Chapter 3

Length, Area, and Volume

GOALS

Measurement is an essential skill that everyone uses every day to make sense of the physical world. In the workplace, you will need to know how to take accurate measurements and how to estimate measurements. In this chapter, you will use your knowledge of fractions, decimals, ratios, and estimation to learn about measurement using both the Système international d'unités (SI) and the imperial system of measurement. You will learn how to

- convert measurements from SI to imperial units and from imperial to SI units;
- calculate perimeter, circumference, and area in imperial units; and
- calculate the surface area and volume of three-dimensional objects in imperial units.

KEY TERMS

- base unit
- capacity
- conversion factor
- foot (ft)
- geometric net
- imperial system
- inch (in)
- mile (mi)
- referent
- surface area
- Système international d'unités (SI)
- volume
- yard (yd)

START TO PLAN

PROJECT OVERVIEW

Have you ever gone ice fishing or seen an ice-fishing shelter? Imagine trying to ice fish for many hours without a shelter to protect you from the cold. In this chapter, your project will be to design an ice-fishing shelter and estimate the cost of its construction.

You will design a small ice-fishing shelter that can seat 2 to 3 people. Your shelter will be sheathed with plywood, and it must have a door, at least one window, a portable propane heater, a flat roof, 2 or 3 fishing holes, interior seating, and a painted exterior.

This ice-fishing shelter seats two people and has a wood-burning stove.

GET STARTED

To begin your project, plan the design of your shelter. You can use the internet and books on construction to research ice-fishing shelters and other shed-like designs. Make a list of all the things you will need to consider. For example:

- What dimensions are needed to fit 2 to 3 people?
- What size of heater will you need for your shelter?
- How will you design your seating?
- Where will you drill the holes in the floor?
- What will be the dimensions of your door and window?

Visualize your shelter and make a list of all the materials you will need to construct the shelter. How will you attach the plywood? How will you attach the door? What other hardware and materials will you need?

FINAL PRESENTATION CHECKLIST

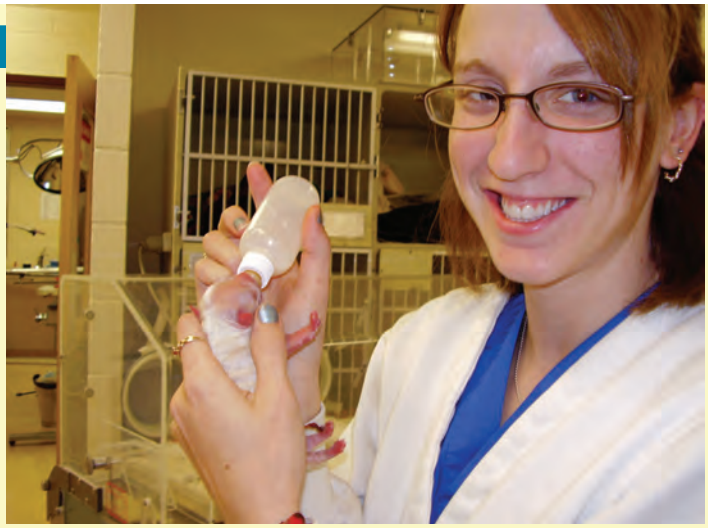
You will make a final presentation when you have completed this project. Your presentation will include the following:

- a description of your materials, quantities, and costs;
- an accurate two-dimensional floor plan; and
- a three-dimensional scale model.

3.1 Systems of Measurement

MATH *ON THE JOB*

Stéphanie Klassen is an animal health technologist at Central Animal Hospital in Saskatoon, Saskatchewan. After graduating from École régionale Saint-Jean-Baptiste in Saint-Jean-Baptiste, Manitoba, she obtained a diploma in animal health technology from Red River College in Winnipeg. As an animal health technologist, Stéphanie knows how to handle and restrain animals, perform initial examination procedures, and identify injuries and possible signs of abuse. She is knowledgeable about animal nutrition, diseases, and illnesses. She takes X-rays, puts animals under general anaesthetic, takes and runs blood tests, and assists the veterinarian with ultrasound examinations. She is informed about animal vaccinations and diets, and she can perform simple surgeries such as neutering cats and extracting teeth.



This newborn Shih Tzu puppy needs to be fed six times a day. Understanding the nutritional needs of animals is an important skill for an animal health technologist.

One task that Stéphanie performs daily is feeding the animals. Stéphanie needs to calculate how much dog food to give a full-grown husky dog. She knows that the husky needs 1250 kilocalories (kcal) of food energy a day. The dog's food contains 254 kcal in 250 mL. How many millilitres of dog food should Stéphanie feed the husky in one day? Round your answer to the nearest mL.

EXPLORE THE MATH

base unit: a unit of measurement on which other units are based

volume: the amount of space a solid occupies

In Canada, we use two systems of measurement: the Système international d'unités (SI) and the imperial system. Although we use the SI most often in our daily lives, the imperial system is used in many trades. For example, plumbers and carpenters typically take measurements in inches and feet, which are imperial units. To work in the trades, you need to be familiar with both the SI and imperial systems.

In the SI, the **base unit** for measuring length is the metre (m). The base unit for measuring **volume** is the litre (L).

The SI is a decimal system because it is based on multiples of 10. Any measurement stated in one SI unit can be converted to another SI unit by multiplying or dividing by a multiple of 10. Multiples of the base units are indicated by SI prefixes. For example, the prefix centi means one-hundredth, so 1 centimetre is one-hundredth of a metre. The prefix kilo means 1000, so 1 kilometre equals 1000 metres. What other SI prefixes do you know?

In the imperial system, the base unit for measuring length is the foot and the base unit for measuring volume is the pint.

The imperial system is not a decimal system. Because the imperial units were developed at different times to meet different needs, each group of units has a particular relationship. For example, there are 12 inches in 1 foot, and there are 3 feet in 1 yard. Figure 3.1 shows the most commonly used imperial units for length.

FIGURE 3.1

Some Common Imperial Units

Length

Unit	*Abbreviation*
inch	in or ″
foot	ft or ′
yard	yd
mile	mi

In order to solve an imperial measurement problem, you may have to convert the given measurements into common units. To convert from one imperial unit to another imperial unit, you use a unit **conversion factor**. A unit conversion factor is a fraction that is equal to 1. The numerator of the fraction contains the units to which you want to convert. The denominator of the fraction contains the original units in which the measurement was taken. What would the conversion factor be for feet to inches?

conversion factor: a number by which a quantity expressed in one unit must be multiplied to convert it to another unit

DISCUSS THE IDEAS

DART

The Disaster Assistance Response Team (DART) is a team of 200 Canadian armed forces personnel that provides assistance to disaster-affected regions around the world. In addition to providing basic medical assistance, DART's main goal is to produce safe drinking water.

1. DART's water purification system can purify 150 000 litres of water a day. If DART has a three-week mission to help in a disaster zone, how many litres of water can it purify?

2. An adult needs at least 4 litres of water a day for drinking, food preparation, and hygiene. For how many days would DART have to be on-site to produce a 3-week supply of clean drinking water for a community of 7000? What assumptions have you made in arriving at your answer? Would the location of the community or the type of disaster change your answer?

3. In the problem above, if DART was only available to be at the disaster location for 3 days, how long would the community have to get its own water systems working again (or to re-establish its access to clean water)?

ACTIVITY 3.1
EXPLORING IMPERIAL MEASUREMENT

In this activity, you will work with a partner to take imperial measurements and create an imperial conversion table.

1. Before you start measuring, look at the division markers on your imperial measuring tools. Imperial rulers and tape measures are marked in fractions of an inch. What is the smallest fraction indicated on each of your measuring tools?

2. With your partner, select 9 objects and distances to measure:

   - 3 objects that fit in your hand
   - 3 objects that are larger than your desk
   - 3 distances that are longer than and outside of the classroom

3. Measure each of your objects and distances and record your answers. Take as many measurements of the objects as are necessary to give the object's dimensions. How did you decide on the appropriate measuring tool to use for each of your measurements? Record your measurements in a table like the one below.

4. How could you estimate these measurements if you didn't have a ruler, measuring tape, or other tool? A referent is an object that represents approximately one unit of measurement. For example, an object that is about one inch long could be used as a referent to estimate inches. Working independently from your partner, find referents that you could use to estimate one inch, one foot, and one yard. Record the referent you used and its approximate length. Compare your referents with your partner's and share your reasons for choosing each referent.

IMPERIAL MEASUREMENTS

Item	*Measurements (imperial units)*	*Referent estimate*	*Difference*

5. Use your referents to estimate the measurements of the objects and distances you selected in question 2. Record your estimates.

6. Compare your estimates with your partner's. Calculate and record the differences between your estimates and the actual imperial measurements. Whose estimates were closer, yours or your partner's? Which referents were most accurate?

7. Copy the table below in your notebook and fill in the missing information to create an imperial conversion table.

IMPERIAL CONVERSION TABLE

1 foot = _____ inches

1 yard = _____ feet = _____ inches

1 mile = 1760 yards = _____ feet

Example 1

Maxime is a finishing carpenter who is replacing the case moulding around a double French door and the baseboards around the 4 walls of a living room. The dimensions of the rectangular living room are 20′ × 15′. The French door is along one of the 20′ walls, and the door frame measures 72″ wide and 84″ high. Case moulding costs $9.50 a linear foot and baseboard costs $4.50 a linear foot. These items must be purchased in whole feet. If Maxime's labour charge is $8.50 a linear foot, what will be the total cost of this job?

HINT

The numerator of the conversion factor is the unit to which you want to convert. The denominator is stated in the original units in which the measurement was taken.

