# Chapter 5

# Angles and Parallel Lines

## GOALS

Every day, in our homes, at work, or when travelling from one place to another, we encounter angles and parallel lines. They can be found in buildings, furniture, graphic designs, city layouts, and many other places. In this chapter, you will learn about how angles are measured and created for workplace applications and how combinations of angles and lines can be used to create parallel and non-parallel lines. You will use your mathematical skills and knowledge to

- measure, draw, and describe angles;
- estimate the measure of angles;
- use certain angles to determine whether two lines are parallel; and
- solve problems involving angles and pairs of angles, and parallel, non-parallel, perpendicular, and transversal lines.

## KEY TERMS

- angle
- angle measure
- degree
- parallel lines
- perpendicular lines
- transversal

## START TO PLAN

### PROJECT OVERVIEW

Three-dimensional (3-D) drawings are found in advertisements, comic books, architectural blueprints, and film mockups. Can you think of other examples?

Although software has been created for the purpose of making 3-D drawings and animations, many artists draw by hand to achieve an effect that is difficult or impossible to create with computer graphics software. For this project, you will learn how to create 3-D perspective diagrams of buildings.

Your task will be to create a 3-D perspective diagram of a city street for the cover of a new comic book featuring a superhero, Theta*. The cover will have Theta standing steadfastly in the centre of a street lined with buildings.


*This is an example of a comic book cover that uses several vanishing points.*

### GET STARTED

Before you start, think about the questions below. Record your decisions in a project file.

- What type of material will you draw on? Paper? Card stock?
- How large will your diagram be?
- What type of pencil(s) or pen(s) will be best for drawing?
- What type of tools will you need to draw straight lines and parallel lines?

### FINAL PRESENTATION CHECKLIST

Make a display that includes the following items:

- all preliminary sketches/diagrams used to develop the final drawing; and
- the final cover drawing itself.

Be sure to sign your name, and the name of any partner(s) involved in the creation of this drawing, in the bottom right corner of each item.

* *Theta is the Greek symbol often used to name angles.*

# 5.1 Measuring, Drawing, and Estimating Angles

## MATH *ON THE JOB*

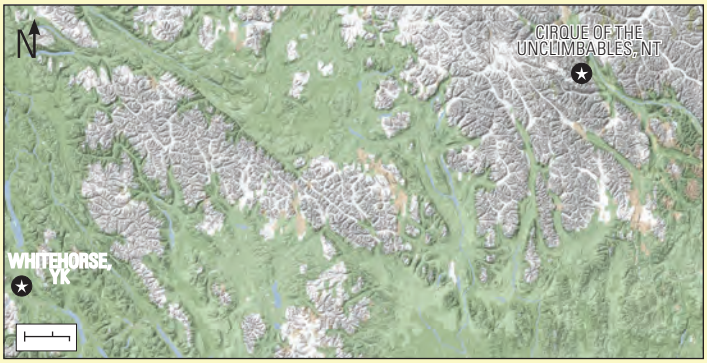
Sunny (Sunje) Petersen and her husband Werner own and run South Nahanni Outfitters, a Whitehorse, Yukon-based company that organizes big game hunting trips to the Mackenzie Mountains. Sunny grew up in northern Germany near the Baltic Sea and Werner was raised in the Austrian Alps.

Sunny helps her husband outfit, organize, and guide hunting expeditions for people from all over the world. To do this, she uses her math skills for bookkeeping, paying wages, calculating the weight and volume of supplies that can fit on a bush plane, determining the weight and amount of fuel the plane needs to reach its destination, writing invoices for clients, and calculating the amount of ingredients she will need when she cooks for large hunting parties. Close encounters with wolves, mountain goats, and moose are also part of the job.

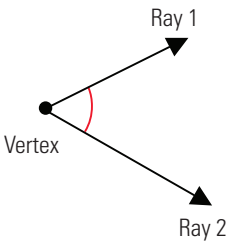
*Sunny Petersen takes people from all over the world heli-hiking in Canada's north.*

Guiding heli-hikers (hikers flown to remote locations in a helicopter) is also part of Sunny's job. Before flying the heli-hikers into the wilderness, Sunny and Werner plot the course the helicopter will take to reach its destination.

As they fly, aircraft change direction many times. Using five or more line segments, plot a course that Sunny and Werner could take in order to reach the towering granite cliffs known as the Cirque of the Unclimbables. Determine the direction of each line segment that comprises your route. Use the eight main directions of a compass to describe the directions used in your course. These are north, northeast, east, southeast, south, southwest, west, and northwest.




**angle:** two rays that meet at a point called the vertex




## EXPLORE THE MATH

Take a moment to look at the structures in your classroom that contain **angles**. Consider who would have been involved in creating the structures that have those angles, for example, architects, designers, surveyors, and carpenters. Angles are also useful to people who do not make structures. Aircraft pilots and boat pilots use angles for navigation. Astronomers use angles to locate objects in the sky.

So, what exactly is an angle? An angle is formed when two rays meet at a common endpoint called a vertex. Angles are measured with tools, such as a protractor, that are marked in degrees.

Visualize an angle that is used to express direction in navigation and mapping, such as east. In this case, the angle is measured relative to true north, which is 0° and may be expressed as a bearing. A **true bearing** describes the number of degrees, measured clockwise, between an imaginary line pointing towards true north (geographic north) and another imaginary line pointing towards an intended direction or along a pathway. East is represented in land navigation and mapping at a 90° angle from true north.

**true bearing:** the angle measured clockwise between true north and an intended path or direction, expressed in degrees

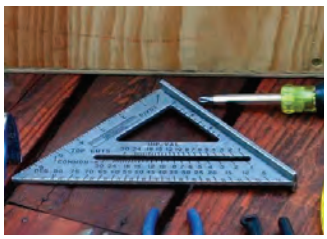

**Angle measures** can be estimated by using **referents**, which are common measurements like 90°, 45°, 30°, and 22.5°.

**angle measure:** a number representing the spread of the two rays of an angle, expressed in degrees

**angle referent:** a common standard of angle measure, for example, 90°, 45°, 30°, and 22.5° used to estimate angles

How can we draw angles? The tools used to measure angles can also be used to draw or replicate angles having specific measures. Tools have been designed to measure and create angles having only one or two specific measures, such as a set square used in technical drawings to draw right angles.

You have used a protractor and ruler to draw angles. You can also draw certain angles with a ruler and compass, and you can replicate any angle with these tools.



*A rafter-angle square is used to lay out and create angles in carpentry.*

### *Example 1*

Use a ruler and compass to create the following angles.

a) Draw a 90° angle.

b) Replicate any existing angle.

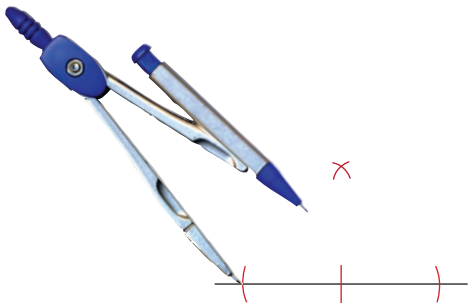
**SOLUTION**

a) To draw a 90° angle, follow these steps.

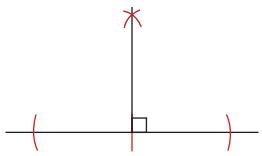
Draw a line segment, then mark where you want the 90-degree angle to go.

Put the compass point at the mark you made. Open the compass slightly and make two more marks on each side of the first mark. Ensure they are the same distance from the first mark.

Widen the compass a bit more, and place the compass point at one of the new marks. Make a small arc, then do the same thing after placing the compass point at the other new mark. Ensure the two arcs intersect each other.

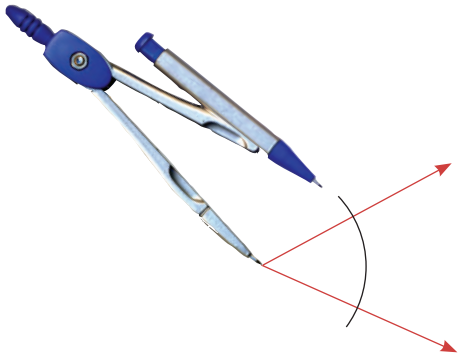


Draw a line segment that goes between or through the point where the arcs intersect and the first mark you made. The two line segments are perpendicular to each other, and therefore form a 90° (right) angle.

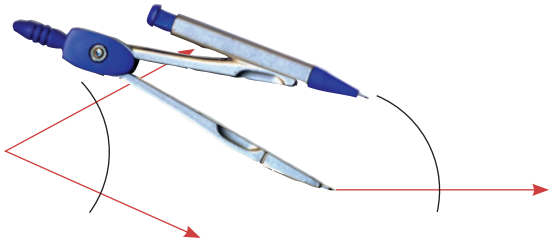


b) To replicate any existing angle, follow these steps.

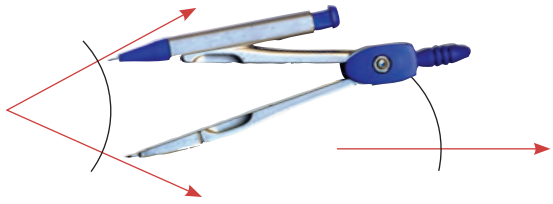
Use a compass to lightly draw an arc centred at the vertex of the original angle.



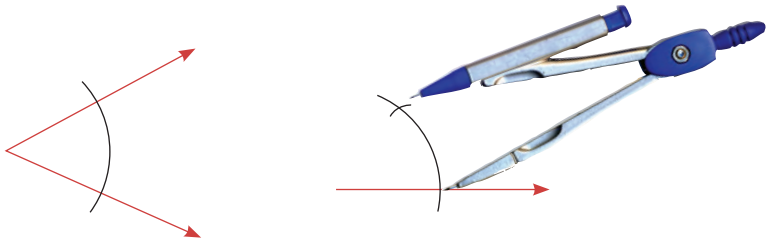
Use a ruler to draw one side of the new angle, and draw an arc of the same radius and arc length as the one you just drew on the original angle.



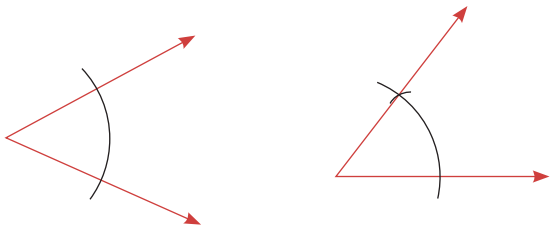
Bring the compass up to the original angle, and set it so that its point and the tip of the pencil touch the points where the original arc intersects the sides of the angle.



Place the compass point over to the point of intersection of the side of the new angle and the new arc. Draw a short arc through the new arc.

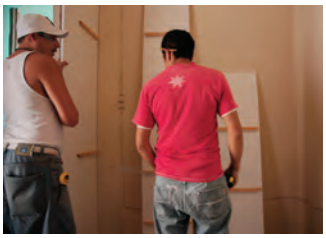


Use the ruler to draw the other side of the angle, from the left end of the first side (the vertex) through the point of intersection of the two arcs. The result is a new angle with the same measure as the original.



## DISCUSS THE IDEAS

### ESTIMATION OR MEASUREMENT?



*Skilled carpenters can estimate the amount of wood, nails, and joist hangers needed to build a set of shelves.*

Many trades require the construction and measurement of angles. Carpenters constantly check angles to ensure, for example, that a foundation is level, a stud is perpendicular to the floor, or a staircase is the right size.

1. Working in small groups, identify five distinct trades that use angles. Start by thinking about the people you know and the trades they work in. For each of the five trades you think of, suggest an example of how the tradesperson creates and measures angles on the job.
2. Do the types of angles your group listed need to be precisely measured and created? This is true in trades such as surveying or manufacturing. Would estimations be adequate, as they may be in landscaping, for example?

## ACTIVITY 5.1
## FIVE ANGLES

Angle measures range from 0° to 360°. On a piece of paper, draw five angles of various measures, labelling the rays, vertices, and angle. (The rays can be about 5 cm long.)

Trade your angle drawings with a partner. Create a chart for drawing, measuring, and describing each of the five angles, as follows:

- angle: measure the angle you've been given and redraw it yourself, to the same angle measure;
- angle measure: state the angle measure in degrees;
- kind of angle: identify the type of angle, for example, "acute" or "obtuse" and explain how you know this;
- example: give an example of where you might see an angle like this in the real world, for example, a rooftop or a vault for gymnastics.

Then check and discuss your work with your partner.

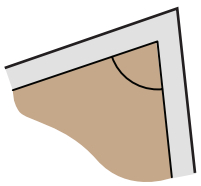
**HINT**

Degrees represent a scale that can be used to measure angles or temperature.

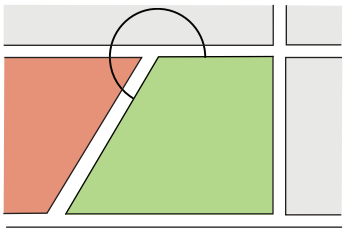
### Mental Math and Estimation

Estimations are made in many trades that use angles. Imagine that you are working as a tradesperson in the situations below and make the following estimations (aim to be within 5°).

