cm and is defined as a similar triangle.

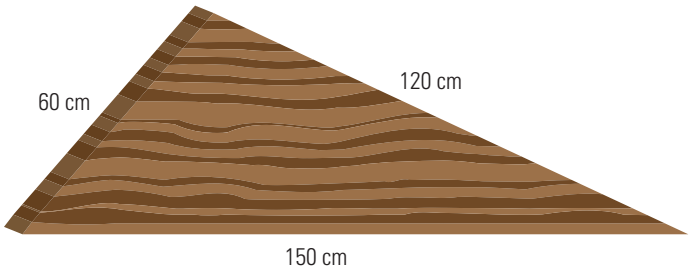
Now Lauren has to figure out the dimensions of the rest of the triangle. She needs to figure out what scale factor her co-worker used. Is there more than one triangle possible?



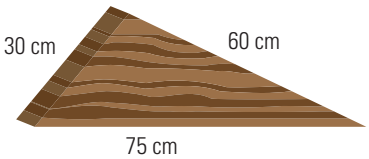
SOLUTION

Since each of the three sides can be multiplied by a number to result in a length of 60 cm, there could be three possible triangles.

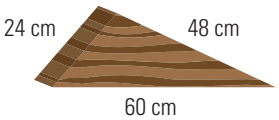
Triangle 1: The 20 cm side can be made into a 60 cm side using a scale factor of 3.



Triangle 2: The 40 cm side can be made into a 60 cm side using a scale factor of 1.5.



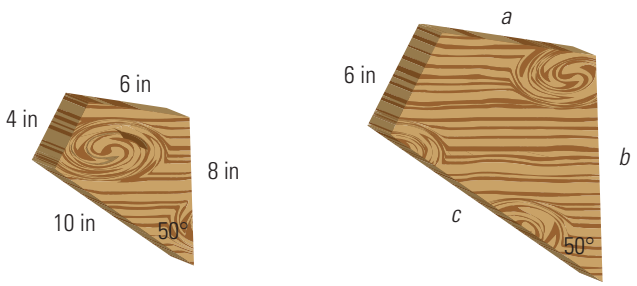
Triangle 3: The 50 cm side can be made into a 60 cm side using a scale factor of 1.2.



Example 2

An artist needs two similar slabs of cedar for a sculpture. They must have the same shape, as sketched by the artist, but two different sizes.

Determine what scale factor was used to create the larger piece and use the scale factor to calculate the missing side lengths.



SOLUTION

The side that measures 4 inches in the first figure corresponds with the side that measures 6 inches in the second figure.

$$\text{scale factor} = \frac{6}{4}$$
$$\text{scale factor} = 1.5$$

You can apply the scale factor to each side length in the first figure to determine the lengths of a, b, and c.

$a = 6 \text{ inches} \times 1.5$
$a = 9 \text{ inches}$

$b = 8 \text{ inches} \times 1.5$
$b = 12 \text{ inches}$

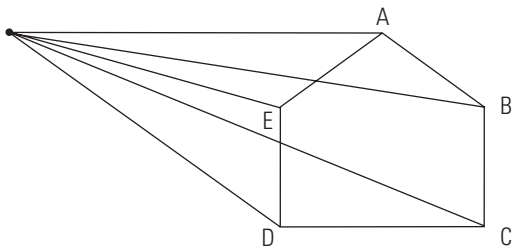
$c = 10 \text{ inches} \times 1.5$
$c = 15 \text{ inches}$

ACTIVITY 6.5
THE RATIO METHOD

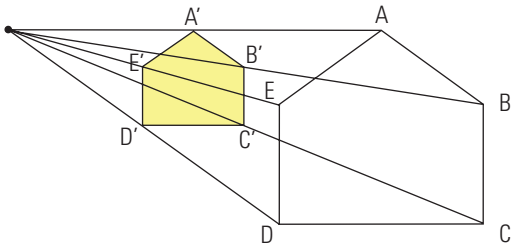
You can create a reduction of many shapes by using the method below. Artists who create paintings and drawings that are true to life may use this method to make perspective drawings.

One method for reducing a figure to create a similar figure is called the ratio method.

1. Choose a point that is external to the shape. This point will become your centre of reduction. Draw lines connecting that point to each vertex of the shape.



2. Choose a scale factor to reduce the image. For this image, we will scale the image by $\frac{1}{2}$. As a result, find the midpoint for each line segment you just created and connect the five points to create a shape that is half the size.



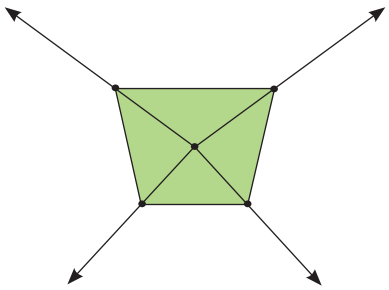
3. Draw an irregular pentagon in your notebook and use the ratio method above to create a similar pentagon that is half the size.