SOLUTION

$20' + 20' + 15' + 15' = 70'$ Calculate the perimeter of the room.

Baseboard will not be installed in front of the French door, so the width of the French door must be subtracted from the perimeter to find the number of linear feet of baseboard needed.

$72'' \times \frac{1'}{12''} = 6'$ Convert the door width from inches to feet.

$70' - 6' = 64'$ Maxime will need 64′ of baseboard.

Case moulding will be installed along the top and two sides of the doorframe.

$72'' + 84'' + 84'' = 240''$

$240'' \times \frac{1'}{12''} = 20'$ Convert 240″ to feet to find the number of linear feet of case moulding.

Maxime will install 64′ of baseboard and 20′ of case moulding. His labour charge is $8.50 a linear foot. Calculate the labour cost.

$$64' + 20' = 84'$$

$$84' \times \$8.50/\text{foot} = \$714.00$$

Labour cost is $714.00.

Calculate the cost of the baseboard and moulding and add it to the labour cost.

$$\$4.50/\text{foot} \times 64' = \$288.00$$

$$\$9.50/\text{foot} \times 20' = \$190.00$$

$$\$714.00 + \$288.00 + \$190.00 = \$1192.00$$

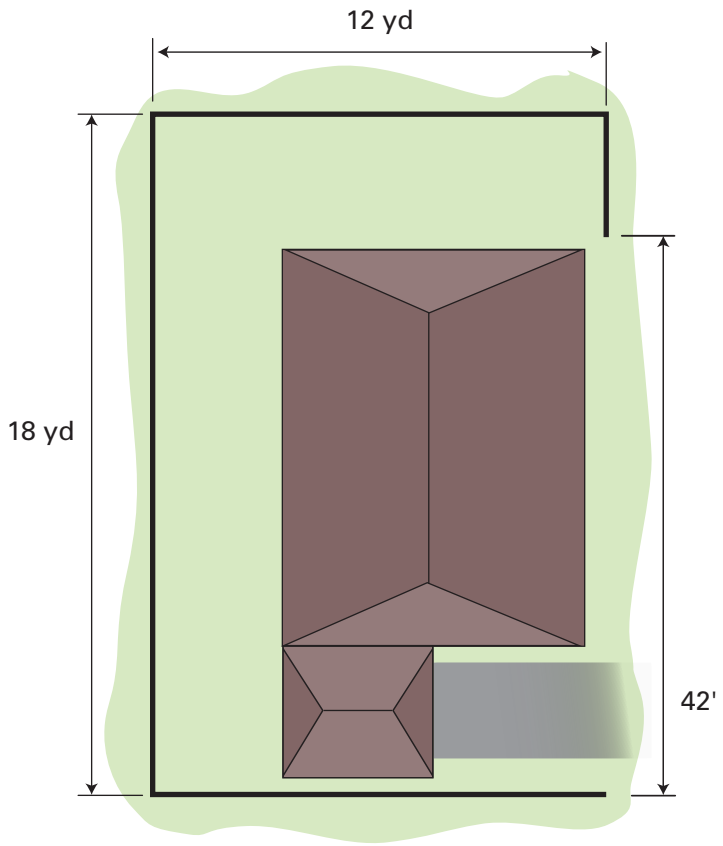
The total cost of the job is $1192.00.

ALTERNATIVE SOLUTION

Conversions can also be done using division.

$72'' \div 12 = 6'$

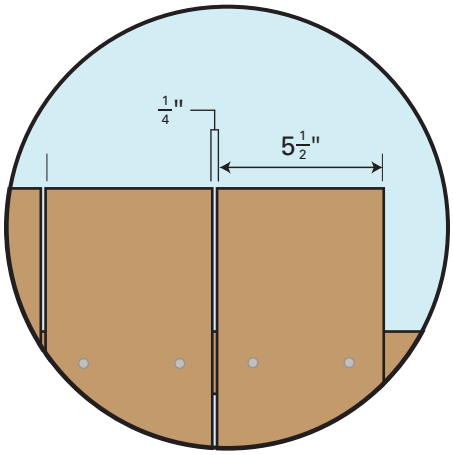
Convert the door width from inches to feet by dividing by 12.



Example 2

Kiri needs to replace the wooden fence that surrounds her yard. She measured her property, and it is 18 yards wide and 12 yards deep. There is no fence in front of her house, and the gap in the fence at the front of the property is 42 feet, as shown in the diagram. Kiri plans to replace the existing fence pickets with 5-foot-long cedar boards placed vertically. The boards are $5\frac{1}{2}$ inches wide and will be spaced $\frac{1}{4}$ inch apart. She placed an order for 275 boards. Did she order enough boards?

SOLUTION



The fence boards will be spaced $\frac{1}{4}''$ apart. Rewrite $5\frac{1}{2}$ as $5\frac{2}{4}$.

$5\frac{2}{4} + \frac{1}{4} = 5\frac{3}{4}$ The $\frac{1}{4}''$ spacing must be added to the width of one board to find the total space needed for each board.

Kiri needs to allow $5\frac{3}{4}''$, or $5.75''$ for each board.

Find the total length of the fence in yards.

The front of the property has a 42-foot gap in the fence.

$42 \times \frac{1}{3} = 14$ Convert 42 feet to yards using the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.

The gap is 14 yards wide.

$18 - 14 = 4$ Subtract the width of the gap from the width of the property.

The fence at the front of the property is 4 yards long.

$18 + 12 + 12 + 4 = 46$ Calculate the total length of the fence.

The total length of the fence is 46 yards.

To find the number of boards needed, first convert 46 yards to inches. This conversion can be done in two steps.

$$46 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 138 \text{ feet}$$

$$138 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 1656 \text{ inches}$$

$\frac{1656}{5.75} = 288$ Divide 1656 inches by the space needed for one board to find the total number of boards needed.

Kiri will need approximately 288 boards, so she has not ordered enough boards.

ALTERNATIVE SOLUTION

You can also find the number of boards needed by converting the board width to yards.

The total space needed per board is $5\frac{3}{4}$ inches, or 5.75 inches.

$5.75 \div 36 = 0.16$	Convert 5.75 inches to yards by dividing by the number of inches in 1 yard.

The total space needed per board is 0.16 yards.

$\frac{46 \text{ yards}}{0.16 \text{ yards}} = 287.5$	Divide the length of the fence by the space needed for one board to find the total number of boards needed.

Kiri will need approximately 288 boards, so she has not ordered enough boards.

Example 3

Bakers need math to create elaborate cakes.

Julie, a baker, specializes in wedding cakes. She would like to calculate the cost of decorating a 3-tiered circular cake with fresh flowers around the base of each level. The bottom cake has a 14″ diameter, the middle layer has a 10″ diameter, and the top layer has a 6″ diameter. All three layers are stacked on top of each other without spacers. If the cost of $1\frac{1}{2}$″ wide red roses is \$0.99 each and the cost of $2\frac{1}{2}$″ wide red roses is \$1.49 each, which size of roses should Julie decorate with to give her client the best price?

SOLUTION

First find the circumference of each cake layer.

$C = \pi d$	Use the formula for the circumference of a circle.
$C = \pi(6 \text{ in})$	Calculate the circumference of the top layer.
$C = 18.85$, rounded to 19 in	Julie would round up the measurement to ensure that enough flowers are ordered.
$C = \pi(10 \text{ in})$	Calculate the circumference of the middle layer.
$C = 31.42$ in, rounded to 32 in	
$C = \pi(14 \text{ in})$	Calculate the circumference of the bottom layer.
$C = 43.98$ in, rounded to 44 in	
$19 + 32 + 44 = 95$	The total circumference of the cake is 95 in.

Find the number of roses needed by dividing the circumference by the width of one rose.

$\frac{95}{1.5} = 63.33$, rounded to 64	Julie would need 64 of the 1.5″ roses to surround the cake.
$64 \times \$0.99 = \$63.36$	The 1.5″ roses would cost \$63.36.
$\frac{95}{2.5} = 38$	Julie would need 38 of the 2.5″ roses to surround the cake.
$38 \times \$1.49 = \$56.62$	The 2.5″ roses would cost \$56.62.

Julie should decorate with the 2.5″ roses.

ACTIVITY 3.2
VISUALIZING A MEASUREMENT

You have baked a rectangular cake that measures 9 in × 13 in. You need to cut 5 large pieces (4 in × 3 in) and at least 12 small pieces (2 in × 2 in) for your guests (whole pieces only).

1. Working with a partner, use an imperial ruler and an 11″ × 17″ sheet of paper to determine how to cut the cake. Do you have enough cake for your guests? Support your answer with a scale drawing.

2. Is there more than one way to cut the cake? Support your answer with a scale drawing.

ACTIVITY 3.3
DESIGNING A TIN CAN LAYOUT

Cylindrical tin cans are manufactured from tin-plate sheets. The body of the can is cut from one tin-plate sheet, and the lid and bottom of the can are cut from another sheet. After the pieces are cut out and shaped, the can is assembled and seals are applied to the top, bottom, and side seams.

You have been hired to design the layout for fabricating cans that are 2.5 inches in diameter and 4.5 inches high. The tin-plate sheets measure $2\frac{1}{2}$ yards by $1\frac{1}{2}$ yards. One sheet of tin plate is used for making lids and bottoms, and two sheets are used for making the body of the can. Three seals are applied to each can. The cost of the tin plate is \$13.20 a sheet, and each seal costs \$0.28 to make.

Work in a small group and use a ruler, compass, and a large sheet of paper to answer the following questions.