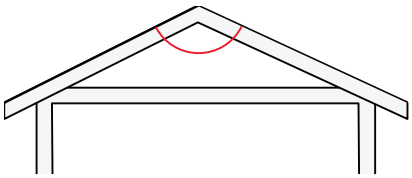
a) a landscaper estimating the angle of the corner of a garden bed



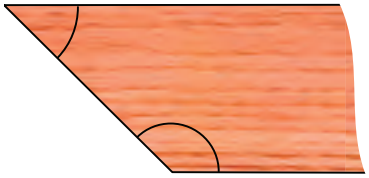
b) a surveyor estimating the angle of a property boundary line on a map



c) a roofer estimating the angle of the peak of a roof



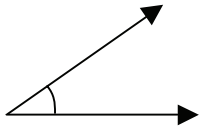
d) a cabinet-maker estimating the angles of two corners of a shelf



## Example 2

*Before this staircase was built, its angle, or incline, was calculated to ensure that it meets building codes.*

Estimate the measure of this angle without using a measuring device.



**SOLUTION**

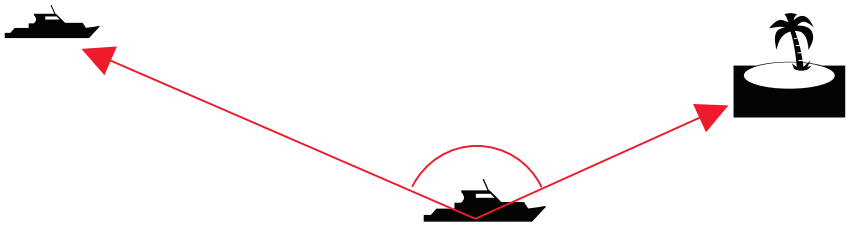
Look closely at the angle, then do some mental mathematics. This is clearly an acute angle since it is less than half the size of a right angle. A close estimate is therefore less than one half of 90°, or about 30° to 35°.

**ALTERNATIVE SOLUTION**

Compare the angle to a known, real-life referent. This angle looks similar to the angle of a standard staircase. Standard staircases are often constructed at an angle of 33° but may be constructed to a maximum of 42°. A close estimate is therefore 30° to 35°.

## Example 3

Estimate the measure of this angle without using a measuring device.



**SOLUTION**

Look closely at the angle, then do some mental mathematics. This is an obtuse angle, since it is bigger than 90° but smaller than 180°. It looks as if it is about one and a half times the size of a right angle. A good estimate is therefore 135° (90° plus 45°).

## ACTIVITY 5.2
## CREATE A REFERENTS DIAGRAM

A referents diagram shows angles from 0° to 360°.

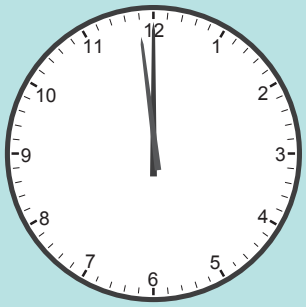
Use only a pencil, ruler, protractor, and/or compass drawing tool. Work with a partner to create your own referents diagram.

Choose an angle whose measure divides exactly into 360° that you think could be useful in estimating angle measures.

Create a diagram featuring referents by drawing a series of radii at angles that are multiples of that angle going from 0° to 360°.

**HINT**

A clock face is an example of a referents diagram.




### *Example 4*

Sort the following angles into pairs of **complementary** and **supplementary angles**.

| | |
|---|---|
| ∠1 = 42° | ∠5 = 121° |
| ∠2 = 107° | ∠6 = 31° |
| ∠3 = 59° | ∠7 = 19° |
| ∠4 = 48° | ∠8 = 73° |

**SOLUTION**

The added measure of angles 1 and 4, and of angles 3 and 6, is 90°, so those two angle pairs are complementary.

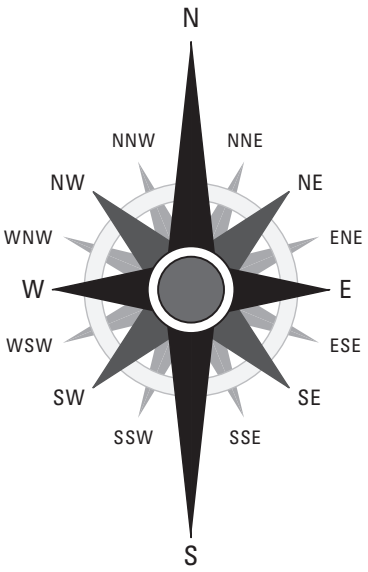
The added measure of angles 2 and 8, and of angles 3 and 5, is 180°, so those two angle pairs are supplementary.

**complementary angles:** two angles that have measures that add up to 90°

**supplementary angles:** two angles that have measures that add up to 180°

## ACTIVITY 5.3
## USING ANGLES IN WEATHER REPORTING

Compass roses, such as the one shown to the right, were originally created to navigate at sea. Although marine navigators now use other technologies, compass roses are still used extensively today to describe weather patterns and wind directions relative to true north.




A compass rose includes the four cardinal directions, N, E, S, and W, plus twelve intermediate directions.

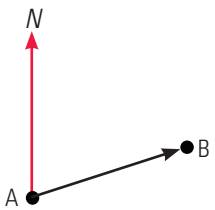
Create your own table of compass-point directions and related bearings stated in degrees. In the first column, list the sixteen cardinal and intercardinal directions, beginning with N (North) and going clockwise to NNW. In the second column, list the true bearing for that direction. The difference in degrees between one point and the next will equal $\frac{1}{16}$ of 360°.

**HINT**

Cardinal direction N is bearing 0°.

### *Example 5*

a) Determine the true bearing between A and B.

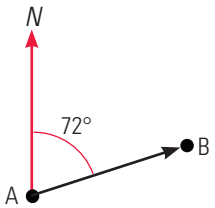



**SOLUTION**

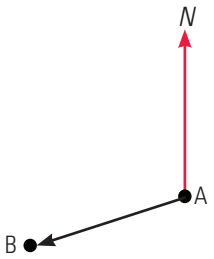
First estimate the angle, using the compass rose above as a guide. It looks as if the bearing is ENE, or about 66°. Using a protractor, measure the angle between direction N and ray AB. The angle measure is 72°, so the true A-to-B bearing is 72°.

**HINT**

Some protractors, such as full-circle and navigational protractors, can measure angles of up to 360°.



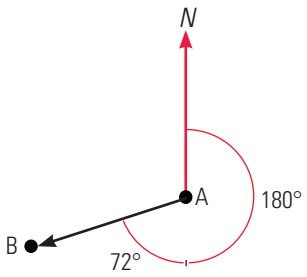
b) Determine the true bearing between A and B.




**SOLUTION**

The common semicircular protractor only includes angles from 0° to 180°. To use it to measure angles greater than 180°, complete the following steps.

1. Mark off an angle of 180° from N.
2. Measure the angle from that mark to ray AB. This angle is 72°.
3. Add the measure of the first angle (180°) and the second angle (72°). The sum is 252°, so the true A-to-B bearing is 252°.




**ALTERNATIVE SOLUTION**

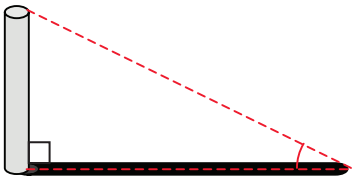
A common protractor usually has two sets of angles, the inner set going from 0° (on the left) to 180° and the outer set going from 180° (on the left) to 0°.

1. Using the outer circle of a protractor, measure the interior angle (108°).
2. Subtract the measure of the interior angle from 360°.
3. The difference is 252°, so the true A-to-B bearing is 252°.

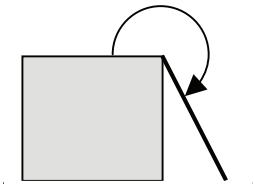
## BUILD YOUR SKILLS

1. Estimate the measure of the indicated angles, to within approximately 5°.

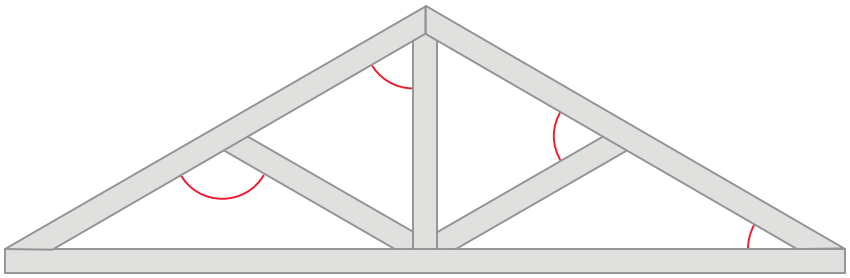
a)



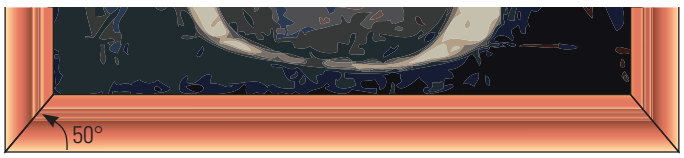
b)



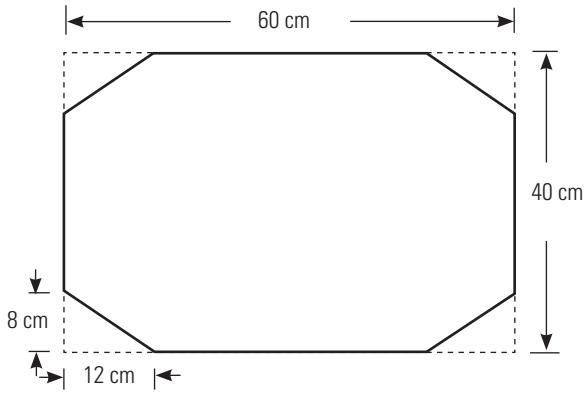
2. Look closely at the indicated angles and predict what relationships may exist among them. Measure the indicated angles in this king post truss to confirm your predictions.



3. Atzuko is making a picture frame for her favourite picture, a reproduction of an abstract painting by Charles Daudelin. She cuts the bottom piece of the wooden frame with 50° angles. If Atzuko wants 90° corners on the frame, at what angle must she cut the bottom ends of the side pieces?




4. A woodworker is given the following diagram of an octagonal panel that will have to be cut from a rectangular piece of wood. At what approximate angles will the cuts have to be made, either to the vertical or the horizontal sides?



5. Draw a vertical ray that points upward to depict the direction north (bearing 0°). From the end point of that ray, draw more rays that depict the following true bearings: 65°, 140°, 220°, and 315°. (The angles should be measured clockwise from north.)

6. Sketch a map featuring several locations, without using any tools. Estimate the distances and directions. First draw an X, which will be the reference point for the other locations. From X, draw the following:

   - a small house symbol 2 km SSW of X;
   - an observation tower 3.5 km with a true bearing of 80° from X; and
   - a hill 5 km to the northwest of X.

   Use a scale for the map in which 1 cm equals 1 km on land. After you have finished, compare your estimated angle and distance measures using a protractor and ruler.


*A compass needle points towards magnetic north, which varies by location.*

## Extend your thinking

7. Magnetic declination is the difference between true north (the direction to the North Pole) and magnetic north (the direction a compass needle actually points towards). A park ranger is using a compass that cannot be adjusted for magnetic declination. From the starting point on her map, she must walk at a true bearing of 54°, then at a true bearing of 195°, and finally in that a true bearing of 107°.

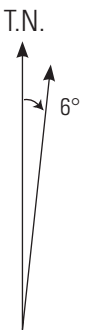
   The magnetic declination on her map is 6° east of true north.

   The park ranger wants to convert the true bearings on her route (which are based on true north) to the compass readings she will read from her compass.

   a) How should the park ranger calculate the compass readings?

   b) What compass reading should she use for each true bearing?

   To visualize the park ranger's route, draw a rough map showing the three bearing directions.



## DRAW A BUILDING IN 3-D

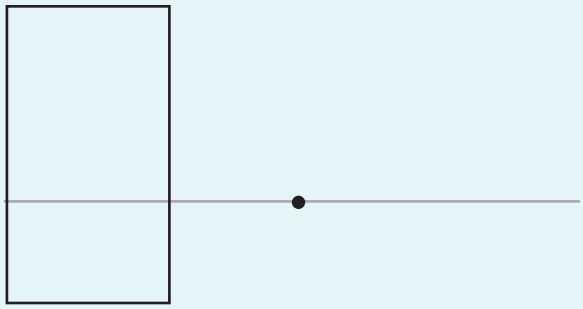
When we look at buildings, the vertical edges remain vertical (and thus parallel to each other). But when we look at horizontal edges they seem to be at an angle to each other, converging at a single point called a vanishing point. When you draw a 3-D diagram of a building in perspective, the horizontal edges appear to converge at a vanishing point.