4. How do you think this method would change if you wanted to reduce a shape by a scale factor of $\frac{1}{3}$?

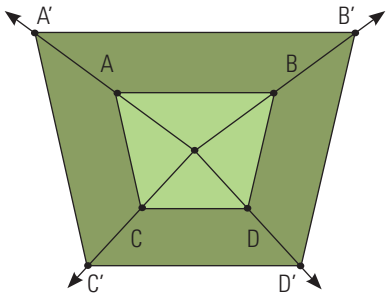
ACTIVITY 6.6
THE PARALLEL METHOD

The ratio method described above works when you want to reduce the size of a figure. There is another similar method you can use to enlarge a figure. This method is called the parallel method. Designers may find this method useful when designing logos and patterns that have concentric objects.

1. Choose a point that is inside the shape that you want to enlarge. This point does not need to be in the centre of the shape. This point will become your centre of enlargement. Draw rays connecting that point to each vertex of the shape and extend the lines. Use this method to enlarge the shape by a scale factor of 2.



2. Measure the distance from the centre of enlargement to each vertex. To double the size of the shape, double that length and measure that length from the centre of enlargement along the ray. This will become a vertex of the new shape. Connect the new points to create your enlarged shape.



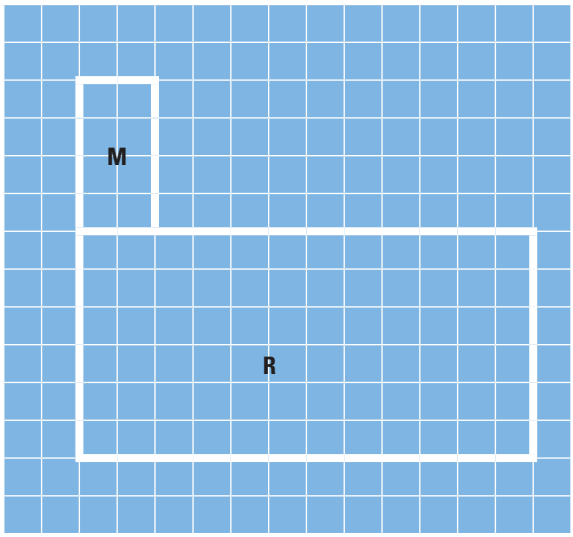
If you've used the method correctly, each side of your new shape will be parallel to the corresponding side in your old shape.

3. Draw an irregular quadrilateral in your notebook and use the parallel method above to create a similar quadrilateral twice the size.

4. How do you think this method would change if you wanted to enlarge a shape by a scale factor of 3?

BUILD YOUR SKILLS

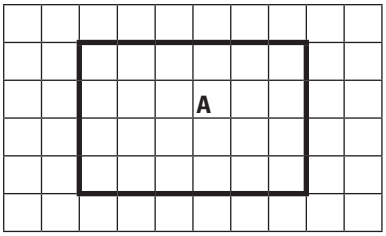
1. The grid below shows a map of two rectangular holes that will be dug to start building a house, labelled R, and a garage, labelled M. The architect drew the two buildings so that their shapes would be similar.



a) Francis and Nipin take a look at the grid and agree that the two rectangles are similar. However, Nipin thinks the scale factor is 2 and Francis thinks the scale factor is $\frac{1}{2}$. Is either of them correct, or both of them? Explain your answer.

b) On graph paper, draw another rectangle that is similar to M and R. Determine the scale factor someone would use to create your rectangle from rectangle M and from rectangle R.

2. Kawa is planning on redesigning her room. She drew rectangle A below on a grid to represent the size of her room.

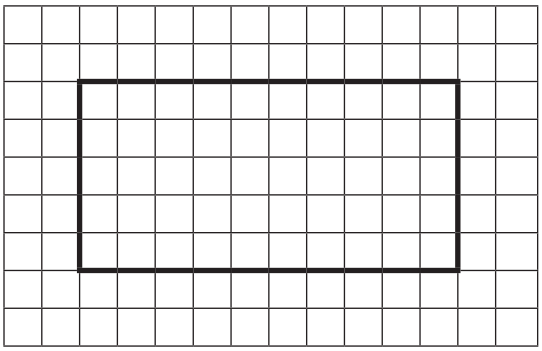


Kawa's brother's room is larger than her room. It is similar in shape, but not identical, to Kawa's room.

a) Suggest two possible dimensions for Kawa's brother's room.

b) Using graph paper, draw one possible rectangle that could represent Kawa's brother's room.

3. Marek is designing a garden in his backyard. He draws a rectangle on grid paper to represent the size of his garden.

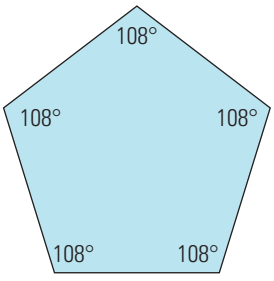


Marek wants to plant 25 different vegetables and herbs in his garden. He decides to divide the garden into 25 plots that are similar to the shape of the garden and are of equal size. On graph paper, trace the figure above and show what could be Marek's plan.