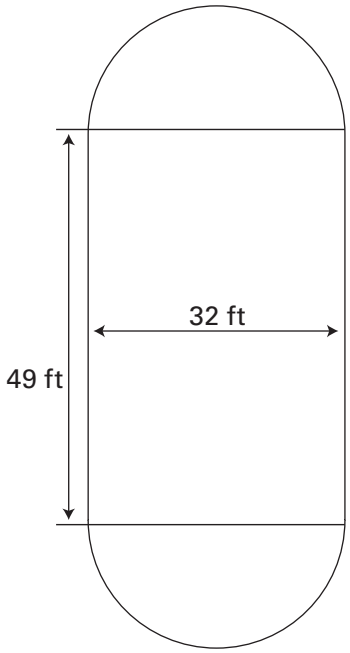
1. How many cans can be made from the three tin-plate sheets? Create a diagram to illustrate your answer. Describe the strategies you used to find your solution.

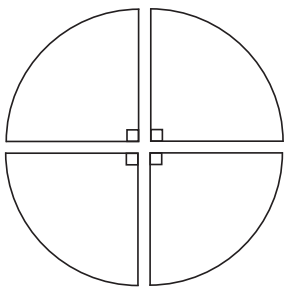
2. What is the cost for each can?

Mental Math and Estimation

The Canadian Football League record for the longest field goal is held by Paul McCallum. In 2001, while playing for the Saskatchewan Roughriders, he kicked a 62-yard field goal. How many feet is that?

BUILD YOUR SKILLS





1. Convert the following measurements.

   a) Convert 3520 yd to miles.

   b) Convert $10' \frac{3}{16}''$ to inches.

   c) Convert $8 \frac{3}{4}$ yards to feet.

2. René is a florist who uses 10 inches of ribbon to tie each bouquet of flowers he creates. He has a roll of ribbon 100 yards long. How many bouquets can he make with the roll of ribbon (assuming no wastage)?

3. Calm Air will accept carry-on luggage if its length, width, and height add up to no more than 46 inches. Beverly's suitcase has the following dimensions: length 2 ft; width 1 ft 4 in; and height 9 in. Will Calm Air allow her to carry her suitcase on board?

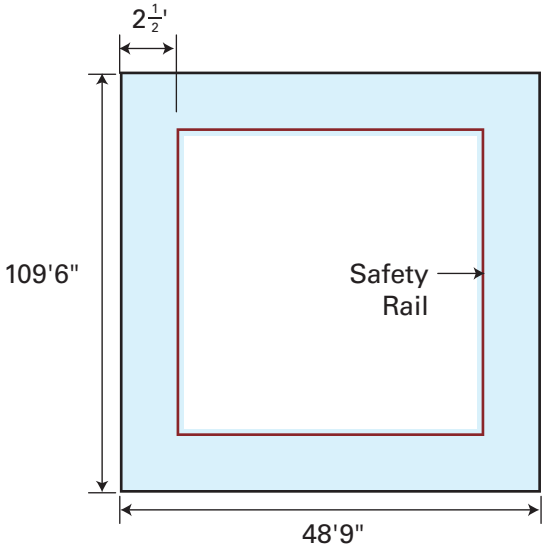
4. You have decided to build a small hockey rink in your backyard, as shown in the diagram. You want to use plywood to build rink boards that are 48″ high. Exterior $\frac{1}{2}''$ plywood is sold in $4' \times 8'$ sheets that cost $14.15 a sheet.

   a) How many sheets of plywood will you need to surround the rink?

   b) What will be the cost of the plywood, before taxes?

5. A landscape gardener has designed a circular herb garden with 4 sectors, shown on the left. The radius of one sector is 4′3″. Each sector will be surrounded with plastic lawn edging that costs $9.99 for a 20′ roll. How much will it cost to put edging around the herb garden? Assume that you cannot buy partial rolls.

6. In professional theatres, there is a catwalk called a fly gallery that runs along the four walls above the stage. Stagehands stand on the fly gallery to raise and lower scenery on and off stage. A structural steel fitter has been asked to replace the inside safety rail of a fly gallery. The space above the stage is 109′6″ long and 48′9″ wide. The fly gallery is $2\frac{1}{2}$′ wide. If the fitter uses rails that are 20 feet long, how many rails will she need?

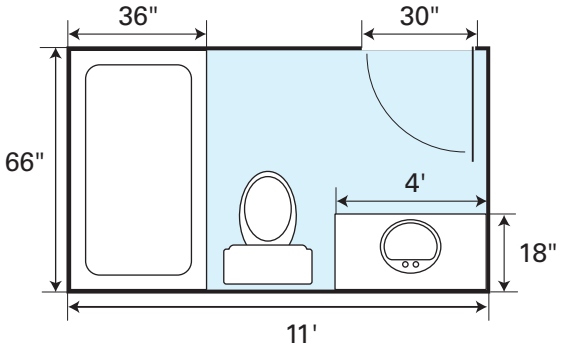


7. Noah is designing new drapes for a client's living room window that is 9′ wide and 4′ high. The finished drapes will have two panels so they can be opened in the centre. The drapes will be twice as wide as the window and they will be 6′ long. The drape fabric is 60″ wide.

   a) How many yards of fabric must be purchased to make these drapes? Disregard seam allowances and hems in your calculations.

   b) How much will the drapes cost if the price of the fabric is $15.00/yard?

8. A finishing carpenter is working on a partial home renovation project, and the homeowner has asked the carpenter how much it would cost to replace the baseboards in the bathroom. The floor plan of the bathroom is shown on the right. The carpenter bills his time at a rate of $45.00/h and he charges a markup of 15% on materials. Baseboard costs $6.50 a linear foot and the carpenter estimates it will take him two-and-a-half hours. How much does he tell the homeowner it will cost? List any assumptions you made in your calculations.



Extend your thinking

9. An insulator has a contract to insulate a residential garage that measures 24′ × 24′. The interior walls are 8′ high, the garage door is 16′ wide and 7′ high, and the door in the wall opposite the garage door is 36″ wide and 80″ high. The garage is framed with wall studs spaced 16″ apart. The insulator is using fiberglass insulation that comes in batts that measure 16″ wide and 47″ long. If there are 18 batts in one package, how many packages of insulation will she need?

THE ORIGINS OF STANDARD MEASUREMENT

The cubit was the unit of measurement the ancient Egyptians used to construct the pyramids.

Humans have always needed to use measurement to make comparisons and to perform tasks such as building shelters or trading goods. But for thousands of years, there was no universal system of measurement. Instead, measurement units developed according to custom and usage.

The first measurement units were based on dimensions of the human body. People used their arms, hands, and fingers to measure length. For example, ancient Egyptians used a unit of measurement called the cubit. A cubit was the distance from a person's elbow to the tip of his or her middle finger. When smaller units of measure were needed, the digit, or width of a finger, and the palm, or width of a hand, were used.

The imperial measurement system we use today also originated from units based on the human body. In England during the Middle Ages, measurement units included the ynce, the foot, the ulna, and the fathom. The ynce was the width of a thumb, and the foot was the length of a human foot. An ulna was the distance from the tip of a person's nose to the end of the middle finger of his or her outstretched arm. A fathom was the distance across a person's outstretched arms, from fingertip to fingertip.

Clearly, the results of this type of measurement would vary depending on who was doing the measuring. Sometimes, that didn't matter. If you built a table for your own use, you could use your hands and arms to take measurements. But what if you were hired to build a table for someone else, and she told you the table had to be eight feet long? Whose "foot" would you use to take the measurements?

In the thirteenth century, King Edward of England decided that his country needed standard measurement units. He decreed that one ynce was equal to the length of three grains of barley, 12 ynces equalled 1 foot, and 3 feet equalled 1 ulna. He created a master ulna made of iron to be used as the standard throughout the country.

Some units of measurement based on the human body are still used today. A horse's height is measured in hands, which has been standardized at 4 inches. Sailors still use the 6-foot fathom to measure the depth of water. And although today we use the words inch and yard instead of ynce and ulna, the ratios between the inch, foot, and yard set by King Edward remain the basis of the imperial system of measurement.

1. Why do we need standard units of measurement?
2. What are the advantages of using non-standard forms of measurement to describe something?

DRAW A FLOOR PLAN

Earlier, you planned the design of your ice-fishing shelter. It is now time to put your plans on paper and draft a floor plan.

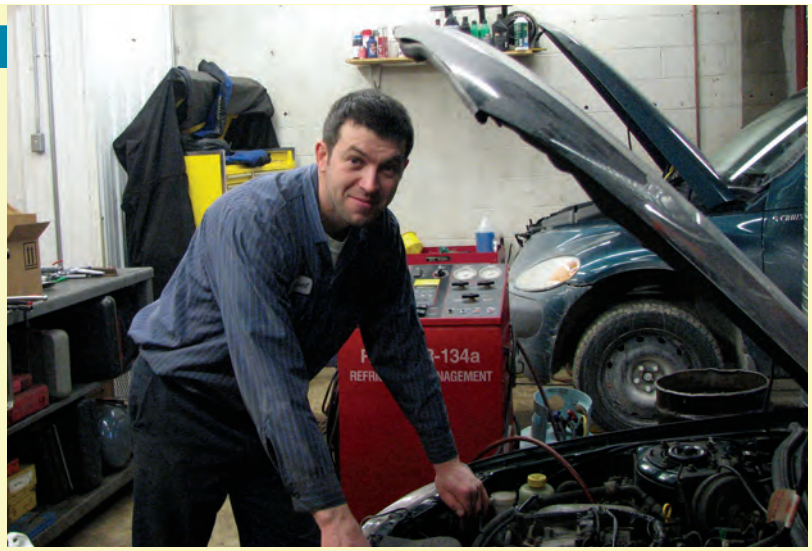
- Write a description of your ice-fishing shelter. Include details about everything that will be inside the shelter and describe the exterior.
- Sketch a rough draft of the floor plan of your fishing shelter. Is your shelter large enough to seat two or three people? Is the shelter small enough to be moved to the fishing location?
- Create an accurate floor plan of the shelter, using your rough sketch as a guide. Show all the measurements of the dimensions of the floor, door, and window openings in imperial units. State your scale.