For this project you will create a one-point perspective diagram, so-called because it has only one vanishing point. In this type of perspective diagram, the horizontal edges of the front and rear faces of buildings remain horizontal, so the faces are rectangular. The horizontal edges of the left and right faces, however, are drawn at various angles that all go towards the vanishing point.

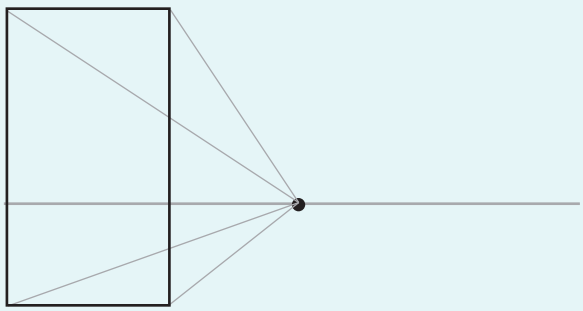
You may or may not have had experience with drawing diagrams in perspective. Here are some simple steps you can follow to get some experience with creating a one-point perspective diagram of one or more buildings.

STEP 1: Very lightly draw a horizon, and mark a vanishing point somewhere along it. (For this project it should be in the centre.)

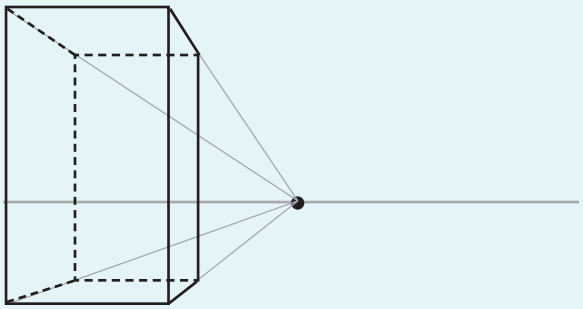
STEP 2: Draw the front face of the building, a rectangle.



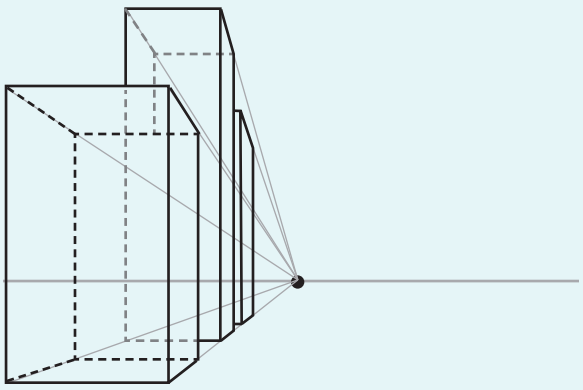
STEP 3: Very lightly draw guide lines going from the four corners of the front face to the vanishing point. These guide lines are horizontal lines drawn in perspective. The top and bottom edges of the right and left faces will be drawn on them.



STEP 4: Draw where the rear of the building is, and draw in the edges that form the faces of the building. (Hidden edges are drawn with dashed line segments, and will eventually be erased. They will help set the widths of buildings behind this one.) Make sure that the vertical edges are parallel to each other.



Draw two more buildings behind this one. You will need to create some space between each building for streets and sidewalks. Also, remember that as the buildings get further away, their faces must be less wide than the closer ones. Your finished drawing should resemble this one. (The relative heights of the buildings in your diagram, however, can be different.)



# Angle Bisectors and Perpendicular Lines

# 5.2

*James spends much of his day cutting pizza wedges.*

## MATH *ON THE JOB*

Yiman (James) Zhan grew up in Bejing, China. He attended No. 58 High School and has a college certificate. He now lives in Vancouver, British Columbia. He and his wife, who trained as a baker, own a small pizzeria, Kinross Pizza, where they make and sell pizzas and desserts. Yiman (James) Zhan bakes dozens of pizzas and cuts hundreds of slices every day at Kinross Pizza.

When he slices pizza, James cuts slices that are all roughly the same size, with angles that are roughly equal in measure. Typically, he cuts large pizzas into 8 slices, medium pizzas into 6 slices, and small pizzas into 6 slices. Customers who order pizzas often ask that the slices be cut in half, usually to make them easier for small children to eat.

One morning James gets an order for one large and one small pepperoni pizza to be picked up for a party at a preschool. He is asked to cut the slices so that they are half their regular size.

What are the approximate sizes of the central angles of regular slices from both a large and a small pizza? What are the approximate sizes of the central angles of the slices from the pizzas for the party order?

## EXPLORE THE MATH

Bisecting an object involves dividing it into two congruent (equal) parts. When you bisect an angle you divide it into two angles of equal measure. For example, when bisected, a 76° angle is divided into two 38° angles. The line, line segment, or ray that separates the two halves of a bisected angle is called the **angle bisector**. Depending on the material that makes the angle, bisecting the angle can be done using measuring tools (such as protractors and compasses) and/or by manipulating the material itself (for example, by folding paper or fabric).

**angle bisector:** a segment, ray, or line that separates two halves of a bisected angle

A right (90°) angle can be thought of as a bisected straight (180°) angle. Perpendicular lines and line segments form right angles. Look around your desk and classroom. Can you identify any perpendicular lines or line segments? You can probably identify several rectangular or square objects that contain them.

**framing square:** a metal tool used to lay out right angles

*A framing square can be used to find the length of a roof rafter, the slope of a set of stairs, or the measure of a bisected angle.*

Perpendicular lines and line segments are drawn using the same techniques that are used to bisect angles. Because perpendicular lines and line segments are so common and so frequently made, specialized tools such as a **framing square** (also known as a carpenters' square) have been developed to make them. Software programs with drawing features often have tools that can be used to create perpendicular lines and line segments.

Finishing carpenters, cabinetmakers, and other woodworkers often make mitre joints, where the ends of two pieces of wood are cut with angles having the same measure. When the ends are joined, the two pieces of wood form a right angle, with the mitre joint acting as a bisector. Mitre joints are made using mitre saws or radial arm saws, because the cuts must be very precisely made. What would the result be if the angles of the cuts were even slightly off? How would this affect the right angle formed by the two pieces of wood?

## DISCUSS THE IDEAS

### WAYS TO BISECT ANGLES

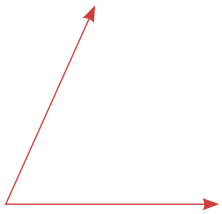
*Mitre saws are used to cut accurate angles. They are often used by people constructing picture frames, baseboards, or doorframes.*

Alexandre is a glazier who cuts and installs glass. He has been hired to replace some windows at Ecole King George in Prince Albert, Saskatchewan. As part of the job, he must cut a square piece of glass into two equal triangles. He will do this by making one cut that bisects the top left angle and bottom right angle of the square. What visual cues could Alexandre use to divide these two angles into approximately equal parts? How could he accurately bisect the angles? What measuring devices could he use to bisect the angles?

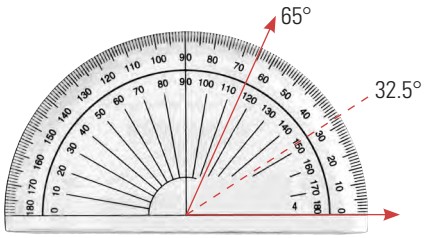
*When two pieces of material meet to form a corner, craftsmen often cut the ends at a 45° angle and join them together to form a mitre joint.*

#### *Example 1*

Accurately bisect an angle like the one shown here.



**SOLUTION**



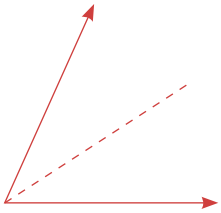

Measure the angle using a protractor. Divide that measure by 2.

The angle measure is 65°.

$65 \div 2 = 32.5$

Use a protractor to measure and mark off a 32.5° angle.

Draw a line segment from the vertex to the mark you made.



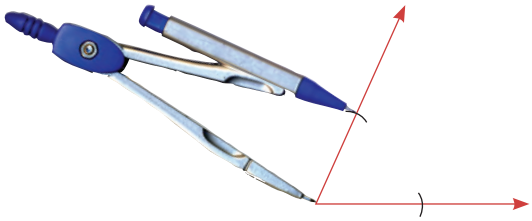
The angle has been successfully divided into two equal parts.

**ALTERNATIVE SOLUTION**

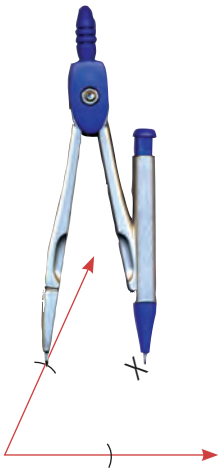
Trace the angle on p. 188 onto a sheet of paper. Place one side of the angle over the other side, creating a fold that goes through the vertex of the angle. The angle has been successfully divided into two equal parts.

**ALTERNATIVE SOLUTION 2**

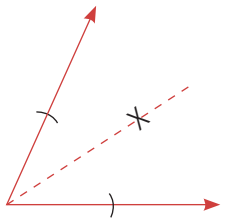
Replicate the angle drawn in the previous solution. Set a compass so that the gap between the pivot point and pencil is a few centimetres. Put the pivot point on the vertex. Mark each side of the angle with the pencil.



Adjust the compass so that the gap between the pivot point and pencil is over half the distance between the two marks on the sides of the angle. Put the pivot point on one mark and mark off a short arc inside the angle. Put the pivot point on the other mark and mark off another short arc inside the angle, to intersect with the first arc.



Draw a line segment from the vertex to the point of intersection.

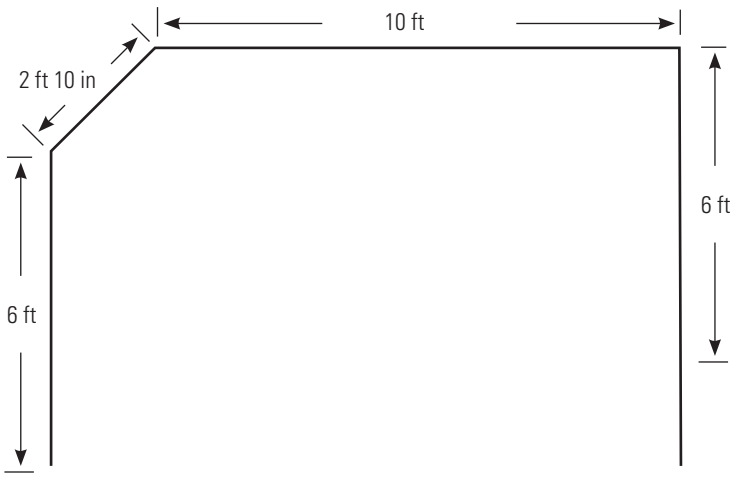


## ACTIVITY 5.4
## DRAW A KITCHEN COUNTERTOP PLAN

*New kitchen countertop can be cut to the correct size with a jigsaw. If the countertop contains a sink, silicone sealant must be applied where the countertop and sink meet.*

House designers, carpenters, and cabinetmakers frequently need to make items with bisected angles. For example, countertops often run along two, three, or four walls in a kitchen, with the ends of the countertop sections meeting at mitre joints that bisect the angles formed in the corners.

In this activity, you and a partner will act as kitchen designers. On a sheet of blank or graph paper, collaborate on drawing a 1:12 scale (where 1 inch equals 1 foot) countertop plan for a kitchen with the dimensions shown below. Show the mitre joints in your drawing.




There is one 90° corner and two 135° corners in this kitchen at which mitre joints will have to be made. The ends of the countertop (the start and finish) will be perpendicular to the walls in your plan. Except for the areas around the mitre joints, the countertop will have, from start to finish, a width of 2 feet.

Follow these steps to make your plan:

a) Draw an outline of the kitchen to scale, using the diagram above as a guide.

b) Draw a perpendicular line from the wall, to scale, where the countertop starts and ends.

c) Bisect the three angles formed by the corners, using a protractor or other tool, and draw the angle bisector lines.

d) Draw four lines parallel to the walls representing the countertop.

e) The angle bisectors show where the sections of countertop will be cut. Write the angle measure of each angle that the sections make at the wall.

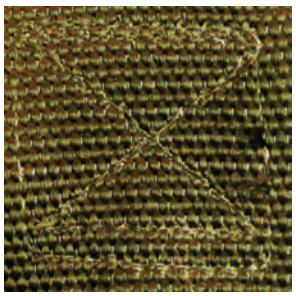
The carpenter will follow your plan to cut the countertop sections with angles that meet your specifications.

Once you and your partner have drawn your countertop plan, check it for accuracy. Ensure that:

- the dimensions of the wall are to scale;
- the extreme ends of the counter are perpendicular to the walls; and
- that the mitre joints bisect the corners.

## ACTIVITY 5.5
## CROSS-BRACING AND CROSS-STITCHING

Cross-bracing is used to erect safe, stable scaffolding. When working high above the ground on a construction project, builders rely on the stability provided by the triangular supports that are used to cross-brace the scaffolding. Box-stitching and cross-stitching are often found in sewn items such as backpacks, postal bags, camera bags, and other bags that are used to carry heavy materials. Box- and cross-stitching enhances durability in the spots that are subjected to a lot of stress from various angles.