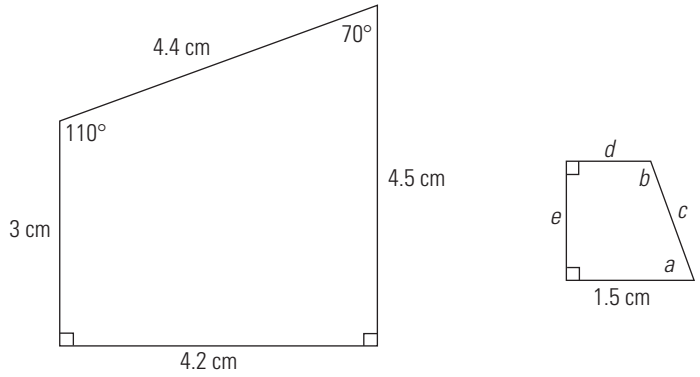
4. Shoshanna designs soccer balls. Each soccer ball is covered with several regular pentagons with a side length of 4 cm. She also creates smaller versions of her soccer balls that use pentagons with a side length of 2 cm.

What would be the perimeter of the a pentagon on the smaller soccer ball? How does this value relate to the perimeter of a pentagon on the larger soccer ball?

regular pentagon: a five-sided polygon where all sides have the same length and all angles have the same measure



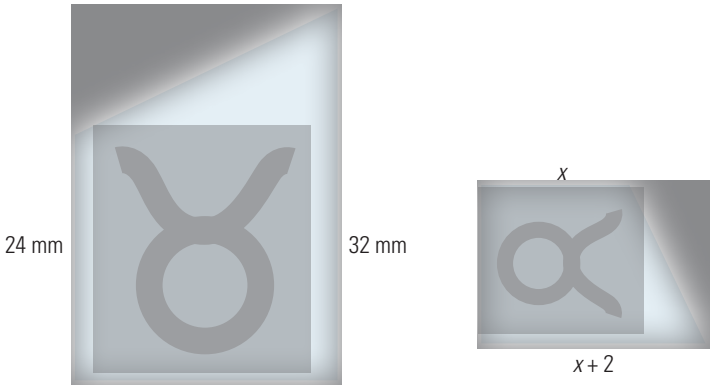
5. Julia drafted two similar logos for a company, one for letterhead and a similar size for envelopes. Unfortunately, she forgot to mark the measurements on the smaller one. Find the missing measurements for her, and write an e-mail to Julia explaining how you solved the problem.



6. A bedroom has dimensions of 9 feet by 12 feet. The living room in the same house is built to be similar in shape to the bedroom, but is larger. The scale factor from the bedroom to the living room is 2.5. What are the dimensions of the living room?

Extend your thinking

7. Carrie designs charms for necklaces and bracelets. She designs a horoscope charm for a necklace in the shape of a trapezoid, and a smaller, but similar, bracelet charm. Determine the side lengths of the smaller charm. Show all the steps as if you were teaching someone how to solve this problem.



PUZZLE IT OUT

RATIONING CHOCOLATE BARS

Many chocolate bars come in the shape of a rectangle and are made up of smaller squares, like the chocolate bars shown below. Determine the minimum number of breaks you need to make to break each of the chocolate bars below into all of its smaller squares. How can you determine how many breaks it will take to break any chocolate bar made up of n squares?

DETERMINING DIMENSIONS

A pool table might be one of the features in a community games room.

At the beginning of this chapter, you were given the role of an interior designer and were asked to design a layout for a rectangular community games room with an area of 70–80 square metres. Your design must have at least four pieces of community game equipment. To create a unique design, include other elements to furnish the room.

On a piece of graph paper, draw top-view sketches of the pieces of equipment and furniture with the dimensions you will use. Once you've determined the dimensions of each piece, use a scale ratio to determine the size of each of the pieces in your drawing. Your drawing must be 50 times smaller than the actual room.

Use a table similar to the one below, which your teacher will provide. Fill in your choice of community game equipment and other pieces of furniture you will be adding to the room, along with your calculations.

SCALING DESIGN ELEMENTS

Type of Equipment	*Length in Room*	*Length in Drawing*	*Width in Room*	*Width in Drawing*

Once you have completed the chart, start drawing a sketch of the room and think about how you will place each piece in the room. Where are the doors and windows in your room? Make sure there is enough room for someone to walk easily between the games.

Similar Triangles

6.4

Maurice Yingst uses knowledge of similar triangles in carpentry and construction work.

MATH *ON THE JOB*

Maurice Yingst is a housing manager for the Wesley Band of the Stoney Nation in Morley, Alberta. He is in charge of housing inspections and renovations. When he lays out the location of new houses to be built, it is helpful to be able to calculate information about triangles with the same angles. He also uses knowledge of triangles in calculations about stairways, trusses, and other structures.

Maurice earned his journeyman carpenter's ticket from the Saskatchewan Institute of Applied Science and Technology in Moose Jaw, Saskatchewan, in 1963. He has been known as a hard worker all his life, and his willingness and ability to perform accurate math calculations have often inspired his bosses to promote him.

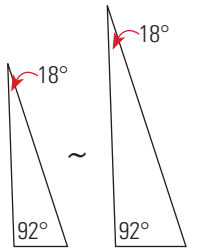
Maurice needs to extend a staircase along the same angle as the existing staircase. The original staircase rises 8 feet over a horizontal distance of 10 feet. What horizontal distance will he need to extend the staircase to make it 6 feet higher?

EXPLORE THE MATH

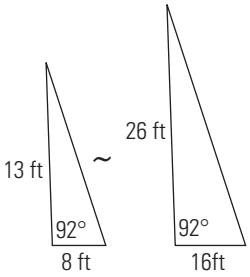
Section 6.2 defined that two figures are similar if their corresponding sides are proportional to each other and their corresponding angles are congruent. The same rules apply to triangles, but you can determine if they are similar by using even less information.