You must measure accurately when you draw a scale floor plan.

3.2 Converting Measurements

MATH *ON THE JOB*

Manuel Marques is an automotive service technician who owns Lube King Service Centre in Winnipeg. Manuel inspects, diagnoses, repairs, and services the mechanical and electrical systems of vehicles. As a business owner, Manuel supervises employees and apprentice automotive technician students. He uses mathematics to calculate resistance and voltage when working with electronic components. He also converts between imperial and SI units when servicing American-made vehicles or using equipment, such as his air conditioning refrigerant capture machine, that was made in the US.



Manuel uses math every time he repairs a vehicle.

Manuel needs to order a supply of hose clamps with a minimum diameter of 32 mm. He found an online auto parts company in the US that sells hose clamps with a maximum diameter of $\frac{7}{8}$ inch. Would the US clamps be the correct size?

EXPLORE THE MATH

In 1983, a Boeing 767 ran out of fuel on a flight from Montreal to Edmonton. The aircraft was able to glide to a safe landing in Gimli, Manitoba, and no one was hurt. An investigation revealed that the aircraft ran out of fuel because the ground crew used the wrong conversion factor to convert gallons to litres. Because of this error, the pilot took off from Montreal with 12 598 litres of fuel, instead of the 27 770 litres of fuel he needed to reach Edmonton.

Not all conversion errors have such potentially serious consequences, but in many jobs you need to know how to convert from SI units to imperial units. For example, the United States, which is Canada's largest trading partner, uses imperial units, so if you are doing business with a US company, you need to know the imperial equivalent of SI units.

In this section, you will use proportional reasoning and conversion factors to convert between SI and imperial units.

ACTIVITY 3.4
CONVERTING BETWEEN SI AND IMPERIAL UNITS

In this activity, you will work with a partner to create a conversion table you can use to convert between linear imperial and SI units. You will need a ruler and metre/yard stick.

1. Compare the metric and imperial units on your ruler. Approximately how many centimetres are in one inch?

2. Compare the metric and imperial units on your metre/yard stick. Which is longer, one yard or one metre? Approximately how many inches are in one metre? Approximately how many feet are in one metre?

3. Proportional reasoning can be used to convert a measurement from imperial to SI units. In a table like the one below, fill in the missing information to create an imperial–SI conversion table.

CONVERSION FACTORS BETWEEN SI AND IMPERIAL UNITS

SI to imperial	*Imperial to SI*
1 mm = ____ in	1 in = ____ mm
1 cm = ____ in	1 in = 2.54 cm
1 m = ____ ft	1 ft = ____ m
1 m = 1.0936 yd	1 yd = ____ m
1 km = ____ mi	1 mi = 1.6093 km

DISCUSS THE IDEAS

INSTALLING A CHANDELIER

Yori was hired to install a chandelier in the foyer of a home with a 9-foot ceiling. The homeowner, who is 180 cm tall, cannot decide on the length of the chandelier that he wishes to hang.

1. Calculate some possible lengths of the chandelier.

2. What factors should the homeowner consider before deciding on the length of the chandelier?

Example 1

Before installing the new carpet, Giselle removes the old flooring.

Giselle would like to replace the carpet in her living room. She used her imperial tape measure to measure the room, and the dimensions were 12 ft by 15 ft. When she went to the carpet store, she found the price of the carpet was \$24.99/m^2 (taxes included). She cannot order less than a full square metre of carpet.

a) How much carpet should she order?

b) How much will the carpet cost?

SOLUTION

a) Convert the room dimensions from imperial units to SI units. Because 1 metre equals 3.2808 feet, the conversion factor used to convert feet to metres is $\frac{1}{3.2808}$.

$\frac{1}{3.2808} = \frac{x}{12}$ Use the conversion factor to convert the width of the room from feet to metres.

$12\left(\frac{1}{3.2808}\right) = 12\left(\frac{x}{12}\right)$ Multiply each side of the equation by 12 to isolate x.

$3.658 = x$

The room is 3.658 metres wide.

$\frac{1}{3.2808} = \frac{x}{15}$ Convert the length of the room from feet to metres.

$15\left(\frac{1}{3.2808}\right) = 15\left(\frac{x}{15}\right)$ Multiply each side of the equation by 15 to isolate x.

$4.572 \text{ m} = x$

The room is 4.572 metres long.

$3.658 \times 4.572 = 16.724$ To find the number of square metres of carpet Giselle needs to order, find the area of the living room.

The area of the room is 16.724 m^2. Giselle must order full square metres of carpet, so she should order 17 m^2.

b) $17 \text{ m}^2 \times \$24.99/\text{m}^2 = \$424.83$ Find the cost of 17 m^2 of carpet.

Her order will cost \$424.83.

Example 2

Samir is the cost estimator for a landscape company. He has to calculate the amount of material needed to construct a circular outdoor patio built from paving stones. The diameter of the patio is 13 m. One bundle of paving stones covers 116 ft^2. Samir has ordered 11 bundles of paving stones. Did he order enough paving stones?

When complete, the patio will look similar to this design.

SOLUTION

Convert the diameter of the patio from metres to feet using proportional reasoning and the conversion factor 1 ft equals 0.3048 m.

$$\left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right) = \left(\frac{x \text{ ft}}{13 \text{ m}}\right)$$

$13\left(\frac{1}{0.3048}\right) = x$ Multiply both sides of the equation by 13 to isolate x.

$42.6509 \text{ ft} = x$ When you are estimating material, it is best to round up to ensure that enough material is purchased.

The diameter of the patio is approximately 43 ft.

Use the formula for the area of a circle to find the area of the patio.

$A = \pi r^2$

$A = \pi(21.5)^2$ The diameter is 43 feet, so the radius is 21.5 feet.

$A = 1452.20$ The area of the patio is 1452.20 ft^2.

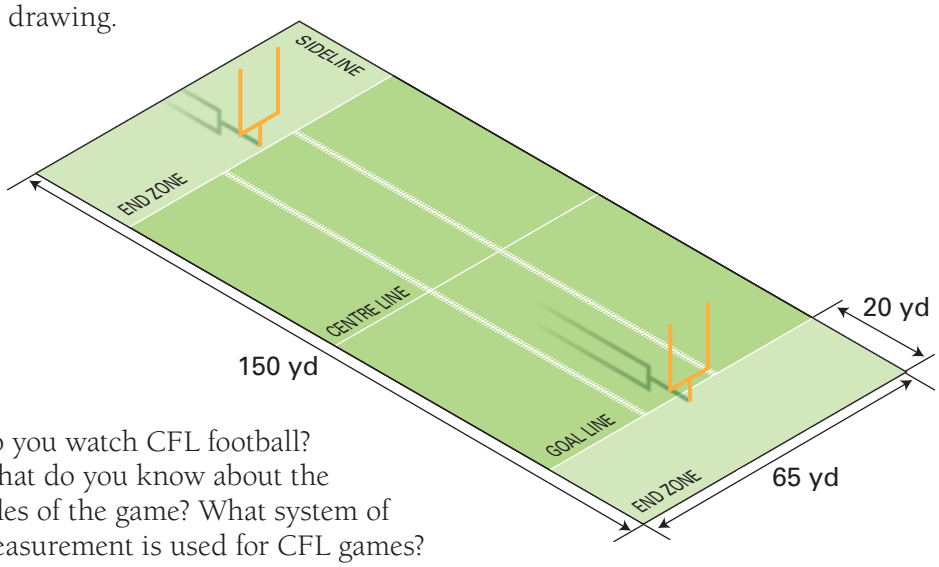
$\left(\frac{1452.20}{116}\right) = 12.5$ Divide the area of the patio by the coverage of 1 bundle of paving stones.

Samir ordered 11 bundles, so he did not order enough paving stones.

ACTIVITY 3.5
DESIGNING A CFL FIELD LOGO

The rules of Canadian football are different from those of American football, but both countries use the same system of measurement in the game.

Most football teams have their logo and team name painted on the surface of the field. Imagine that your high school has been chosen to create the field logos for the next Canadian Football League (CFL) Grey Cup Championship. Working with a partner, you will design field logos for a CFL team and create a scale drawing.



1. Do you watch CFL football? What do you know about the rules of the game? What system of measurement is used for CFL games?

2. Why do you think the CFL has not converted its game to the SI? Make a list of the impacts converting to the SI would have on the game.

3. The CFL playing field is a rectangle that is 150 yards by 65 yards. At each end is an end zone, with a goal line 20 yards from the end of the field. How many yards long is the playing field between the two goal lines? How many yards is it from the goal line to the centre line?

4. Create a scale diagram of the CFL field on 1-inch graph paper. Record the scale you used.

5. Design your logos and place them on your scale diagram using the following design specifications:

   a) 2 end zone logos, each 10 yards × 35 yards

   b) 2 large field logos, each 10 yards × 10 yards

   c) 2 small field logos, each 5 yards × 10 yards

   d) 1 centre field logo, centred on the 55-yard line, with a maximum radius of 5 yards

6. Calculate the amount of paint that will be needed to paint all the logos if one 3.8-litre can of paint covers 37 m^2.

DISCUSS THE IDEAS

LAKE WINNIPEG

Lake Winnipeg is the world's tenth largest freshwater lake. In 1999, researchers began documenting the effects of the buildup of blue-green algae blooms caused by excess nitrogen, phosphorus, and other pollutants in the lake. Researchers analyze water samples, observe lake water colour, and document marine species to determine changes to the toxicity of the lake. For some calculations, researchers need to know the area of the lake, but because it is an irregular shape, its area cannot be found using a standard formula.