*This fabric has been strengthened by cross-stitching.*

Imagine that you are designing a piece of scaffolding that contains cross-bracing for a construction site. Work with a partner. Have one person draw a square and the other a rectangle, on separate sheets of paper. Then draw line segments connecting the opposite corners of the shape. Identify and name each angle with a unique number. Then identify the following items.

1. the pairs of adjacent angles that are complementary;
2. the pairs of adjacent angles that are supplementary;
3. the pairs of line segments that are perpendicular;
4. the line segments (if any) that are angle bisectors.

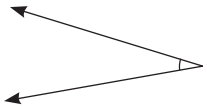
Switch papers with your partner and confirm with each other that your answers are correct.

## BUILD YOUR SKILLS

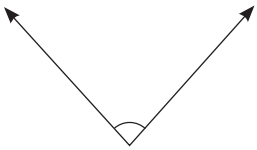
1. Luisa is a tile setter who works in The Pas, Manitoba. She is installing some tile in a science lab at the University College of the North. When installing tile, Luisa has to cut pieces of tile to fit the surface she is covering. Below are shown the angles of some pieces of tile that Luisa must cut.

   For each of these angles, measure the angle. Then determine the measure of the resulting angles after each of the original angles has been bisected.

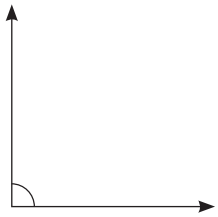
   a)

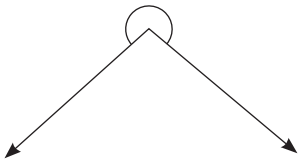
   b)

   c)

   d)

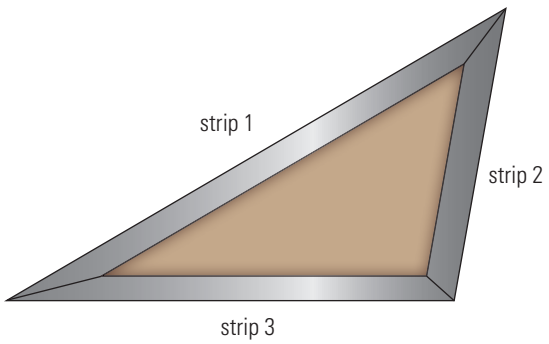
   

2. Jung Min has worked as a sheet metal worker for over 10 years. He has been hired to install work tables in the kitchen of St. Boniface General Hospital. These work tables need stainless steel edging bolted to their edges in order to reduce wear. Below is a scale (1:10) drawing of the stainless steel edging Jung Min will bolt over the top edges of one table. The edging has three mitred joints.

   a) Measure each angle.

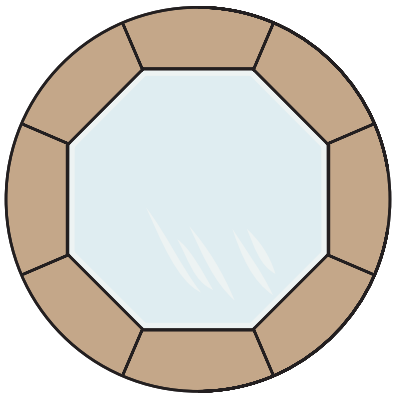
   b) Determine the measure of the bisector of each angle.

   c) Determine at what angle each end of each strip should be cut.

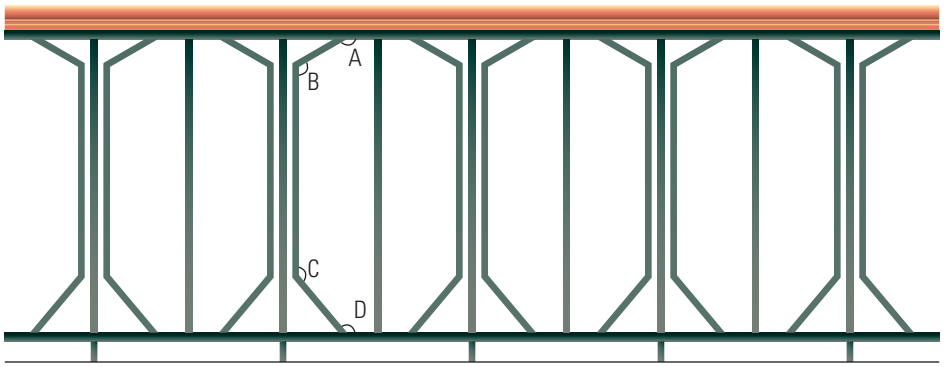


3. Imagine that you are a furniture-maker and have been asked to build a wood and glass tabletop according to the design shown here. Your client wants the outside to be made of wood, with an octagonal piece of glass in the centre. The tabletop is to be made of eight identical pieces of wood that have an arc on the outside edge, a straight side on the inside edge, and angled ends.

   The pieces of wood have to be cut so that their ends form mitre joints. If the mitre joints bisect the reflex angles outside of the octagon, at what angle relative to the straight sides of the wood must the ends be cut?

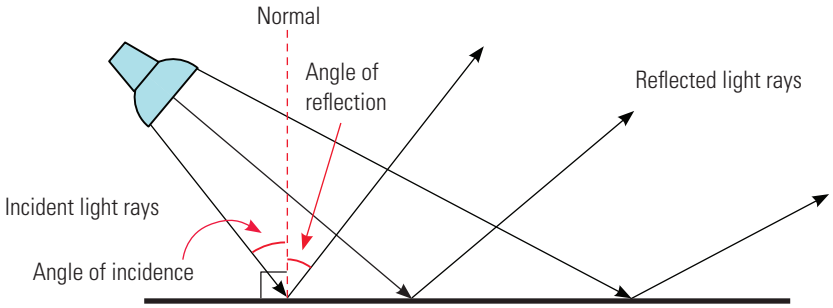


4. Jurek is a welder working on the construction of a pedestrian bridge in Cold Lake, Alberta. He is welding together bridge railing. As a decorative element to the rail, Jurek will weld four metal strips on each side of the figure, from the middle of each of the four marked angles to the edge of the piece of metal in the centre. A scale drawing of the plan is shown. Determine the measure of each angle bisector for angles A, B, C, and D.

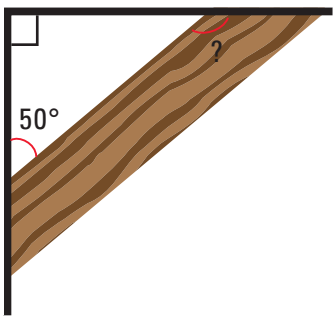


5. When a ray of light is reflected from a flat surface, the light strikes the surface at an angle (the angle of incidence) that is equal to the angle of reflection. Both of these angles are measured from a line that is perpendicular to the surface (called the normal).

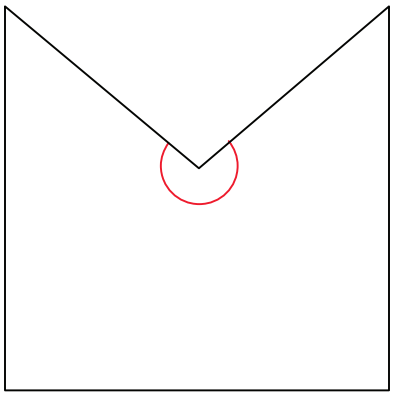
   Using the diagram below, measure the total angle between each pair of incident and reflected rays in the left, middle, and right rays. Then determine the angles of incidence and reflection for each ray.




6. A carpenter needs to cut a 2-by-4 piece of wood that will fit in a corner, as shown in the diagram below. If one end of the wood forms a 50° angle with one wall, at what angle must the other end be cut (at the indicated angle) to lay flat against the other wall? Why must that end be cut to that measure?



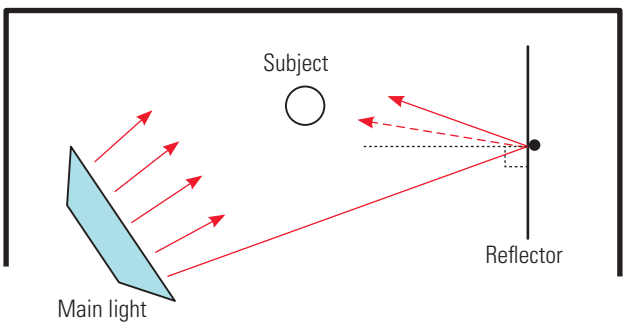
7. Suppose that you are creating a graphic design for a logo that is made up of a reflex angle at the top of a square area, similar to the one shown below, with the reflex angle divided into sixteen equal angles. Copy the design below onto a piece of paper. Use your knowledge of bisecting angles to draw line segments that divide the reflex angle into sixteen equal angles. What strategy will you use?



## Extend your thinking

8. Lan works as a portrait photographer in Watson Lake, Yukon. When taking photographs, Lan uses a flat reflector to brighten the shadow areas. The main light illuminates the front and left side of the subject, but leaves the right side in shadows. The reflector is supposed to bounce some of the light rays from the far right side of the main light source towards the right side of the subject (shown by the dashed ray). With the reflector in its present orientation (parallel to the right wall), those light rays will miss the subject (shown by the solid reflected ray).

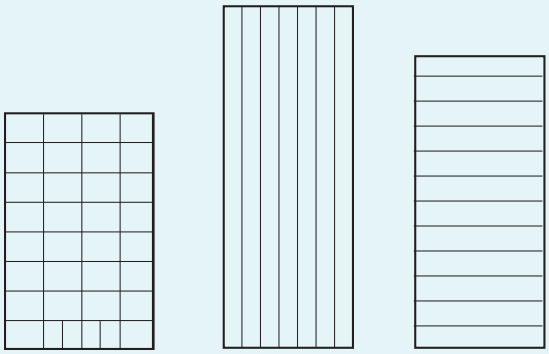
   Assume that the diagram below is an accurate scale diagram of Lan's studio. Determine how many degrees and in what direction (clockwise or counterclockwise) the reflector should be swivelled to ensure the right side of the subject's face is properly illuminated? (Consider how to change the direction of the normal line to the reflector so that the angle of incidence equals the angle of reflection depicted by the dashed ray.)



## DRAW DETAILS OF BUILDINGS IN 3-D

Buildings in city centres typically have features that are vertical and horizontal. The closest buildings in a perspective diagram should have a fair amount of detail. Buildings that appear further away will be too small for you to effectively draw fine details. Details on those buildings can be less fine.

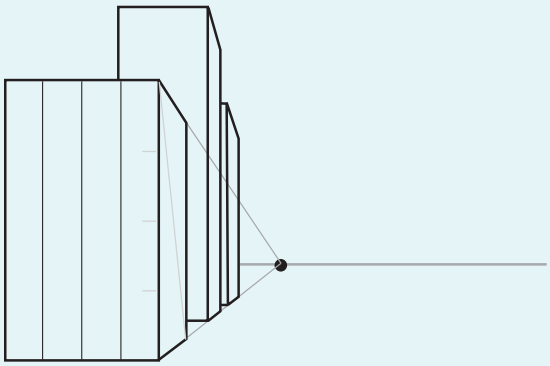
To help determine what detail to draw on buildings, create a profile sketch of the buildings with details (as shown in the example below). Notice that the building on the left (closest to the viewer) has more detail than the other two buildings.



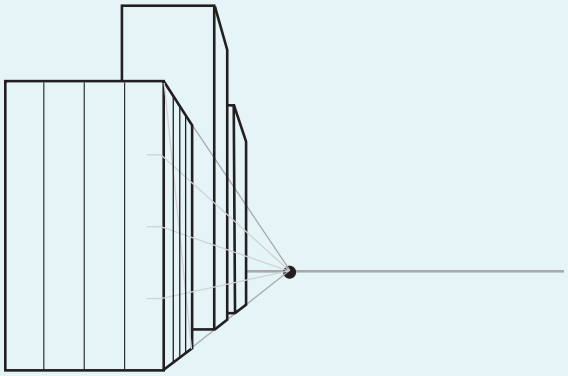
CLOSEST FURTHEST

Go back to the sketch of the buildings you previously made. Here is how you can draw vertical and horizontal details of a building in perspective.

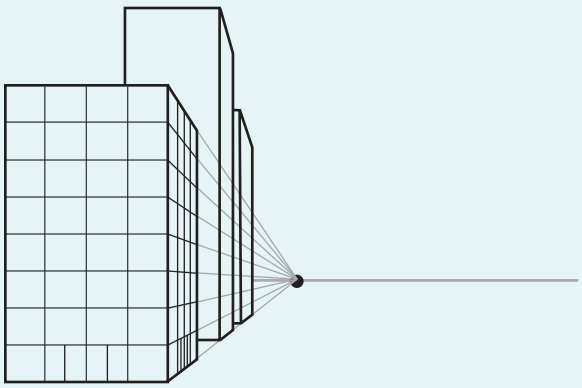
STEP 1: Draw the main vertical details on the front face. Then start drawing the verticals on the right face by very lightly drawing a diagonal guide line between opposite corners on that face. Then very lightly draw three evenly-spaced marks on the right front vertical edge of the building.