Two triangles are similar if one of the two following requirements is true.

- Any two of the three corresponding angles are congruent.
- One pair of corresponding angles is congruent and the corresponding sides adjacent to these angles are proportional.

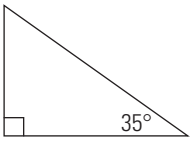


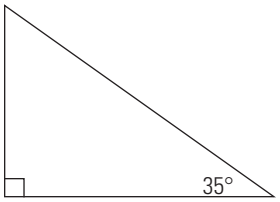
Determining whether right triangles are similar is even more straightforward. A right triangle always has one known angle, the 90° angle or right angle. Because the two remaining angles add up to 90°, they are both always less than 90°, or acute angles.



Two right triangles are similar if the following requirement is true.

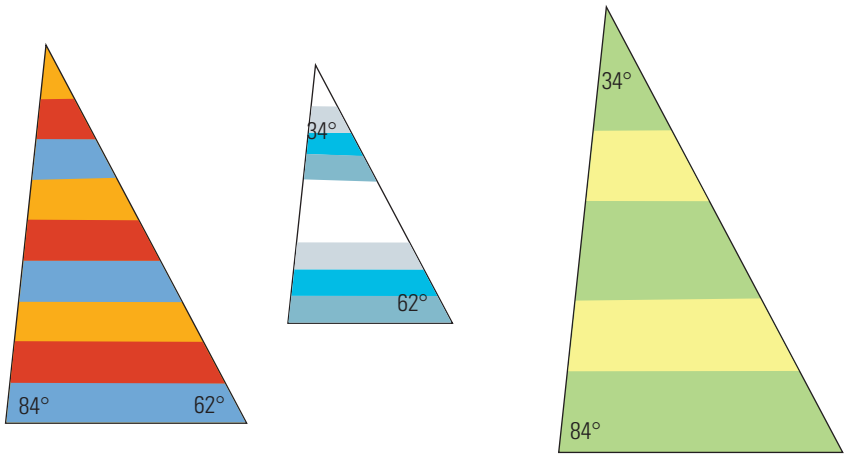
- One pair of corresponding acute angles is congruent.





Mental Math and Estimation

Are any of the three triangular boat sails pictured here similar? How do you know?



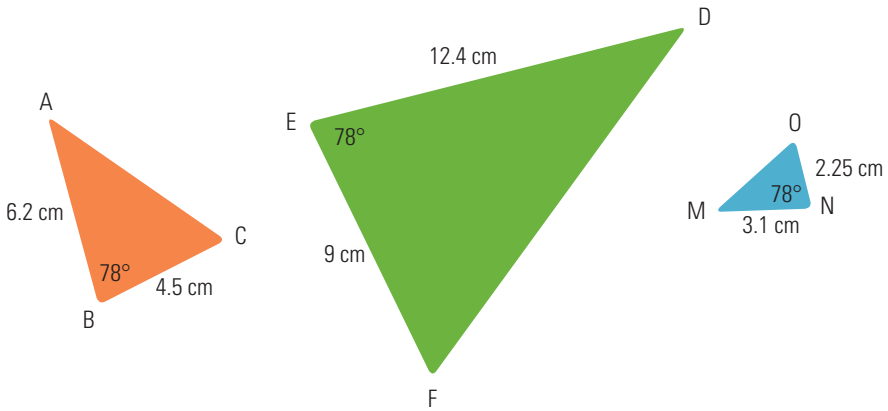
DISCUSS THE IDEAS

COMPARING TRIANGLES

Work in pairs to discuss the following questions.

1. Does the size of two or more triangles influence whether or not they are similar? Explain your reasoning.

2. Look at the three triangles shown here. Are they similar? Explain why or why not.



3. In triangle ABC, the side length of AC is 7 cm. What is the length of DF and MO?

Example 1

Adsila is designing a T-shirt and wants to use several triangles in her design. She drew triangle ABC below to represent the triangular shape she wants to use in her design. The side lengths of the triangle are as follows.

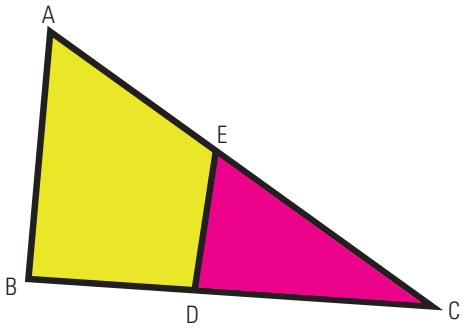
AB = 4

BC = 5

AC = 6

DC = 2.5, and

EC = 3



Adsila thinks that if she draws line ED so that it is parallel to side AB, then triangle ABC will be similar to triangle EDC. Prove that she is right and determine the length of ED.