Given a map of Lake Winnipeg, what strategies could you use to find the approximate area of the lake in both km^2 and mi^2?

This map of Lake Winnipeg shows its irregular shape.

Mental Math and Estimation

The distance a plane travels from Calgary to Edmonton is approximately 277 km. Estimate this distance in miles.

BUILD YOUR SKILLS

1. A low bridge has a posted maximum vehicle height of 7′6″. Your truck is 2.3 m high. Will it fit under the bridge?

2. David works for a company that is developing a high-accuracy altimeter for skydivers that measures and records their velocity and other measurements during a jump. The system that it has developed uses SI measurements. David must give a presentation on the device in San Francisco, showing the data the company obtained from several jumps from an altitude of 4200 metres. What is this altitude in feet?

3. Valerie wants to apply for a driver's licence. The application asks her to state her height in cm. Valerie is 5′8″ tall. What is her height in cm?

4. Sandy has been asked to give an estimate for replacing a countertop in a client's kitchen. The countertop measures 2′ × 6′ and the client wants Sandy to install 4″ × 4″ tiles that cost $3.50 each. Sandy has estimated her labour charge will be $350.00. What is the total cost of tiles and labour?

5. The Vancouver Parks Board wants to install grass sod on a playground that measures 20 m × 40 m. Two companies have bid on the job. Company A's bid was $4.00/$yd^2$ installed. Company B put in a bid of $2.50/$m^2$ plus $2000.00 for installation. Which company should get the job based on the best price?

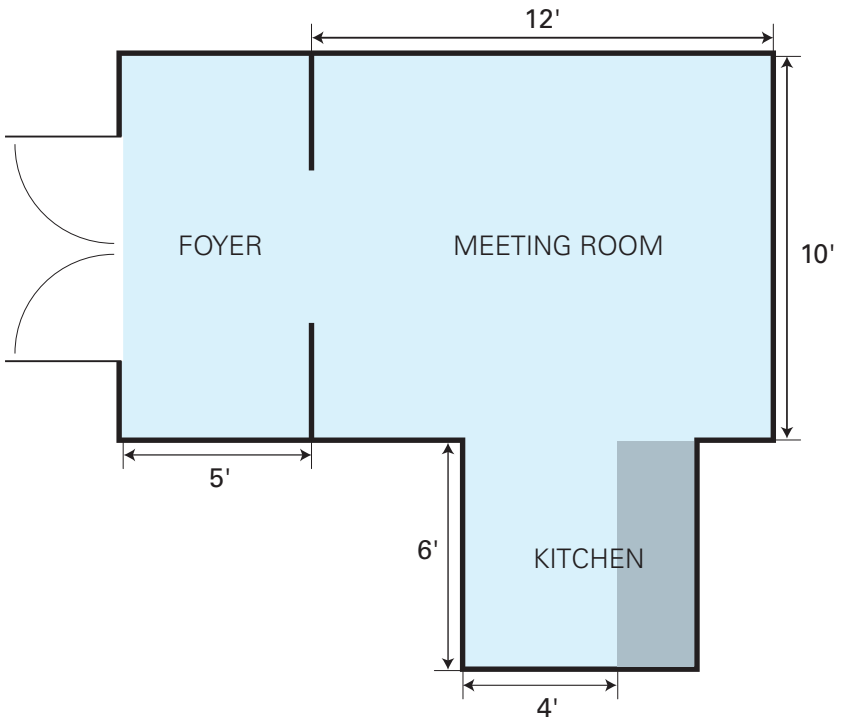
6. Shelley is trying to decide whether to put hardwood flooring or carpet on her living room floor. The dimensions of the room are 22 ft by 16 ft. The hardwood flooring costs \$18.99/m^2 with an installation cost of \$1500.00. The carpet costs \$21.95/yd^2 with an installation cost of \$1350.00. Which type of flooring costs less?

7. Irina purchased a farm in Saskatchewan. She wants to plant balsam fir seedlings, and she estimated the field measures 72 yards by 65 yards. The tree nursery manager told her that each seedling requires an area of 64 ft^2 to grow properly. The seedlings cost \$0.65 each. The nursery sells them in bundles of 20, and she cannot order partial bundles.

   a) How many seedlings can Irina plant on her acre of land?

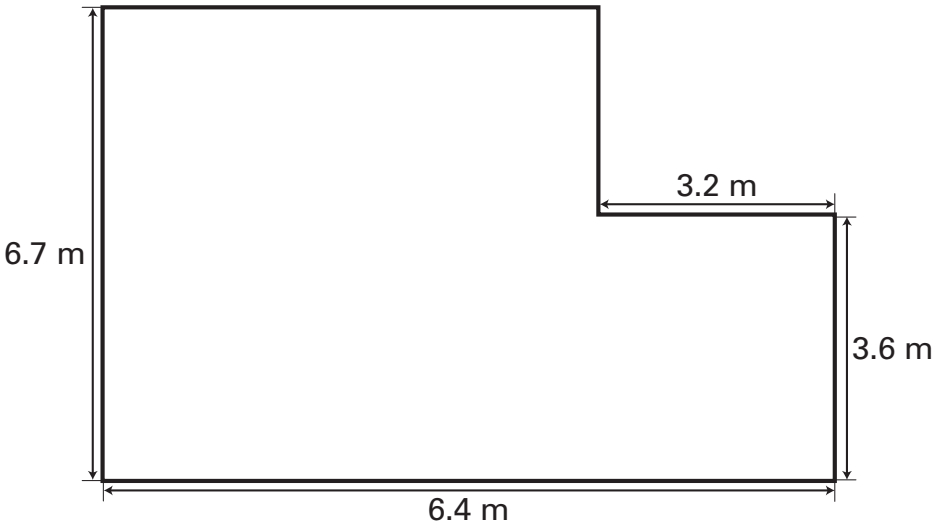
   b) How much will it cost to purchase the seedlings?

8. Dejan was hired to lay vinyl flooring in the kitchen, meeting room, and foyer of the local Friendship Centre. He used the measurements in the diagram below to estimate the job. The flooring material comes in rolls 10 ft wide and is sold by the running foot, and Dejan needs to purchase enough flooring to ensure the pattern will match in all three rooms. He will also add 15% to his order to compensate for wastage. One running foot of vinyl flooring costs \$12.50. Dejan estimates his labour charge will be \$560.00. What is the total estimate for vinyl flooring and labour?



Extend your thinking

9. Lesa plans to carpet her sunroom with indoor-outdoor carpeting. She measured the room and made a sketch, shown below. When she went to the store, she discovered that the bolts of carpeting are 12 feet wide and cost \$22.95/yd^2. The carpet will be attached with double-sided tape around the perimeter and along all seams. A 30-foot roll of double-sided tape costs \$4.85. Lesa wants to lay out the carpet so the nap is running in the same direction, with the minimum number of seams. What will be the total cost of tape and carpet?



ESTIMATE MATERIALS AND COSTS

In the first part of the project, you listed the materials you would need to build your ice-fishing shelter. In this project activity, you will estimate the quantities and the costs of the materials you will need. Record your calculations and research on charts similar to the ones below. You may want to use spreadsheet software to organize your information.

Use your floor plan to calculate how much paint and plywood you need.

1. First, review your lists to ensure that you have listed all the materials you will need. Is there anything else that you thought of while you were drawing your floor plan?
2. The exterior of your shelter, the roof, and the floor will be sheathed with plywood. Plywood is sold in 4 ft by 8 ft sheets. Estimate the number of sheets of plywood you need to buy.
3. A one-litre can of paint will cover 100 ft^2. Estimate the number of litres of paint you need to buy to paint the exterior of your shelter with one coat of paint. Will you paint the door?
4. Using the internet, local newspapers, or flyers, find the cost of your materials. How much will your shelter cost to build? Are there ways that you can save money?

SHELTER DIMENSIONS

	Length	*Width*	*Area*
Wall 1			
Wall 2			
Wall 3			
Wall 4			
Floor			
Roof			
Door			
Window(s)			

SAMPLE

MATERIALS AND COSTS

Materials	*Quantity needed*	*Unit price*	*Cost before taxes*	*Taxes (GST and PST)*	*Total cost*
Paint					
Plywood					
Propane heater					
Window					

SAMPLE

3.3 Surface Area

Measuring and budgeting are two of the ways that David Kattegatsiak uses math in his job.

MATH *ON THE JOB*

David Kattegatsiak is the community economic development officer for Chesterfield Inlet, Nunavut. David provides information to his community about government resources and writes business proposals for local community members. He assists management with grant proposals and acts as a liaison between the community, municipal government, and various consultants. David helps plan and facilitate hamlet council meetings, creates budgets, and analyzes cash flow financial statements. One of David's duties is to do preliminary planning work with surveyors, architects, and engineers for construction projects in the community. In order to receive government grants to construct or renovate a building, David must take measurements at the site, calculate how much material or landfill is needed for the site, and create a budget for the project.

David is creating a budget for a renovation project. He needs to calculate how much paint to buy to paint the walls of a school classroom. The classroom is 32 feet long and 25 feet wide, with a 10-foot ceiling. Paint is sold by the litre, and one litre of paint covers 12 square metres. Approximately how many litres are needed for one coat of paint?