STEP 2: Finish the verticals on the right face by very lightly drawing guide lines from those marks on the right front edge towards the vanishing point. Draw the verticals through the points where these new guide lines meet the diagonal. Drawn correctly, the verticals on the right face should get progressively closer together as you go from left to right. When finished, carefully erase the diagonal.



STEP 3: Draw the horizontal details. There are seven horizontals spaced evenly apart. Draw them first on the front face. Then draw the horizontals on the right face in perspective, going towards the vanishing point.



STEP 4: Draw in any smaller details, like the doors.

Now, draw details on the other two buildings. You could draw primarily vertical details on the second building, and primarily horizontal details on the third building.

## THE ROOTS OF MATH

### GEOMETRIC PERSPECTIVE IN ART

*The School of Athens*, completed by Raphael in 1511, is famous for its use of perspective.

Panels painted with olive trees and placed behind stage actors were some of the first artistic attempts to make closer objects appear larger and distant objects appear smaller, or to produce perspective. This occurred in fifth-century Greece. Medieval and Byzantine art also incorporated perspective in paintings.

Renaissance artists such as Michelangelo are most frequently celebrated for their use of perspective, since they were the first to use the same principles of perspective and scale that artists use today. Between the fourteenth and seventeenth centuries, Renaissance artists used geometric perspective to create the appearance of three-dimensional space within two-dimensional paintings.

Geometric perspective uses four elements to create a three-dimensional effect, the first being a horizon line. It is often found at the viewer's eye level, and represents the horizon. The second is a vanishing point, a spot on the horizon line where the parallel lines in the painting converge and seem to disappear. Perspective lines—those drawn from the edges of objects and leading back into the distance—and angular lines are also used. Geometric scale also allows artists to create perspective by accurately representing the size of one object in relation to another.

1. Think about a photograph you took, a painting you like, or a poster you own. In what ways does it represent geometrical perspective? Identify parallel lines, a vanishing point, or a horizon line that it contains.
2. How could a picture have more than one vanishing point? Is it possible for a picture not to have a vanishing point? Explain your reasoning.
3. How would you define the term "parallel lines"? Describe a method you could use to prove whether or not two lines are parallel.

# 5.3 Non-Parallel Lines and Transversals

*Using her client's ideas and her own skills, an interior decorator can transform office space, hotel lobbies, or rooms in a house.*

## MATH *ON THE JOB*

An interior designer needs to know which textures and colours complement each other. Being able to place furniture so that it makes a large living room seem cozy or a cramped kitchen seem spacious is another necessary talent. Listening to and incorporating a client's ideas and preferences is also important—hot pink suede couches and leopard print rugs aren't for everyone. Michelle Diaz helps customers improve their living spaces by using her design, communication, and math skills. Michelle manages a showroom of designer fabrics in Winnipeg, Manitoba.

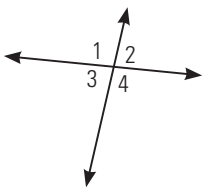
After going to school at Winnipeg's West Kildonan Collegiate, Michelle obtained her interior decorating diploma. Today, she works in sales, customer service, and order administration. Her job involves calculating the yardage needed for drapery and upholstery orders, calculating the length of drapery tracks, setting prices, and writing up invoices. As an interior designer, Michelle also needs to be able to produce polished drawings of design ideas to present to clients.

For an order, Michelle must cut square pieces of coloured cloth into six asymmetrical sections. The pieces will be stitched together to form multi-coloured cushion covers. To cut the fabric accurately, Michelle uses chalk to mark off the angles of the shapes she will cut.

The first line Michelle draws is horizontal and bisects the square. Next, she draws two vertical lines. The first vertical line forms four angles where it intersects with the horizontal line. Michelle draws this line so that the upper left angle that is formed measures 115°. She draws the second vertical line so that the lower right angle that is formed with the horizontal line measures 72°. What are the measures of the other six angles?

## EXPLORE THE MATH

**vertically opposite angles:** angles created by intersecting lines that share only a vertex




A variety of objects and materials such as trusses, railroad tracks, and fabrics contain intersecting lines. The measures of certain angles created by intersecting lines and the ability to identify types of angles can indicate whether these lines are parallel or non-parallel.

When two lines intersect each other, four distinct angles are created. The angles that share a side are adjacent angles. Angles that share only a vertex are **vertically opposite angles**.

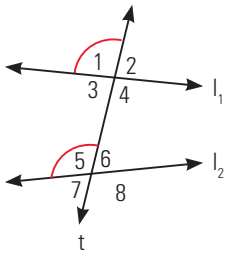
Consider this diagram. Pick an angle and identify the two angles that are adjacent to it and the one that is vertically opposite to it. What type of angle do any two adjacent angles make? How do the measures of vertically opposite angles seem to compare to each other?

Suppose that there are two main lines ($l_1$ and $l_2$) and a third line ($t$) intersects both of them. That third line ($t$) is a **transversal**. Pairs of angles formed by the intersections of these lines fit into categories based on their relative positions to each other.

**transversal:** a line that intersects two or more lines

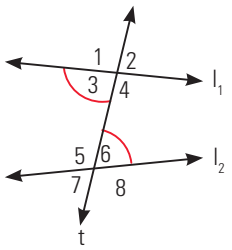
Angles that have the same corresponding positions at the two intersections are corresponding angles. In the diagram below, one pair of **corresponding angles** is highlighted. Identify the three other pairs.

**corresponding angles:** angles that occupy the same relative position in two different intersections

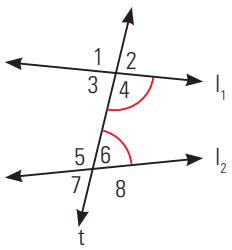



Angles between the two main lines are interior angles. Two interior angles that are on alternate sides of both the transversal and the interior of the main lines are called **alternate interior angles**. One pair of alternate interior angles is highlighted in the diagram below. Identify the other pair.

**alternate interior angles:** angles in opposite positions between two lines intersected by a transversal

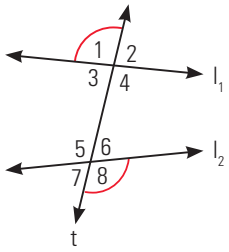



In the following diagram, two interior angles on the same side of the transversal have been highlighted. Identify the other pair of interior angles on the same side of the transversal.

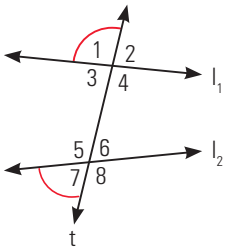


**alternate exterior angles:** angles in opposite positions outside two lines intersected by a transversal

Angles outside the two main lines are exterior angles. Alternate interior angles have exterior counterparts called **alternate exterior angles**. Alternate exterior angles are angles on the outside of two lines intersected by a transversal and are highlighted in the diagram below. Identify the other pair of alternate exterior angles.




Two exterior angles on the same side of the transversal are highlighted in the diagram below. Identify the other pair of exterior angles on the same side of the transversal.




## DISCUSS THE IDEAS

### LINES AND TRANSVERSALS

Lacrosse has many other names, including *baggataway* (Anishinabe) and *tewaarathon* (Mohawk). This game was first played by First Nations peoples. Today, many people play lacrosse for the great cardiovascular workout it provides. They also play to develop the coordination needed to manipulate the lacrosse stick and pass and shoot the ball accurately.

Lacrosse sticks contain parallel and non-parallel lines as well as transversals. These can be found in the top of the stick that holds the ball, which is called the head.

Work with a partner. You will each draw a diagram of a lacrosse stick, complete with netting (which looks similar to cross-bracing), like the one shown.

On one of the diagrams pick any two of the line segments as the main line segments, and one more that is a transversal to the other line segments. Next, identify the following types of angles (if they exist) based on those line

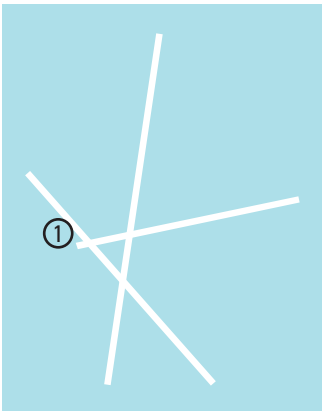
segments and transversal: two corresponding angles, two alternate interior angles, and two interior angles on the same side of the transversal.

Follow the same steps for the second diagram, but pick a different pair of main line segments and transversals. Measure the angles you have identified and discuss the following questions.

1. Are the measures of corresponding angles of non-parallel line segments and a transversal equal or different? Explain your reasoning.
2. Are the measures of alternate interior angles of non-parallel line segments and a transversal equal or different? Explain your reasoning.
3. Are the interior angles formed on the same side of the transversal by non-parallel line segments supplementary? Explain your reasoning.

## ACTIVITY 5.6
## MAPPING AN AIRPORT RUNWAY

Large airports often have multiple runways positioned so they run parallel to the direction in which winds typically blow. It is essential for airplanes to take off into the wind to help them gain lift during takeoff. The runway with the direction that most closely matches the direction of the prevailing wind at a given time is the one that is used for takeoffs and landings. The diagram shows the runway layout of a Winnipeg airport.



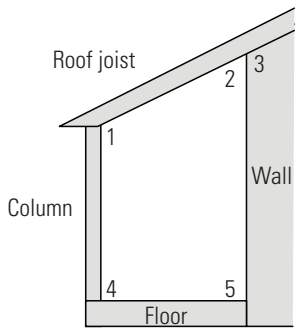

Winnipeg's James Armstrong Richardson International Airport has three intersecting runways.

Work with a partner to complete the following steps.

1. Place a sheet of tracing paper over the runway diagram and copy it.
2. Starting from 1, number each of the exterior and interior angles on your copy of the runway layout, going from left to right and top to bottom.
3. Select two of the runways to be main line segments and label them $l_1$ and $l_2$. The remaining runway will be the transversal, $t$.
4. Identify and list all pairs of the following types of angles: corresponding, alternate interior, interior angles on the same side of the transversal, alternate exterior, exterior angles on the same side of the transversal, and vertically opposite angles. Explain your reasoning for identifying each pair.
5. Measure the exterior angles, then compare the alternate exterior angles. Do any two alternate exterior angles have the same angle measure? Explain your conclusion.
6. Add up the measures of the exterior angles on the same side of the transversal. Are the angles in each pair supplementary angles? Explain your reasoning.

### *Example 1*

Below is a side diagram of a verandah that is attached to a house.




For each pair of angles listed below, identify the kind of angle pair as well as the parts of the verandah that make up the angle pair's lines and transversals.

a) $\angle 1$ and $\angle 4$

b) $\angle 3$ and $\angle 5$

c) $\angle 1$ and $\angle 3$

**SOLUTION**

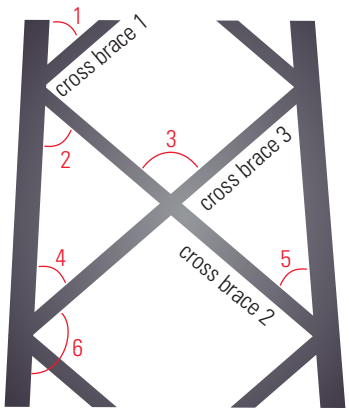
a) $\angle 1$ and $\angle 4$ are inside the lines and on the same side of the transversal, so they are interior angles on the same side of the transversal. The roof joist and floor are the lines and the column is the transversal.

b) $\angle 3$ and $\angle 5$ are on alternate sides of a transversal, so they are alternate interior angles. The roof joist and floor are the lines and the wall is the transversal.

c) $\angle 1$ and $\angle 3$ are in the same position relative to two lines and a transversal, so they are corresponding angles. The column and wall are the lines and the roof joist is the transversal.

## *Example 2*

Lattice towers are free-standing structures that have cross-bracing to give the structures the strength and rigidity needed to stand by themselves without additional support. The lattice that exists on each side of a lattice tower is essentially a series of pairs of line segments and transversals. The diagram below shows a small part of one such tower.




Transmission towers often use a lattice steel pylon design to support cables and conductors.

The following list contains three pairs of angles and the type of angle each pair is. Determine which two parts of the tower make up the main line segments, and which part makes up the transversal that forms each of these pairs of angles.

a) Angles 3 and 4 are corresponding angles.

b) Angles 2 and 5 are alternate interior angles.

c) Angles 1 and 6 are exterior angles on the same side of the transversal.