SOLUTION

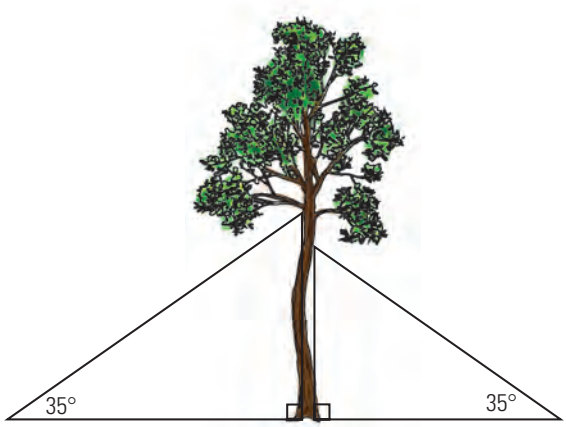
The triangles are similar because ABC and EDC share a common angle and the ratio of CD to CB is equal to the ratio of CE to CA. The sides that form that angle are proportional, meaning that the triangles have a congruent angle and two sides and are similar.

The scale factor used to create triangle EDC from triangle ABC is $\frac{1}{2}$. Since AB equals 4, the length of ED must be 2.

Example 2

Roberto and Marcos tie wires to either side of an artificial tree as part of the set-up of a concert stage. They decide to attach the wires so that they both make a 35° angle within a right triangle.

Are the right triangles created by the wires similar triangles?



SOLUTION

Since two pairs of corresponding angles are given, the triangles are similar. The third angle would equal 55°, making all three pairs of corresponding angles congruent. Therefore the triangles are similar.

ACTIVITY 6.7
PROVING SIMILARITY OF TRIANGLES

Three students are working in a group to build model houses for a woodworking class. Each of them is designing the front view of the roof of their house in the shape of a triangle. As they sketch, they decide to make their roofs similar in shape to each other. They each come up with a plan for how they will build their roofs so that they are all similar triangles.

Student 1: "Let's build triangles that have their corresponding angles equal."

Student 2: "All the roofs should be isosceles triangles."

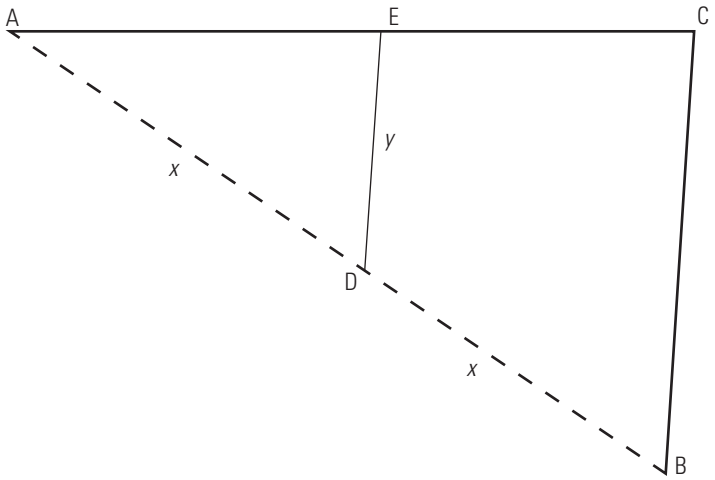
Student 3: "The roofs should have corresponding sides that are proportional."

Work in groups of three. Each person chooses one statement and tries to prove whether it will lead to the creation of similar triangles. Use diagrams to test whether the rule works. After you've shown whether your statement will or will not result in similar triangles, try to prove or disprove the other two statements. As a group, you will need to come to an agreement about which of the statements are true and which are false.

ACTIVITY 6.8
REDUCING TRIANGLES

A farmer in Manitoba keeps cows on an irregularly-shaped pasture resulting from the highway right-of-way across a corner of his property. As part of his pasture management plan he wants to keep the cows off a portion of the pasture, and puts a fence across from D to E parallel to pasture edge BC.

Show that the two triangles ABC and ADE are similar. What is the length of BC?



Some farmers build temporary fences to divide their land. For example, cattle can be allowed to graze on different fenced-off sections of land throughout the year.

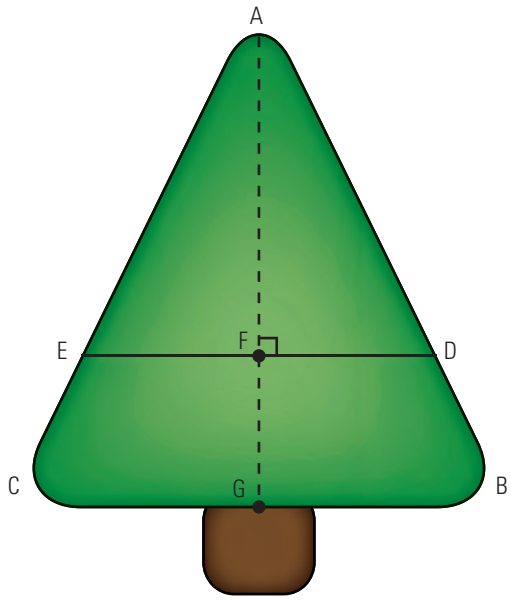
BUILD YOUR SKILLS

1. Hillary and Kuruk have designs for building sets of nesting triangular tables. To ensure that the tables in a set match each other, the triangles must be similar in shape. Hillary cuts two sets of table tops with the angles shown below. Determine if the tops are similar.

   Set 1: Table 1: 90°, 45°
   Table 2: 45°, 45°

   Set 2: Table 1: 133°, 11°
   Table 2: 35°, 11°

2. Kaia baked a holiday cake in the shape of a tree. When it was done baking, she realized it was too big for her platter. She decided to cut off the stem, and trim the cake along line DE, and replace the stem.