EXPLORE THE MATH

Have you ever noticed the many types of packaging used to contain household goods? Cereal, detergent, and tissue are sold in rectangular boxes. Soup and tuna come in cylindrical cans. What other packaging shapes have you seen? Why do you think manufacturers choose different packaging shapes for their products?

In order to know how much packaging material is required for a product, its **surface area** must be calculated. Tradespeople such as machinists also need to calculate surface area to determine the amount of material they need to fabricate parts.

One way to find the surface area of a three-dimensional object is to create a **geometric net**. A geometric net is created by imagining that you are cutting open a three-dimensional object and laying it out flat to create a two-dimensional figure. The surface area can then be found by summing the areas of each side, or face, of the two-dimensional net.

surface area: the total area of the surface of a three-dimensional object

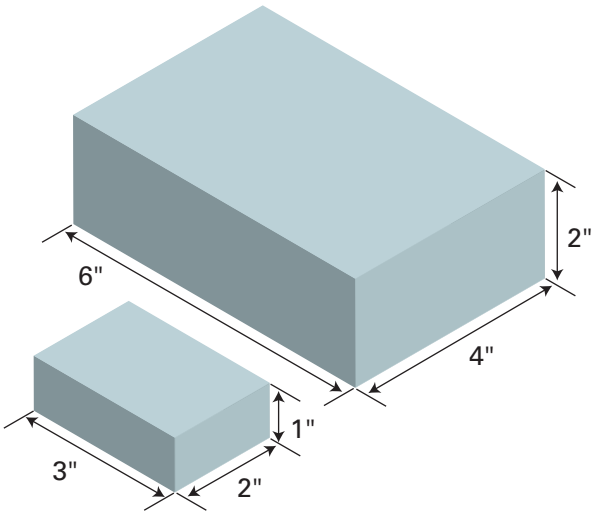
geometric net: a two-dimensional pattern used to construct three-dimensional shapes

DISCUSS THE IDEAS

SCALE FACTOR

scale factor: the ratio of the lengths of corresponding sides of two polygons

Sarita manufactures cardboard boxes. One of her clients would like her to double the lengths of the sides of a box that is currently 3 in long, 2 in wide, and 1 in high.



1. What **scale factor** will Sarita use to create the new box?
2. What will be the surface area of the new box?
3. What effect does doubling the length of the sides have on the surface area of the box?
4. What effect would tripling the length of the sides have on the surface area of the box? What if the lengths were quadrupled?
5. Write a formula that you can use to find the surface area of a scaled object when you know the original surface area and the scale factor.

ACTIVITY 3.6
DESIGNING A TOOL BOX

A welder needs to make a rectangular aluminum tool box for the bed of a pickup truck. The inside dimensions of the truck bed are $78\frac{5}{8}''$ long × $62\frac{3}{8}''$ wide × 21″ high.

a) Working in a group of 3, design a tool box for the truck bed. What factors influenced your design?

b) Sketch your tool box design and label its dimensions.

c) How many square feet of aluminum would be needed to make your tool box? What formula did you use to arrive at your answer?

ACTIVITY 3.7
SURFACE AREA FORMULAS

In this activity, you and your partner will examine the geometric nets of common three-dimensional shapes and develop formulas for their surface area. Your teacher will provide you with the required nets.

1. Cut out the geometric nets for the rectangular prism, triangular prism, cylinder, and cone. Describe the nets. How many faces are the same on each net? Record your answers in a table like the one below.

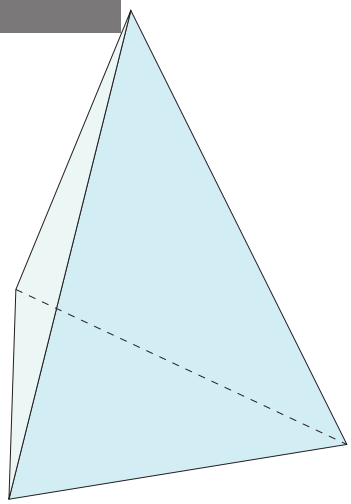
DESCRIBING GEOMETRIC NETS

Description	*Rectangular prism*	*Triangular prism*	*Cylinder*	*Cone*
Total number of faces				
Number of each type of face				
Area of each type of face				
Total surface area				
Formula for total surface area				$\pi rs + \pi r^2$

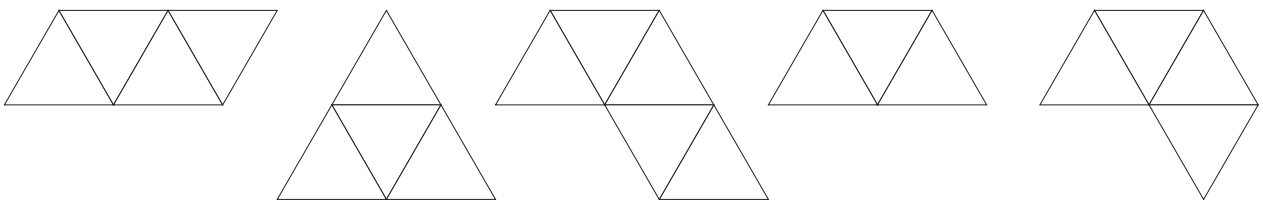
2. Measure the nets with an imperial ruler and calculate the area of each face. How can you use the area of the faces to find the total surface area of the object? Calculate the total surface area of each shape and record it in your table.

3. Assemble the geometric nets, using tape to hold them together. Write a formula that you can use to find the total surface area of a rectangular prism, triangular prism, and cylinder.

4. A triangular pyramid, or tetrahedron, is a pyramid with a triangular base, as shown.



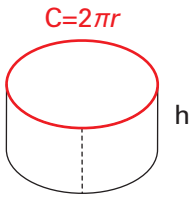
   a) Which of the following are geometric nets of a triangular pyramid (all are congruent triangles)?



   b) If the area of one face of a triangular pyramid is 8 in^2, what is the total surface area of the triangular pyramid?

Example 1

A cannery has redesigned the size of the can for its canned salmon. The diameter of the new can is 4″ and its height is 5.5″. How much tin will be needed to construct one can?

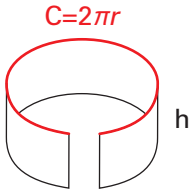


SOLUTION

A can is a cylinder. To find the amount of tin, you need to find the surface area of the cylinder.

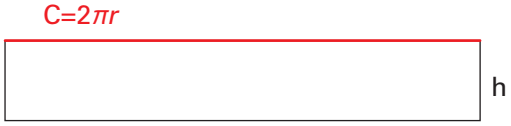
$A = \pi r^2$	Use the formula for the area of a circle to find the area of the top of the can.
$A = \pi(2)^2$	The radius is half the diameter. The diameter of the can is 4″, so the radius is 2″.
$A = 12.57 \text{ in}^2$	
$2(12.57) = 25.14 \text{ in}^2$	Multiply the result by 2 to find the total area of the top and bottom of the can.

The side of a cylinder is a rectangle. The length of the rectangle is equal to the circumference of the top of the cylinder.



$C = 2\pi r$	Use the formula for the circumference of a circle to find the length of the rectangle.
$C = 12.57 \text{ in}$	The rectangle is 12.57 in long.
$A = 12.57 \times 5.5$	Use the formula for the area of a rectangle to find the area of the side of the can.
$A = 69.14 \text{ in}^2$	
$25.14 + 69.14 = 94.28$	Find the sum of the areas of the top, bottom, and side of the cylinder.

The amount of tin needed to construct one can is 94.28 in^2.



Example 2

Taizo manufactures and sells farm implements in Winkler, MB. One piece of equipment that he sells is a 3-point spreader that attaches to a tractor to spread grass seed, wheat seed, or fertilizer. The hopper of the spreader is a cone with a diameter of 40″ and slant height of 45″. How many square feet of plastic is needed to form one hopper?

This is an older style hopper made of metal instead of plastic.

SOLUTION

The hopper is a cone without a base, so to calculate the amount of plastic, you only need to calculate the surface area of the side. The formula for the surface area of the side of a cone is $A = \pi rs$, where r is the radius of the base and s is the slant height, or length of the side.

Convert the hopper dimensions from inches to feet using the conversion factor $\frac{1 \text{ ft}}{12 \text{ in}}$.

$\frac{1 \text{ ft}}{12 \text{ in}} = \frac{x \text{ ft}}{20 \text{ in}}$ Convert the radius from inches to feet. The diameter of the hopper is 40″, so the radius is 20″.

$20\left(\frac{1}{12}\right) = 20\left(\frac{x}{20}\right)$ Multiply both sides of the equation by 20 to isolate x.

$1.67 = x$ The radius of the hopper is 1.67 ft.

$\frac{1}{12} = \left(\frac{x}{45}\right)$ Convert the slant height from inches to feet.

$45\left(\frac{1}{12}\right) = 45\left(\frac{x}{45}\right)$

$3.75 = x$ The slant height of the hopper is 3.75 ft.

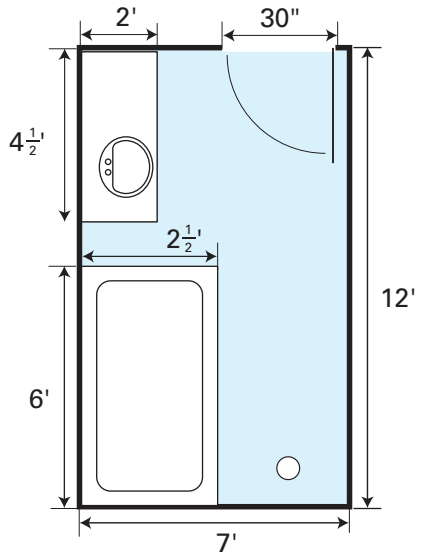
$A = \pi(1.67)(3.75)$ Rewrite the formula, substituting the known values, and multiply.