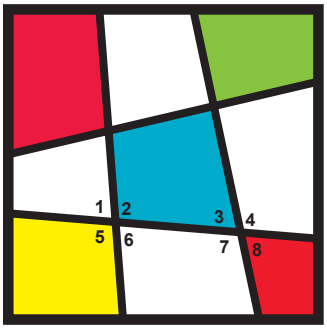
**SOLUTION**

a) Angles 3 and 4 are both above cross brace #3, and to the right of cross brace #2 and the left side of the tower respectively. The transversal is cross brace #3, and the main line segments are cross brace #2 and the left side of the tower.

b) Angles 2 and 5 are in between the left and right sides of the tower. They are also on alternate sides of cross brace #2. In this case, the transversal is cross brace #2, and the two main line segments are the left and right sides of the tower.

c) Angle 1 is above cross brace #1, and to the right of the left side of the tower. Angle 6 is below cross brace #3 and also to the right of the left side of the tower. This makes the left side of the tower the transversal, and cross braces #1 and #3 the main line segments.

## BUILD YOUR SKILLS




1. Aimée is an artist who makes stained-glass items. She lives and works in the small town of Jean Marie River, NT, but sells many of her items over the internet. Currently, Aimée is working on a window whose outer dimensions are 0.5 m by 0.5 m. The design consists of 9 glass panels of different colours bound by the square frame and non-parallel dividers, as shown in the diagram.

   The two dividers going from the top side to the bottom side of the frame are the main line segments, and the bottom divider going from the left to the right side of the frame is the transversal.

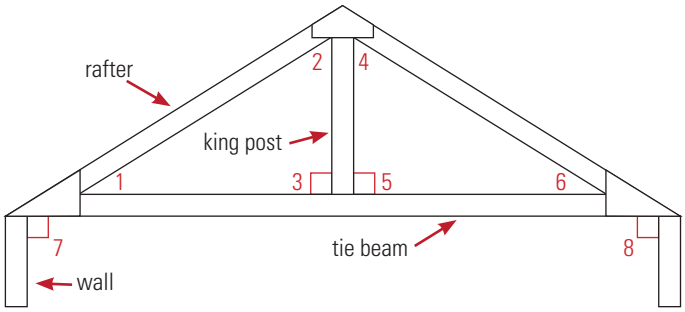
   a) Determine which angle is vertically opposite to angle 1. Measure it, along with angle 1. Is the measure of angle 1 equal to its vertically opposite angle?

   b) Determine which angle is the alternate exterior angle to angle 4. Measure these two angles. Do these alternate exterior angles have equal measures?

   c) Determine which angle is the corresponding angle to angle 6. Measure the two angles. Are the measures of these corresponding angles equal?

   d) Angle 2 is one of a pair of interior angles on the same side of a transversal. Which angle is the other angle in this pair? Measure both angles. Are the measures of these angles supplementary or not?

2. Pitseolak lives in Pelly Bay, Nunavut. She is an avid kayaker and is building a small shed to store her new kayak and fishing equipment in. Before building, she draws a plan for the shed. Consider her sketch of the shed's roof, which will be supported by a king post roof truss.




a) If the right rafter and the left rafter make up the two main line segments, name one possible transversal for the rafters.

b) What type of angles are angles 3 and 5?

c) If the walls are the line segments and the tie beam is the transversal, what type of angles are angles 7 and 8?

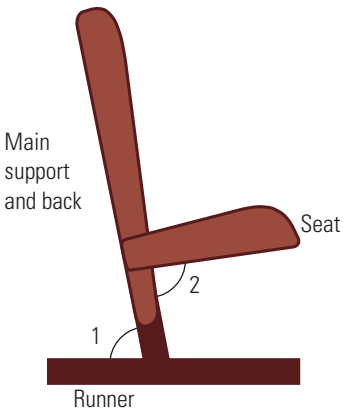
d) If angles 1 and angles 2 and 4 combined are a pair of interior angles on the same side of a transversal, which components of the truss must form the two main line segments and the transversal?

3. Laurence is an apprentice furniture-maker trying to make a chair with a side profile like the one below. The seat and runners are not parallel to each other. The main support and back is to be slanted 12° from an imaginary vertical line, and the seat should be 7° above an imaginary horizontal line. Do the following.

a) Consider what parts would make up the two main line segments and the transversal.

b) State the type of angles (corresponding, alternate interior, interior angles on the same side of the transversal, alternate exterior, or exterior angles on the same side of the transversal) that angles 1 and 2 are.

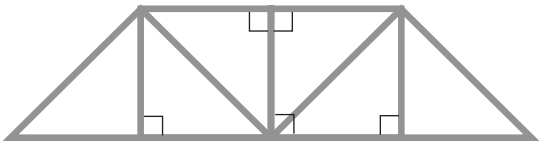
c) Determine what the degree measures of angles 1 and 2 should be. (Do not measure directly with a protractor because the diagram is not to scale.)



## Extend your thinking

The railings and supports of this bridge are reinforced with right triangles.

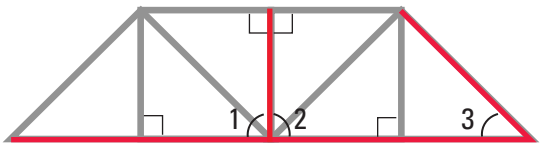
4. Below is a diagram of a Pratt truss, which is used in the construction of many bridges. A Pratt truss contains components that are parallel and non-parallel, perpendicular and non-perpendicular.



a) Consider these components—a vertical and a diagonal member form the two main (non-parallel) line segments, and the bottom line segment forms the transversal. Measure angles 1, 2, and 3, then do the following analysis. Explain your reasoning for each question.

i) Compare the measures of the corresponding angles, ∠1 and ∠3. Is ∠1 equal to, less than, or greater than ∠3?

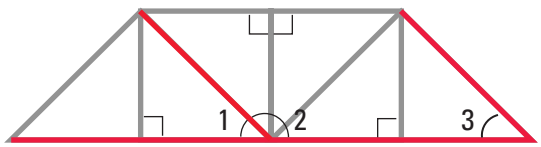
ii) Add the measures of the interior angles on the same side of the transversal (∠2 and ∠3) together. Are angles 2 and 3 supplementary angles or not? Explain your reasoning.




b) Consider these components—two diagonal members form the two main (parallel) line segments, and the bottom line segment forms the transversal. Measure angles 1, 2, and 3, do the following analysis, and explain your reasoning for each.

i) Compare the measures of the corresponding angles, ∠1 and ∠3. Is ∠1 equal to, less than, or greater than ∠3?

ii) Add the measures of the interior angles on the same side of the transversal (∠2 and ∠3) together. Are angles 2 and 3 supplementary angles or not? Explain your reasoning.



c) Consider the results for parts a) and b).

i) What did you notice about the measures of corresponding angles for parallel line segments that was not true for non-parallel line segments?

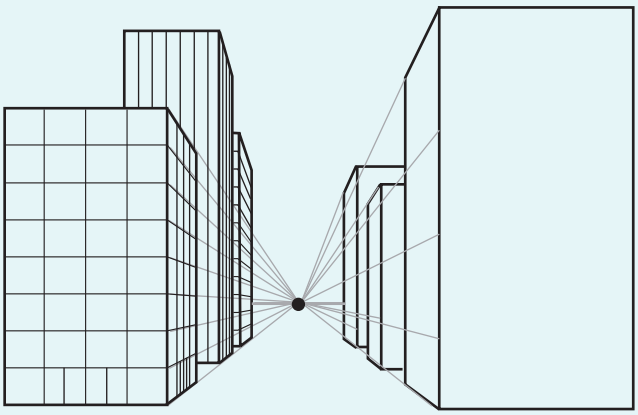
ii) Are either pair of interior angles on the same side of the transversal, one formed by two parallel line segments and one formed by non-parallel line segments, supplementary angles or not? Explain your reasoning.

*The Victoria Bridge in Saskatoon, Saskatchewan, crosses the South Saskatchewan River.*

## DRAW A STREET SCENE IN 3-D

The drawings you have made so far have been of buildings on one side of the street. This project requires an entire street scene to be done. Complete the street scene by drawing the right side of the street, which will primarily show the left faces of the buildings drawn in perspective. The same vanishing point must be used. The spacing between buildings should be the same as the other side of the street, so that streets crossing the one in the diagram are aligned.

Draw details on these buildings as you see fit. Use the same techniques for drawing horizontal and vertical detail you used earlier. Draw a sidewalk on each side of the street. When finished, use this diagram as the basis of your project diagram.



# Parallel Lines and Transversals

# 5.4

## MATH *ON THE JOB*

June Peterson is a member of the Sturgeon Lake Cree Nation and lives in Sturgeon Lake, Alberta. After graduating from Hillside High School, she took the pre-trades program through the Northern Alberta Institute of Technology (NAIT). Since she has always loved building things, she focussed on the carpentry aspect of the program and worked with a local contractor on a new housing project at Sturgeon Lake.

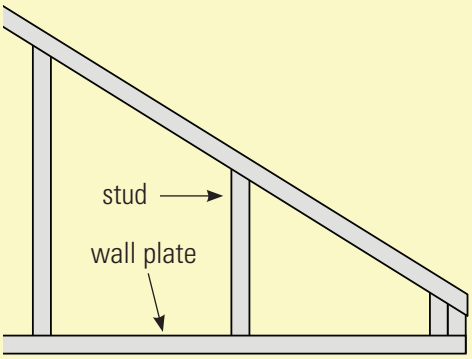
June often works on wood-framed houses and buildings. The frames are made of studs (parallel, vertical pieces), and wall plates (pieces that are attached along the top and bottom of the studs). Frames are usually constructed on the ground or floor and then erected. The wall plates hold the studs in position.

Part of June's job is to make sure that the studs are exactly perpendicular to the bottom wall plate and parallel to each other. To do this, she uses a measuring device such as a carpenters' square, which is used to measure and mark off 90° angles.

*Carpenters often make rough sketches of structures they work on.*

June is constructing a partial wall for the side of a staircase. The top of the wall follows the slope of the staircase. A partial diagram of the framing for the staircase is shown here.

- Decide upon a reasonable angle for the staircase. Staircase angles range between 33° and 42°.
- To make the studs parallel, what angle measure will June need to make between the studs and the bottom wall plate?
- To make the ends of the studs align with the top wall plate, what angle will June need to make between the studs and the top wall plate?




## EXPLORE THE MATH

Two lines are parallel if they never intersect each other. This only happens when the lines are a constant distance from each other. Imagine a case where you find parallel lines, then imagine what would happen if those lines were not parallel. For example, lines drawn on roads to define lanes should be parallel. What would happen if those lines became closer and closer together? In houses, the studs that support walls should be vertical and parallel to each other. What would happen to a wall if the stud at one end of a wall was vertical but the other studs were not parallel to this one and to each other?

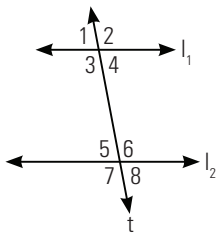
Craftsmen use tape measures to measure and create accurate angles.

If two lines are parallel and are intersected by a transversal, the corresponding angles, alternate interior angles, interior angles on the same side of a transversal, alternate exterior angles, and exterior angles on the same side of a transversal all have certain properties. Whether lines are parallel or not can be confirmed if these angles have certain measures.

Look at the pair of corresponding angles, ∠1 and ∠5, in the diagram below. Measure them. How do the measures of the angles compare? Are they equal or different? Is this also true of the other pairs of corresponding angles?

Look at the pair of alternate interior angles, ∠3 and ∠6, in the diagram below. Measure them. Are the measures the same or different? Is the same true of the other pair of alternate interior angles? What about the pairs of alternate exterior angles?

Look at the pair of interior angles on the same side of the transversal, ∠3 and ∠5, in the diagram below. Measure them. What is the sum of the angle measures? Is this true of the other pair of interior angles on the same side of the transversal, and the two pairs of exterior angles on the same side of the transversal?




**HINT**

In a diagram of two lines and a transversal, if the corresponding angles are equal, the lines are *not* parallel.

What you should have found by measuring the angles above is that when two lines are parallel and intersected by a transversal:

- The measures of corresponding angles, alternate interior angles, and alternate exterior angles will be equal. (If such angles do not have equal measures, then the lines are not parallel.)
- Interior and exterior angles on the same side of a transversal will be supplementary. (If they are not, then the lines are not parallel.)

## DISCUSS THE IDEAS

### DRAWING PARALLEL LINE SEGMENTS

Wooden drying racks are used by many peoples to dry and smoke fish and meat. The racks are made of parallel rods, over which strips of scored meat or filleted, scored fish are hung. A smoldering smoky fire is lit on the ground in the middle of the structure. It can take from twelve hours to several days for the fish or meat to be ready.

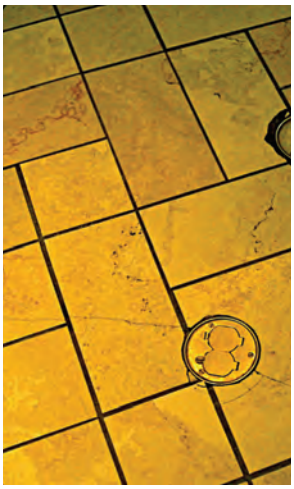
Drying fish was traditionally used as a way of preserving meat for consumption during the winter.