These are some of the cake's measurements.

AD = 9″
BC = 10″
DE = 8″

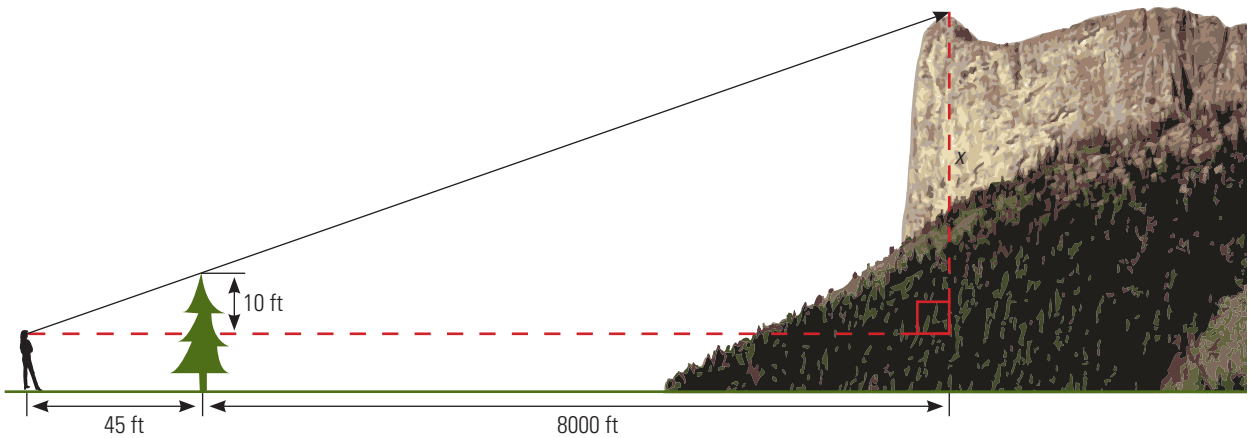
Given these measurements, what is the length of side AB?

3. Referring to the diagram in question 2, if

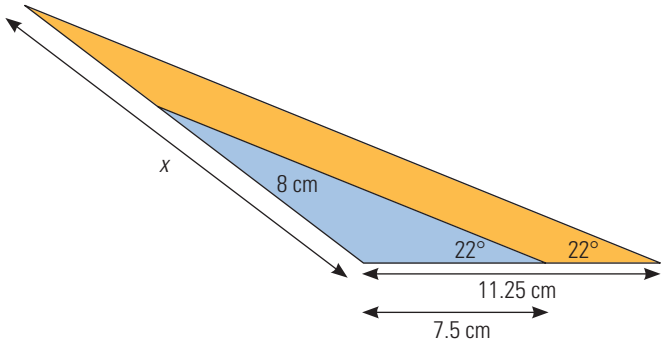
AF = 8″
FG = 2″
AE = 9.5″

What is the length of EC?

4. Tryna is hiking near Seebe, Alberta. She sees the top of Mount Yamnuska (Yamnuska means "cliff" in the Stoney language) in line with the peak of a tree, whose height she estimates to be 10 ft above her eye level. If the distances correspond as shown in the diagram, how tall is the face of the mountain above where Tryna is hiking?



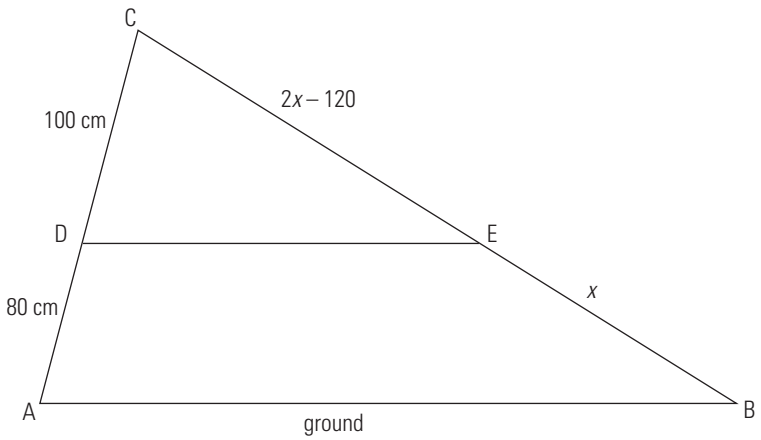
5. Mukako and Maya are constructing a quilt piece for their high school Fashion Studies class. They have some of the measurements they need, but they need to know the length of x below to ensure that the larger and smaller triangles are similar in their measurements. What should be the length of x?



Extend your thinking

6. A company designs a slide for a children's playground near Kinngait, Nunavut. They make a sketch of the slide as a triangle. They place a beam parallel to the ground, labelled DE in the diagram below, to support the slide. Find the lengths of CE and EB in the diagram.

Playground equipment, such as a slide, is designed to meet safety standards.



PRESENT YOUR DESIGN

When designing a community games room, make sure there is enough room for people to move around.

You are now ready to present your design for the community games room to the deciding committee at the meeting.

Decide how to present the project. Will you use computer sketching software, build a three-dimensional scale model, or use sketches on paper? What are the selling points of your design? What does your plan have that other designing committees may not offer?