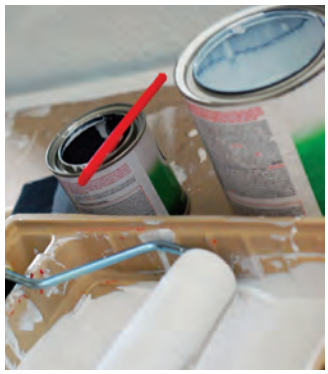
$A = 19.67$

The amount of plastic in one hopper is 19.67 ft^2.

ACTIVITY 3.8
A REDECORATING PROJECT

You have added a new bathroom in your house, shown in the diagram below. You have installed the vanity and bathtub. You want to tile the floor before the toilet is installed and then paint the walls. The walls are 9 feet and the door is 7 feet high. The height of the bathtub is $1\frac{1}{2}'$ and the height of the vanity is 36″. Work with 1 or 2 partners to answer the following questions.





Painters calculate surface area to determine how many cans of paint they need to buy.

1. The floor will be tiled with 12″ × 12″ tiles. The area under the tub and vanity will not be tiled. Tiles cost $6.99 each. What will be the total cost of the tiles?

2. The door and the wall area that is covered by the tub and vanity do not need to be painted. The walls require 2 coats of paint. The second coat will require $\frac{2}{3}$ as much paint as the first coat. If a 4-litre can of paint covers 400 ft^2 at a cost of $46.99, and a 1-litre can covers 100 ft^2 at a cost of $17.99, what combination of cans of paint should you buy to minimize the cost of painting the bathroom?

3. You have a budget of $600.00 for the entire bathroom. After adding taxes (GST and provincial or territorial PST) to your materials cost, do you have any money left over to purchase a new mirror for the vanity?

Mental Math and Estimation

The surface area of a cube is 24 ft^2. Find the area of one face and the length of one side of the cube.

BUILD YOUR SKILLS

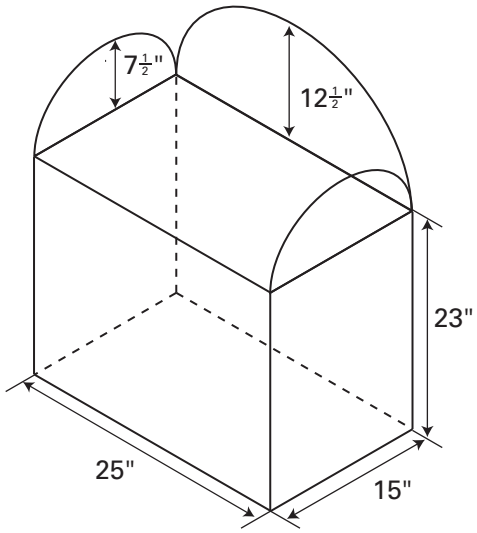
1. Vedran is a landscaper who builds custom garden ponds. Most of his clients want a circular pond that is 1.5 feet deep and has a 4-foot diameter. Vedran lines the ponds with synthetic rubber liner, which he buys in 10′ × 15′ rolls that cost $149.00.

   a) How many pond liners can be made from one roll?

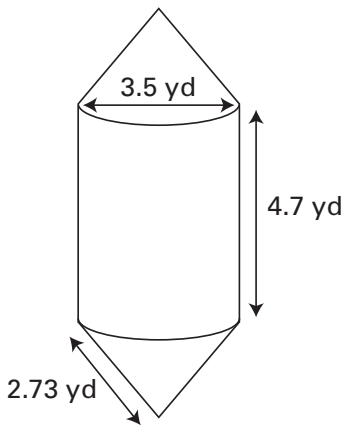
   b) How much does it cost to line one pond?

2. A company manufactures aluminum beverage cans in two sizes: Can A has a diameter of 2.5 inches and a height of 4.5 inches. Can B has a diameter of 3 inches and a height of 3 inches. Which can requires more aluminum to manufacture?

3. Jashandeep is refinishing a wooden child's storage bench. She has two options for finishing the bench. Her first option is to treat the bench with 2 coats of wood stain. A one-litre can of stain costs $12.99 and will cover 15 m^2. Her second option is to paint the bench with 2 coats of latex paint. A one-quart can of paint costs $14.99 and will cover 100 ft^2. She drew the sketch at the right so she could estimate how much paint or stain she will need. Which option is less expensive?



4. A seed hopper in the shape of a cylinder has a diameter of 3.5 yd and a height of 4.7 yd. Both ends of the cylinder are capped with a cone that has a slant height of 2.73 yd. Find the number of square yards of sheet metal required to build the hopper (assume the hopper is hollow inside).



5. Francine wants to build a rectangular barbeque in her parents' backyard. The barbeque will be 40″ wide, 32″ deep, and 26″ high and will be open at the front. She has decided to use bricks for the three walls and the floor. Bricks cost $0.80 each. Francine learned that bricklayers often estimate the number of bricks they need by calculating 48 bricks/yd^2 and then adding 5% for cutting and breakage.

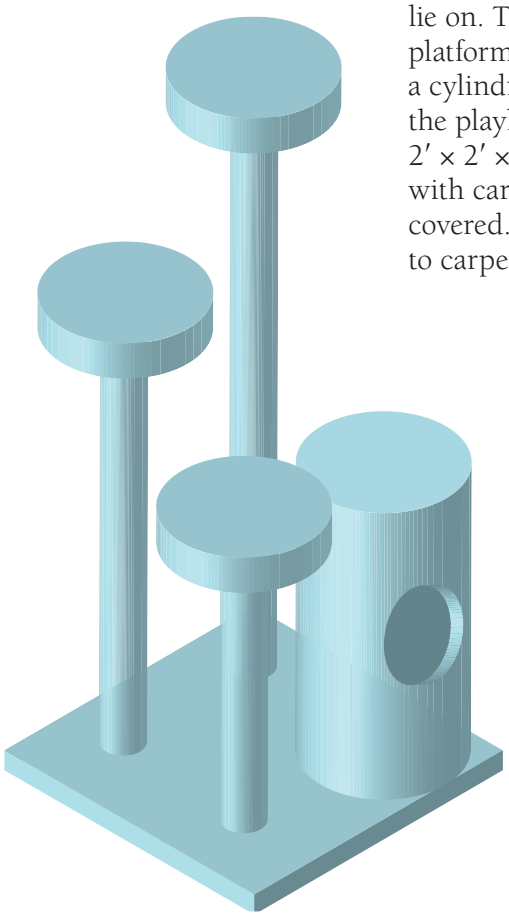
   a) How many bricks should Francine buy?

   b) How much will the bricks cost?

6. A packaging company received a special order to design and manufacture boxes that measure 3″ long, 1″ wide, and 1″ high. The boxes will be cut from sheets of cardboard that measure 1 yd × 1 yd. Draw a diagram to illustrate how you could cut 72 boxes from 1 sheet of the cardboard.

Extend your thinking

7. Benoit built an elaborate scratching post for his cats. It consists of three poles of different lengths topped with circular platforms for the cats to lie on. The poles are 2″ in diameter and 2′, 3′, and 4′ long. Each circular platform is 12″ in diameter and 3″ high. At the base of the structure is a cylindrical playhouse $1\frac{1}{2}$′ high with a 12″ diameter. The cats get into the playhouse through a 6″ diameter hole. The base of the structure is 2′ × 2′ × 2″. Now Benoit wants to completely cover the scratching post with carpet. All sides of the poles, platforms, playhouse, and base will be covered. The carpet he has chosen costs $5.60/ft^2. How much will it cost to carpet the scratching post?



BUILD A THREE-DIMENSIONAL MODEL

A three-dimensional model will help you visualize your ice-fishing shelter. Here is one way that you can build a scale model without a roof (so that you can see inside it). You will create a two-dimensional geometric net of your ice-fishing shelter, and then you will fold the net to create a three-dimensional model.

- Obtain a large piece of Bristol board or cardboard and measure its dimensions.
- Choose a scale that will enable you to copy your floor plan in the centre of the Bristol board and draw your 4 walls connected to the floor. Draw the window and door. Your drawing should look like the geometric net of a rectangular prism without the top (roof).
- Cut out your two-dimensional geometric net and fold up the walls to create a three-dimensional model. Tape the sides together.
- What is inside your shelter? Use the same scale factor to create any of the amenities that you plan to include. For example, include the seats, the heater, and the fishing holes in the floor. You can also provide additional materials to illustrate your design, such as paint samples or photos.

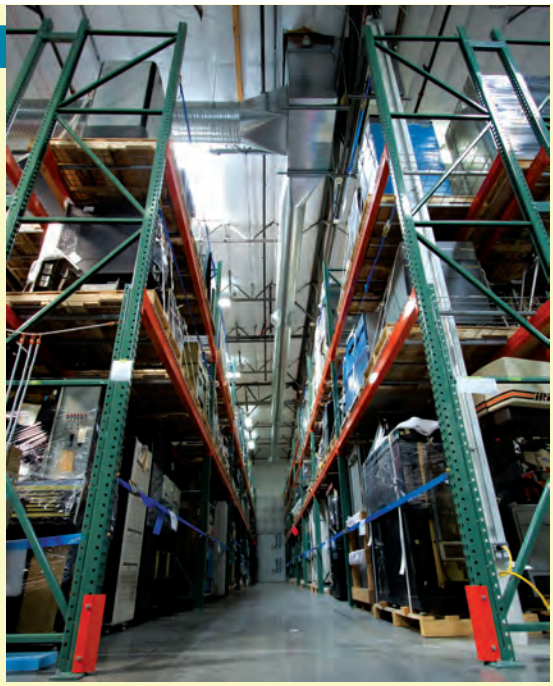
This student is creating a three-dimensional model based on her floor plan.