Some First Nations peoples build drying racks in square or rectangular shapes. Square-shaped drying racks are made of a square held up by four vertical posts. Parallel horizontal poles are fastened across the square. Triangular drying racks look like a triangular tent with parallel poles fastened horizontally, end-to-end.

Imagine you are out on a fishing trip and catch many fish. You would like to smoke some of it at your camp, in order to preserve it. Would you build a triangular or rectangular drying rack? Explain your reasoning.

Work with a partner. Choose the shape of rack you would build and draw a construction diagram. One partner can draw the horizontal line segments and the other can draw the parallel lines. You can draw the sides of the racks separately or in three dimensions. Choose a scale factor to use, such as 1:48.

While you are working on this task, determine how to ensure that the parallel line segments remain parallel. Discuss how much space you need to leave between each segment of the rack that fish will be draped over, so that air and smoke can circulate evenly. (Fish fillets are often around 30 cm long.) What would be the best type of material to make your rack from? How would you fasten the poles together? How could you discourage animals and insects from coming near the fish?



Different-sized tiles are used to set tile in the French pattern.

## ACTIVITY 5.7
## DESIGN A FRENCH PATTERN TILE FLOOR

When tile is laid in a French pattern, the result is a mosaic that connects square and rectangular tiles of different sizes. The pattern is random, but each stone must be flush with the ones beside it. The vertical sides of all the tiles should be parallel with each other, as should the horizontal sides. Traditionally, marble and travertine, a type of limestone, were used to make a French pattern tile floor. A French pattern tile floor can be made using different-sized squares that were made to the same scale.

Imagine you are installing tile on a kitchen floor in a French pattern. On a sheet of graph paper, measure off a square that is 15 cm by 15 cm. Draw in tiles in the French pattern. Start with one tile that is 3 cm by 4 cm. Use a scale of 1.5:1. (You can round your measurements up to one decimal place.) Add two tiles that were enlarged using this scale and two that were reduced. Add a tile that is 1 cm by 1 cm. Use a scale of 3:1. Add two tiles that were enlarged using this scale. Fill in the rest of the paper with tiles of different sizes. Make sure the horizontal and vertical sides are parallel.

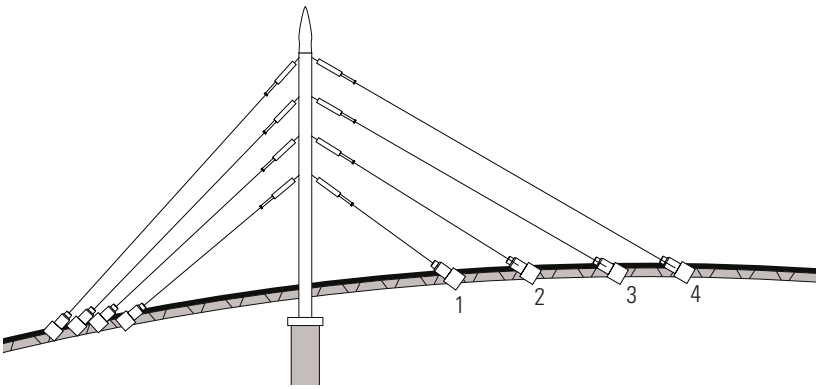
Think about how a craftsperson would make sure the horizontal and vertical sides of the tiles are parallel to each other. What tools could you use to help you do this? Ensure that the lines are parallel by making the interior and exterior angles on the same side of a transversal add up to 180°.

### *Example 1*

Winnipeg's Esplanade Riel is a side-spar cable-stay bridge designed for pedestrian use. It crosses the Red River to connect downtown Winnipeg with St. Boniface.

A cable stay bridge is made of a support tower and cables that reach down to the bridge deck. The cables, which can be parallel or angled, suspend the bridge above water. One of Canada's most well-known cable stay bridges is the Esplanade Riel in Winnipeg.

Consider the diagram below. Determine which of the four indicated cables are parallel to each other and which ones are not. Explain how you came to your conclusion.



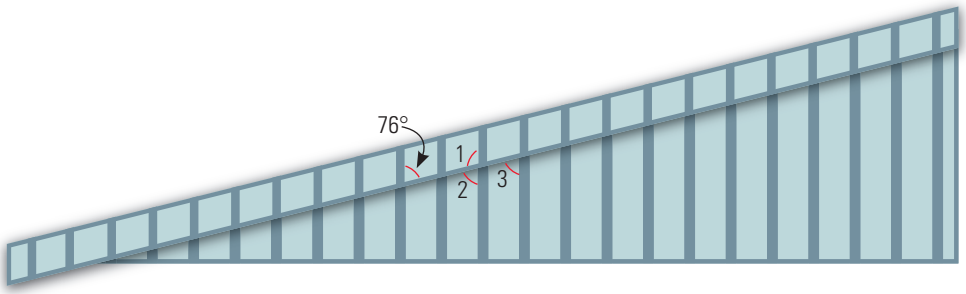
**SOLUTION**

You could measure the bottom angles between the right side of the vertical tower and each cable. Since these angles all have the same positions relative to the points where the cables intersect the tower, these angles are corresponding angles. Those angles, from bottom to top, should have the measures 54°, 58°, 60°, and 60°. The only two corresponding angles that have equal measures are the ones under Cable 3 and Cable 4 (both 60°), making Cables 3 and 4 the only cables that are parallel to each other. The other cables are not parallel to each other because the measures of their corresponding angles do not equal the measures of other corresponding angles.

## *Example 2*

Danielle is a sheet metal worker. She specializes in installing sheet metal roofs in her hometown of La Ronge, Saskatchewan. The sheet metal roofing Danielle uses contains parallel ridges. It can be purchased in segments, snapped into place, and secured with screws or nails.

Below is a diagram showing two segments of sheet metal roofing. The horizontal line, or transversal, represents where the two segments meet. The vertical lines are ridges. One angle is given. State the measures that angles 1, 2, and 3 must have if the ridges are parallel to each other. Explain why they must have those measures.




**SOLUTION**

Angle 1 forms a pair of interior angles on the same side of a transversal with the 76° angle. These two angles must be supplementary, so the measure of Angle 1 must be 180° minus 76°, which equals 104°.

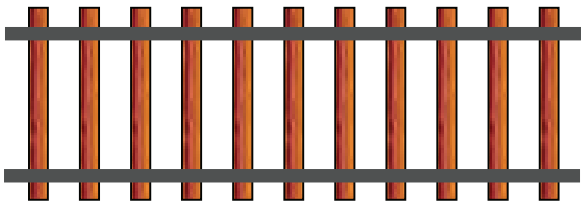
Angle 2 is an alternate interior angle to the 76° angle. These two angles must have equal measures, so the measure of Angle 2 must equal 76°.

Angle 3 is another alternate interior angle to the 76° angle, and is a corresponding angle to Angle 2. These angles must all have equal measures, so the measure of Angle 3 must also equal 76°.

## BUILD YOUR SKILLS

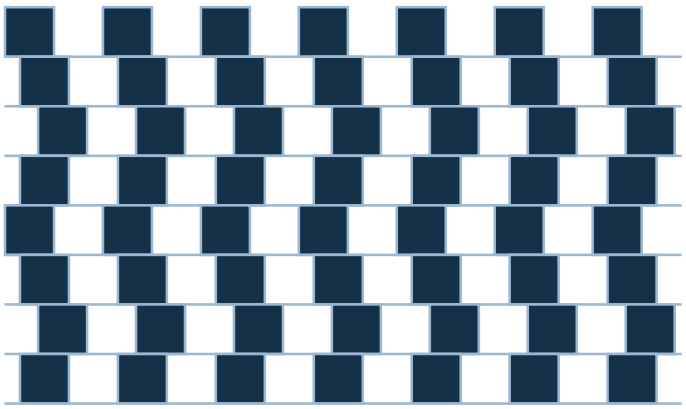
1. After graduating from high school, Jamila worked for her uncle's flooring business in Port Hardy, British Columbia, and learned how to install electric radiant heating. Electric radiant heating consists of criss-crossing parallel lines of conductive wires that are installed underneath the floor. When she works, Jamila needs to ensure the wires are parallel, so that the heating is distributed evenly. Sketch how a radiant heat system might look by drawing three lines that are parallel. Then, draw three parallel transversals.

2. Railroads in Canada are typically made out of parallel steel rails that are held in place by wooden ties. State two ways that rail workers could ensure that the rails run parallel to each other.

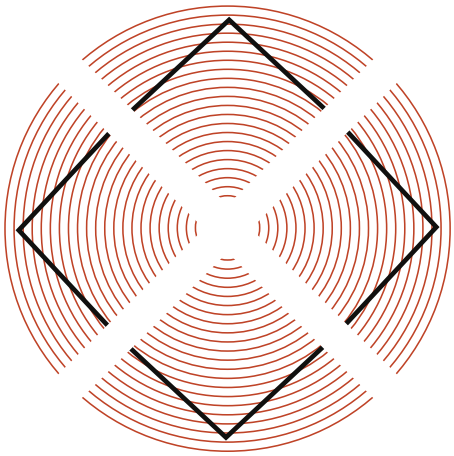


3. Sometimes optical illusions can make lines look parallel when they are not, and non-parallel when they are. Look at two line segments in each of the drawings below. Judge whether they are parallel. Describe two different techniques you would use to determine whether or not they are parallel.

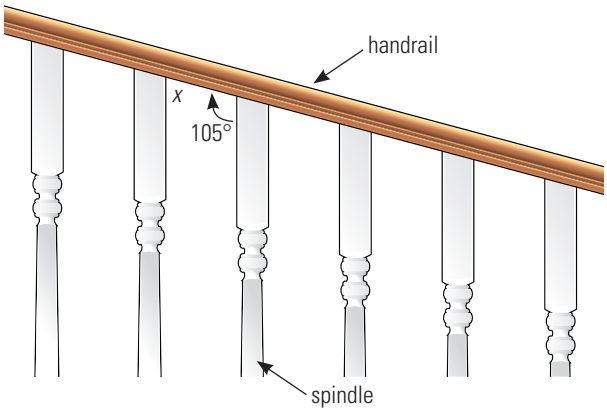
   a)



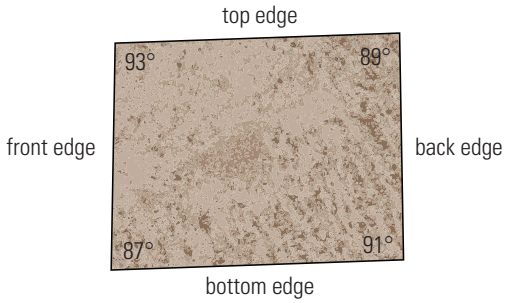
b)



4. To make sure his grandmother can get upstairs safely, Robert is installing a new banister on her stairs. Vertical posts, called spindles, attached to the handrails of a banister must be parallel to each other. What is the degree measure of $x$? Explain how you determine that measure.

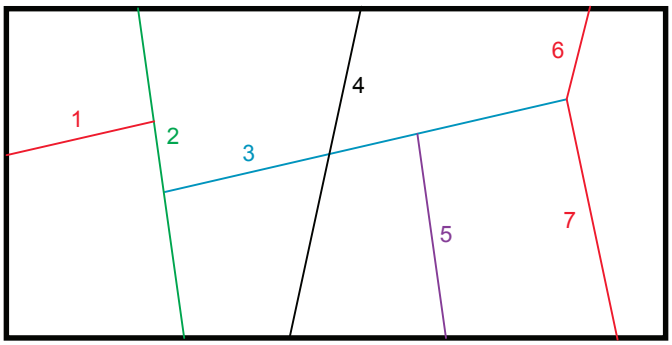



5. The front face of a concrete block is shown in the diagram below. Consider the indicated angle measures to determine which edges (if any) are parallel, and which edges (if any) are not parallel.



6. Giang gives boat tours along the Yukon River. When on the river, he tries to stay 10 m away from, and run parallel to, other boats on the river for safety reasons. If Giang's boat is travelling on a bearing of 193° while another boat is travelling at a bearing of 201°, by how many degrees starboard (right) or port (left) must Giang adjust his boat's course to make it parallel to the other boat's course?

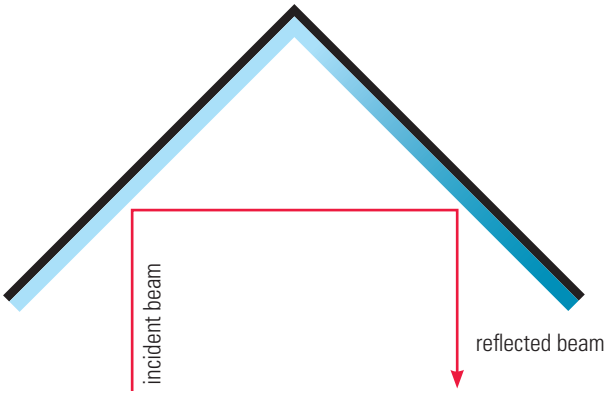
7. An architect designs a wall of a building with line segments that make up a lattice. Which of the numbered line segments is parallel to other(s)? In each case, state how you can tell the line segments are parallel.