Remember that your project must include:

- A sketch, computer sketch-up, or 3-D model of your room with all of the required elements in your design.
- A separate sheet of paper or spreadsheet with your calculations.

Be prepared to explain how you determined the measurements of each piece of furniture and community game equipment in your scale drawing.

REFLECT ON YOUR LEARNING

SIMILARITY OF FIGURES

Now that you have finished this chapter, you should be able to

- ❒ determine if polygons are similar by their corresponding angle measures;
- ❒ determine if polygons are similar by their corresponding side lengths;
- ❒ explain why two polygons are not similar;
- ❒ find the scale factor between the corresponding sides of similar polygons;
- ❒ draw a polygon that is similar to another polygon; and
- ❒ explain why right triangles with one shared acute angle are similar.

You will also have finished a chapter project that allowed you to apply these skills in a practical way to a real-world task.

PRACTISE YOUR NEW SKILLS

1. Rayne is an industrial designer who designs food storage containers. He has designed a container with a width of 24 cm and a length of 36 cm. He now wants to design a nesting set of containers proportional to the first one. If he uses each of the scale factors below, what will be the new dimensions of each container?

   a) $\frac{2}{3}$

   b) 1.25

   c) 25%

   d) $\frac{5}{6}$

Stacking plastic containers are used in many workplaces. They might be used by chefs storing food, movers transporting goods, or hotel housekeepers organizing cleaning supplies.

2. Chenna scaled a polygon by a factor of $\frac{3}{4}$.

   a) What is the ratio of any two corresponding sides?

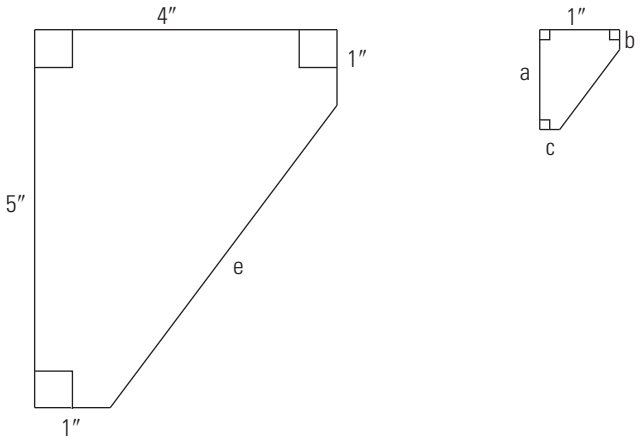
   b) What is the ratio of the measures of any two corresponding angles?

3. Bill is ordering molding stock for carpentry work. He uses similar cross-section shapes for crown moldings and for chair rail moldings and trim.

   a) Determine what scale factor was used to create the smaller shape from the larger one.

   b) Use the scale factor to calculate lengths a, b, and c.

   c) Does Bill need to know length *e* to make the smaller shape?



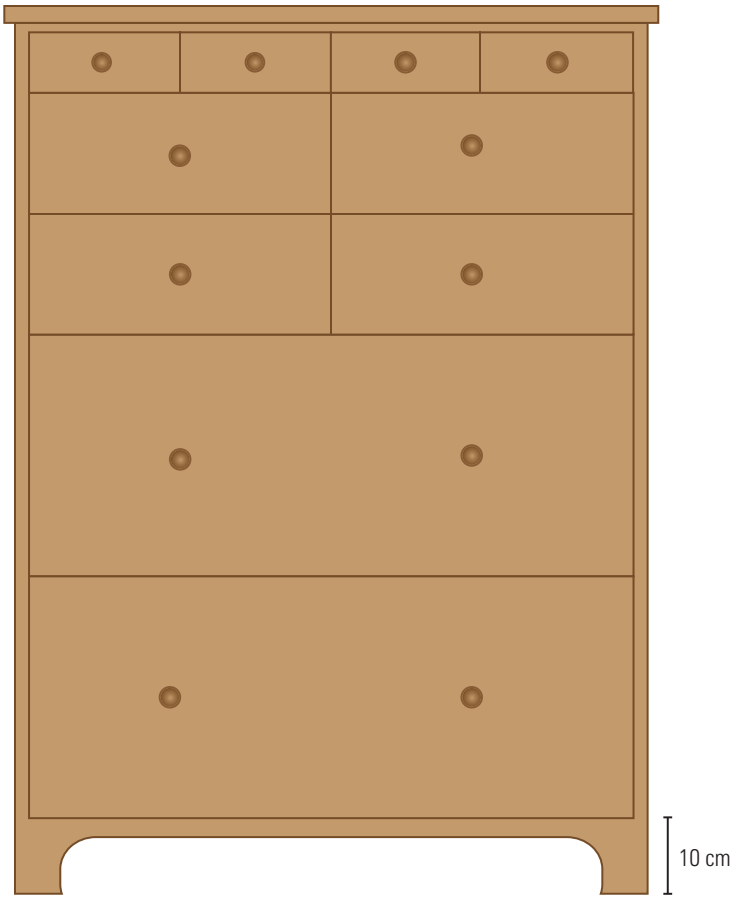
4. Sherelle is designing a dresser with three sizes of drawers. She decides the largest drawer will be 80 cm wide by 32 cm high.

   a) For the middle size drawer she reduces the big drawer by a factor of $\frac{1}{2}$. What are the dimensions of the middle-size drawer?

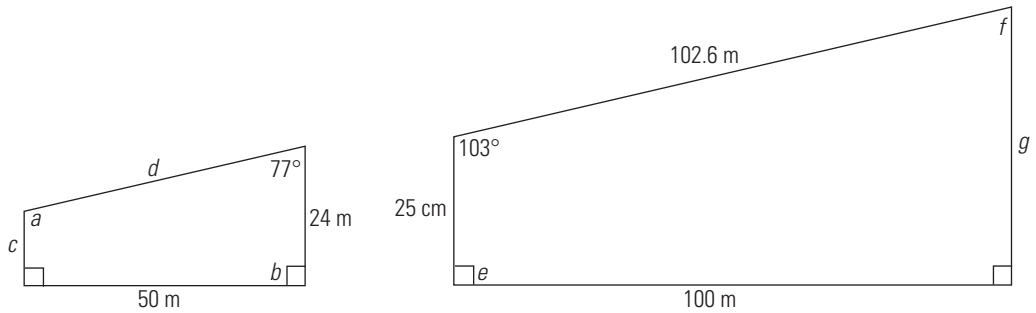
   b) For the smallest drawer she reduces the middle drawer by 50%. What are the dimensions of the smallest drawer?

   c) What scale factor is used on the biggest drawer for the smallest drawer?

   d) If the drawers are arranged as shown in the diagram, what will be the height of the dresser if the feet are 10 cm high?



5. Tim is a park designer in Carrot River, Saskatchewan and is planning two fields outside a community building. He wants each field to be similar in shape, but one field needs to be larger than the other field, while each makes the best use of the available space. He creates the two diagrams below to represent the two fields.



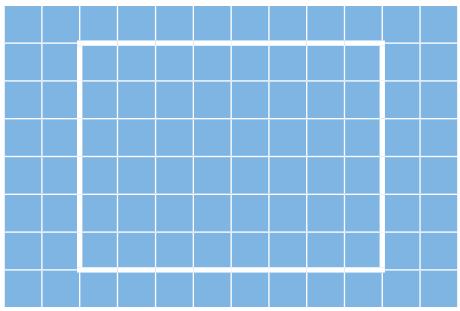
a) What scale factor did he use on the dimensions of the first field to create the dimensions of the second field?

b) If the two fields are to be similar in shape, what should be the angle measurements at a and b? What should be the missing side lengths, c and d?

c) In the larger field, what should be the measurements of the angles e and f? What should be the length of side g?

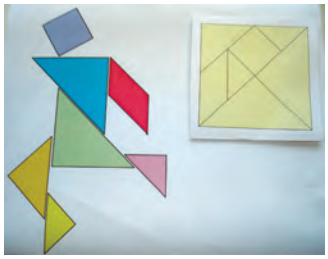
6. Elise works at a museum gift shop. A customer has asked for a poster that reduces the size of their favourite painting (measuring 57″ wide by 76″ long) by no more than a scale factor of $\frac{5}{8}$. Elise finds a poster that is 36″ wide by 48″ long. Will this satisfy the customer? Justify your response.

7. Ryan and Cyndi's parents have offered to supply materials for them to build a playhouse if they create a reasonable design. Ryan drew the rectangle below on 1 centimetre grid paper to use as the outline of the walls.



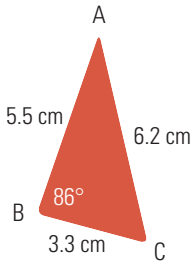
Cyndi drew another rectangle on 1 centimetre grid paper. She says that she used a scale factor of 0.5 on Ryan's plan to create the new plan. Her playhouse plan has an area of 24 cm^2. Could the rectangles be similar? Justify your answer.

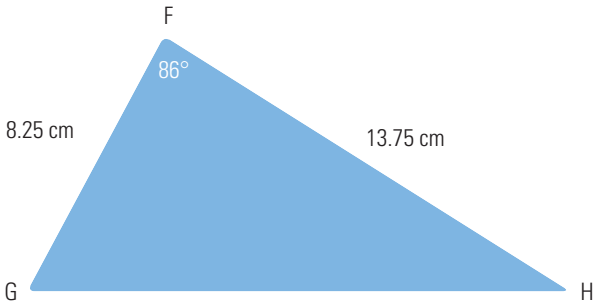
8. Jeremy is building a coffee table that separates into two triangles. He makes the first triangle using angle measures of 85° and 32°. He plans to make the second piece using angle measures of 63° and 32°. Will the two triangular pieces have a similar shape? Explain your response.



The tangram puzzle pieces combine to make a figure of a person running.

9. A tangram puzzle contains plastic triangular pieces for combining into different pictures. A company produces two pieces with the measurements shown below. Are the two pieces similar in shape? Explain your answer.





10. Petra is building an obstacle course for her daughter in their backyard. She designs a sketch of the course so that it is in the shape of a triangle. She labels the triangle ABC and uses side lengths so that AB equals 10 ft, AC equals 20 ft, and BC equals 25 ft. When she goes to build the obstacles, she finds that she has fewer materials than she thought she had. She decides to make a new sketch using triangle FGH so that $\frac{AB}{FG}$ equals 2.5. What is the measure of FH?