## Extend your thinking

8. A client has asked a shop that specializes in mirrors to create a full-length roof mirror to be used by people standing in front of it. (A roof mirror is so named because it is shaped like the roof of a house). A roof mirror reverses a reflection so that people looking at themselves see their reflections oriented as others see them rather than oriented as a standard mirror image. (The left side of your body will appear on the right side of the reflection.)

   A roof mirror can be created by joining two full-length rectangular mirrors vertically so that their reflecting surfaces are perpendicular and facing inwards. The perfectness of the right angle can be tested by shining a narrow beam of light straight towards the roof mirror. If, when viewed from above, the reflected beam is parallel to the incident beam (as shown in the ray diagram below), then the mirrors are perpendicular to each other, and a roof mirror will work as expected.




a) Think about how you would ensure the mirrors could be held in place to form a 90° angle.

b) Draw a ray diagram similar to the one above, but with an angle between the mirrors measuring less than 90°. Do you think that the same orientation effect can be achieved? Explain your reasoning.

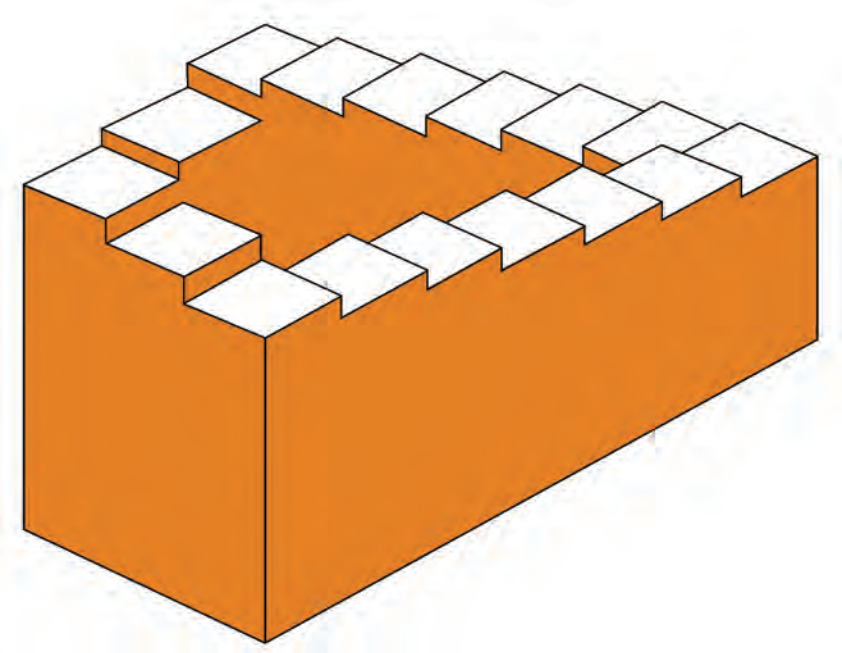
c) Do the same as in part b) of this question, but use an angle that is greater than 90°.

If possible, obtain two small rectangular mirrors to see how light is reflected off of them.

## PUZZLE IT OUT

### THE IMPOSSIBLE STAIRCASE

This drawing of an impossible object, the first of its kind, was created by the English geneticist Roger Penrose. It is a two-dimensional depiction of a staircase in which the stairs make four 90° turns as they ascend or descend. Yet they form a continuous loop, meaning that a person could climb them forever and never get any higher (or lower). The staircase is impossible in three dimensions, but looks possible in two dimensions because of distorted perspective.



Follow the staircase around. Can you determine the lowest or highest step? What happens when you go around in a clockwise direction? In a counter-clockwise one? Try creating your own neverending staircase (without tracing this image).

Explain how this illusion works, using what you have learned about parallel lines.

## CREATING 3-D PERSPECTIVE

You must now create the street scene that will appear on the front cover of THETA. Use your preliminary drawings as guides for making the final drawing. Create a cover drawing that is as close to publishable quality as you can make it. Ensure that all edges and details are easy to see, and that all temporary lines are fully erased.

Make a display that includes the following items:

- all preliminary sketches/diagrams used to develop the final drawing; and
- the final cover drawing itself.

Be sure to sign your name, and the name of any partner(s) involved in the creation of this drawing, in the bottom right corner of each item.

*A graphic artist's studio.*

# REFLECT ON YOUR LEARNING

## ANGLES AND PARALLEL LINES

Now that you have finished this chapter, you are able to

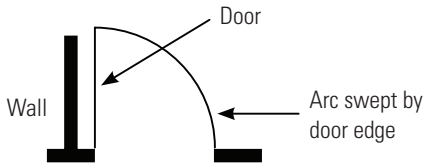
- ❒ draw, measure, and describe angles of various measures;
- ❒ determine whether pairs of angles are complementary, supplementary, or neither;
- ❒ use referents to estimate angle measures;
- ❒ bisect angles;
- ❒ identify corresponding, alternate interior, and alternate exterior angles around lines and transversals;
- ❒ identify interior and exterior angles on the same side of a transversal; and
- ❒ determine whether or not lines are parallel.

You will also have finished a chapter project that allowed you to apply these skills in a practical way to a real-world task.

## PRACTISE YOUR NEW SKILLS

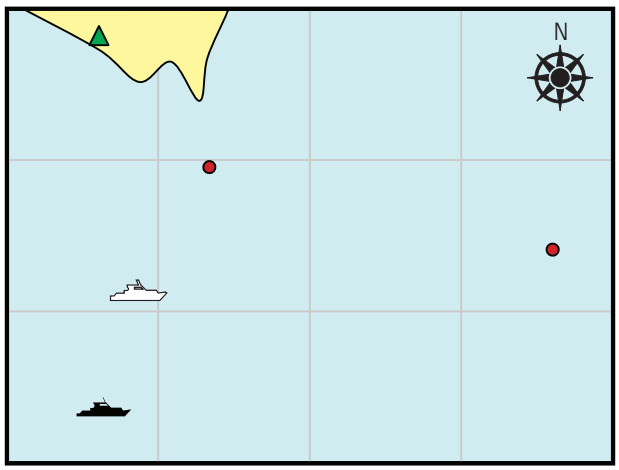
1. As her first job as a designer, Martine was asked to create a large decorative panel for a café in Medicine Hat. The panel will be suspended from a ceiling by cables, to partially hide exposed pipes and electrical conduits. To show what the panel will look like from the front of the coffee shop, make a 1:24 scale front-view diagram in which 1 inch equals 2 feet. The diagram should depict the panel suspended 2 feet below the ceiling. The room is 16 feet wide and 12 feet tall. The panel is to be 11 feet wide by $\frac{1}{2}$ foot tall. Two cables will hold up the panel on each of its sides, which will have 95° and 110° angles (relative to the top of the panel) when viewed from the front.

2. A standard door in a floor plan is drawn as a line segment with the arc swept by the door's edge (typically a quarter circle representing 90°) when it is fully swung open.




   On a sheet of paper, draw a similar diagram of a door that will open as wide as 125° to rest along a wall that is at a 125° angle to the adjoining wall, rather than the standard 90°.

3. Suppose that you are the skipper of the boat depicted by the white symbol in the map below. Place a sheet of paper over the map. Draw a clockwise course made up of three legs that will take you just past each of the buoys (marked in red), to a spot right beside the boat (depicted in black). Use the vertical gridlines, which point north, to determine what true bearing the boat must be on during each leg, and record those bearings.



4. Garnett is a draftsman, and part of his job consists of making floor plans. He has been asked to create a small meeting room to add to the Makkuttukkuvik Youth Centre in Iqaluit. Imagine that you have Garnett's job. Draw a sketch so you can show the staff of the centre what you propose.

   - The front wall has a length of 16 feet.
   - The left wall has a length of 12 feet, and the right wall a length of 10 feet. Both walls are perpendicular to the front wall.
   - The back wall also has a length of 12 feet and is perpendicular to the left wall.
   - There is a fifth wall that joins the back and right walls at an angle.

   Use this information to draw that sketch. Then analyze the diagram to estimate the length that the fifth wall will have, along with the measures of the angles it makes with the back and right walls. Also draw in a door on the side of the fifth wall that will open fully to the maximum angle. What is that angle?

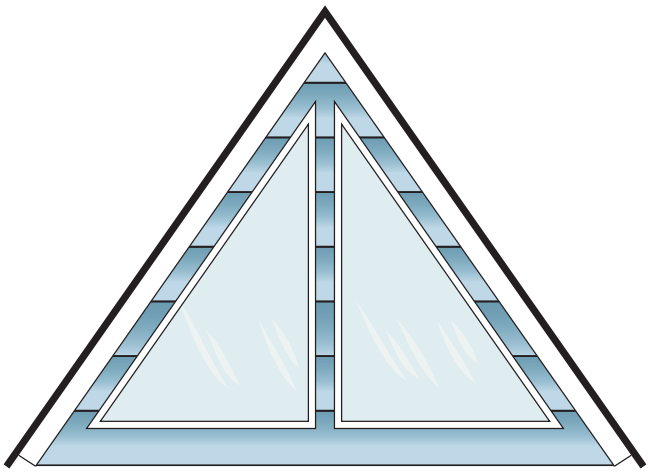
5. You are a contractor in charge of building a house with exterior walls that face the four cardinal directions—N, E, S, and W. A meteorologist's report shows that strong winds in that area most likely come in from the NW, WSW, and SSE directions. You had intended to have double-pane windows installed in all exterior walls. But to make the house more energy efficient, you now want to install triple-pane windows with extra insulating properties on the two walls most likely to be hit by cold winds. Draw a sketch of the house, and label the directions from which the strong winds typically blow. Then determine which of those two walls should get the triple-pane windows.

*Installing multi-paned windows will make a house more energy efficient.*

6. Binh makes custom windows. He is asked to make a pair of windows shaped like right triangles that will be installed side by side in the top floor of an A-frame house. The two halves of the ceiling meet at a 70-degree angle. The sloped side of each window runs parallel to the ceiling. The diagram below roughly depicts the arrangement. Determine the following measures:

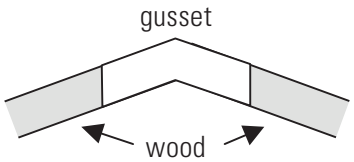
   a) the measures of the top and bottom angles; and

   b) the measures of the angles needed to cut the pieces of window frame to form mitre joints at each vertex.

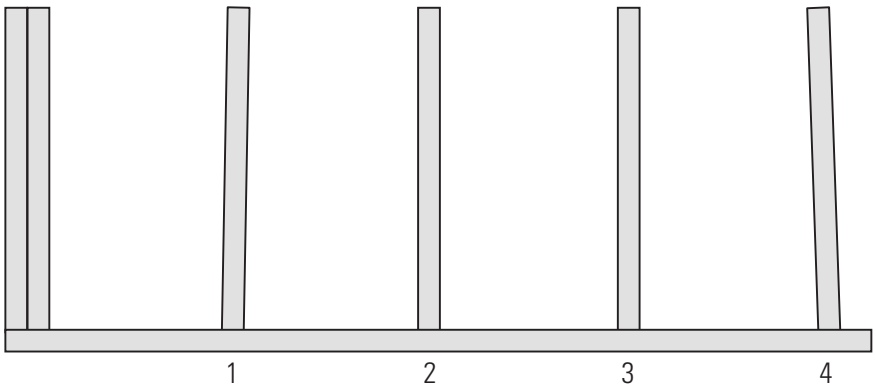


When mitre joints are under pressure or need added stability, gussets are used to reinforce the joint.

7. A gusset is an angled reinforcing plate that is glued and either nailed or bolted onto a joint where pieces of wood meet end to end. A gusset helps reinforce this type of joint. Suppose that you are a roofer who wants to make a series of identical gussets like the ones depicted below from a sheet of $\frac{3}{4}$-inch plywood. To ensure that the gussets have a consistent shape, make a template on paper or cardstock. The template will be used to draw outlines of the gussets on the plywood. Use your knowledge of parallel lines and angle bisectors to draw a gusset template that meets the following criteria. Each gusset is to precisely fit over two 2-by-6 boards that meet in a mitre joint at a 120° angle. Each "wing" of the gusset must be exactly $5\frac{1}{2}$ inches wide, and at least 8 inches long. The ends of the gusset should be parallel to each other.



8. The following diagram shows part of the framing in a wall, with 2-by-4 studs nailed into a bottom wall plate. The double stud on the left side is perfectly perpendicular to the wall plate. Which of the numbered studs are perpendicular to the bottom plate? Which of the studs need to be moved before they are nailed to the top plate? Explain how you identified which studs are parallel to the left side, and which are not.




9. Suppose that one aircraft travels at 120 km/h due west, and another at 100 km/h due south. They take off from the same airport at roughly the same time.

   a) Calculate how far each of the aircraft will travel in 30 minutes.

   b) Without using measuring tools, draw a rough sketch showing the approximate positions of the aircraft from the airport after 30 minutes.