3.4 Volume

MATH *ON THE JOB*

Anthony is a warehouse technician in the City of Yellowknife Public Works Department. His job is to manage storage, shipping, and inventory of government assets such as furniture and office supplies. He inspects the items that arrive at the warehouse, decides where to store them, and uses a computer database to keep track of the number and location of the items in the warehouse. In order to make the best use of his warehouse space, Anthony uses math to plan how and where to store his inventory.

Anthony needs to know how many boxes can be stored in a storage bay in his warehouse. The storage bay is 24 feet long and 12 feet wide. The maximum height that boxes can be stacked is 9 feet. Each box is 24 inches × 36 inches × 18 inches. What is the maximum number of boxes that will fit in the storage bay?



Anthony needs to know the dimensions of the warehouse in order to store the inventory efficiently.

EXPLORE THE MATH

capacity: the maximum amount that a container can hold

In the SI, the base unit for measuring volume is the litre. The litre is also the base SI unit for measuring **capacity**. A litre is one-thousandth of a cubic metre. Why is volume measured in cubic units? What is the formula for calculating volume?

In the imperial system, the base unit for measuring volume and capacity is the pint, but volume can also be measured in cubic inches, cubic feet, or cubic yards.

In Canada, we use the term imperial units to mean British imperial units. The United States also uses an imperial system, but the sizes of its units for volume and capacity are different from the British units. Britain redefined its volume measurements after the United States had become an independent country that set its own standards. A British gallon is equal to 4.54609 litres; a US gallon equals 3.785 litres. Because British and US gallons are different sizes, so are all the related volume and capacity measures. For example, a British pint is 20 fluid ounces, while a US pint contains 16 fluid ounces.

Figure 3.2 lists the abbreviations for some common imperial units of volume and capacity.

FIGURE 3.2

Imperial Units of Volume and Capacity

Unit	*Abbreviation*
ounce	oz
fluid ounce	fl oz
pint	pt
quart	qt
gallon	gal

In many industries, volume and capacity are measured in imperial units rather than SI units. For example, in the food industry, ingredients are often measured in teaspoons, cups, and ounces. Other industries use both the imperial system and the SI. A building contractor estimating the amount of concrete needed for a job may find one supplier who delivers concrete by the cubic metre and another who delivers it by the cubic yard. The contractor would need to know how to convert between the two systems to ensure that she is getting the best price for her client.

In this section, you will develop methods to convert between the imperial system and SI for volume and capacity measurements.

DISCUSS THE IDEAS

PACKAGING

Over the years, beverage companies have marketed soft drinks in cans and bottles with different capacities. In Canada, a standard-size can of pop contains 355 mL. Why do you think pop is sold in containers of this size?

1. The table below lists three of the pop can sizes sold in the US and the equivalent sizes of cans sold in Canada. Working with a partner, copy the table in your notebook and fill in the ratios for the three sizes of cans.

RATIOS OF POP CAN SIZES

US pop can	*Canadian pop can*	*Ratio fl oz: mL*
8 US fl oz	237 mL	
12 US fl oz	355 mL	
16 US fl oz	473 mL	

2. Using the ratios from question 1, create a formula you can use to convert from mL to US fl oz. Do your results suggest a reason for the Canadian packaging sizes?

3. Copy the table below in your notebook and use your information from questions 1 and 2 to fill in the missing information and create a conversion chart.

CONVERTING US IMPERIAL TO SI UNITS

US Imperial	*SI*
1 fl oz	____ mL
1 pt = 16 fl oz	____ L
1 qt = 2 pt	____ L
1 gal = 4 qt	____ L

ACTIVITY 3.9
CONVERTING A RECIPE

You are making a batch of raisin bannock to take to a community feast. Your grandmother has given you her recipe, but the ingredients are in imperial units and you only have SI measuring equipment.

1. Examine your teaspoon and measuring cup. What SI and imperial markings are on them? Use the two items and work with a partner to convert the following recipe.

RAISIN BANNOCK RECIPE

Imperial	*Ingredients*	*SI*
3 cups	flour	____ mL
$1\frac{1}{2}$ teaspoons	baking powder	____ mL
$\frac{1}{2}$ teaspoon	salt	____ mL
$\frac{1}{4}$ cup	shortening	____ mL
$1\frac{1}{4}$ cups	water	____ mL
1 cup	raisins	____ mL

2. Copy the table below in your notebook and fill in the missing information to create a conversion chart.

CONVERTING COMMON COOKING UNITS

Imperial	*SI*
$\frac{1}{4}$ teaspoon	____ mL
$\frac{1}{2}$ teaspoon	____ mL
1 teaspoon	____ mL
1 tablespoon (3 teaspoons)	____ mL
1 cup	____ mL
1 pint	568.2614 mL
1 quart (2 pt)	1.1365 L
1 gallon (4 qt)	4.5461 L

Example 1

Nigel imported a vehicle that was made in Britain. The capacity of the gas tank is 22 gallons. If the price of gasoline is $1.20 a litre, how much will it cost Nigel to fill his tank when it is empty?

Nigel may find it difficult to drive his British, right-hand drive car on Canadian roads.

SOLUTION

Because 1 British gallon equals 4.5461 litres, the conversion ratio used to convert gallons to litres is $\frac{4.5461}{1}$.

$$\frac{4.5461 \text{ litres}}{1 \text{ British gallon}} = \frac{x \text{ litres}}{22 \text{ gallons}}$$ Set up a proportion.

$$22\left(\frac{4.5461}{1}\right) = 22\left(\frac{x}{22}\right)$$ To isolate x, multiply both sides of the equation by 22.

$$22 \times 4.5461 = x$$

$$100.0142 = x$$

The capacity of the gas tank is 100.0142 L.

$100.0142 \times \$1.20 = \$120.02$

It will cost $120.02 to fill Nigel's gas tank.

Example 2

Antifreeze prevents water from freezing and expanding in the cooling system.

The cooling system of a car's 6-cylinder, 250-cubic-inch displacement engine has a capacity of $3\frac{1}{2}$ gallons. To protect the engine against freezing temperatures, an antifreeze solution of $\frac{3}{8}$ ethylene glycol and $\frac{5}{8}$ water is added to the cooling system. If the cooling system is filled to capacity with the antifreeze solution, how many quarts of ethylene glycol are in the cooling system?

SOLUTION

First find the capacity of the cooling system in quarts. Because 4 quarts equals 1 gallon, the unit conversion factor is $\frac{4 \text{ quarts}}{1 \text{ gallon}}$.

$\frac{x}{3.5} = \frac{4}{1}$ Convert $3\frac{1}{2}$ to a decimal and set up a proportion.

$\frac{x}{3.5}(3.5) = \frac{4}{1}(3.5)$ Multiply both sides of the equation by the common denominator to isolate the variable.

$x = 4 \times 3.5$

$x = 14$

The cooling system has a capacity of 14 quarts.

Find the number of quarts of ethylene glycol the cooling system contains.

$0.375 \times 14 = 5.25$ Convert $\frac{3}{8}$ to a decimal and multiply by the capacity of the cooling system.

The cooling system contains 5.25 quarts of ethylene glycol.

Example 3

Reshma built 24 wood planters for her garden. The inside of each planter measures 4′ long, 2′ deep, and $1\frac{1}{2}$′ wide. She needs to order soil to fill the planters. At K & R Soils, potting soil sells for \$17.00/yd^3, while Bob's Best Buy sells potting soil for \$21.50/m^3. Where should Reshma buy her soil?

SOLUTION

To find the cost of buying the potting soil from K & R Soils, convert the dimensions of one planter from feet to yards.

$\left(\frac{x \text{ yd}}{4 \text{ ft}}\right) = \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)$ Set up a proportion to convert the length to yards.

$$4\left(\frac{x}{4}\right) = 4\left(\frac{1}{3}\right)$$

$$x = \left(\frac{4}{3}\right)$$ Convert the fraction to a decimal.

$$x = 1.33 \text{ yd}$$

$$\left(\frac{x \text{ yd}}{2 \text{ ft}}\right) = \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)$$ Set up a proportion to convert the depth to yards.

$$2\left(\frac{x}{2}\right) = 2\left(\frac{1}{3}\right)$$

$$x = \frac{2}{3}$$ Convert the fraction to a decimal.

$$x = 0.67 \text{ yd}$$

Reshma will be able to add a wide variety of flowers to her planters.

$$\left(\frac{x \text{ yd}}{1.5 \text{ ft}}\right) = \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)$$ Convert the fraction to a decimal and set up a proportion to convert the width to yards.

$$1.5\left(\frac{x}{1.5}\right) = 1.5\left(\frac{1}{3}\right)$$

$$x = 0.5 \text{ yd}$$

$V = 1.33 \times 0.67 \times 0.5$ Use the formula for volume to calculate the cubic yards of soil needed for one planter.

$V = 0.45 \text{ yd}^3$ One planter requires 0.45 yd^3 of soil.

$24 \times 0.45 = 10.8$ Multiply by 24 to find the cubic yards of soil needed to fill all the planters.

Reshma would need 10.8 yd^3 of potting soil.

$10.8 \times \$17.00 = \$183.60$ The soil from K&R Soils would cost $183.60.

To find the cost of buying the potting soil from Bob's Best Buy, convert the dimensions of one planter from feet to metres.

$$\frac{x \text{ m}}{4 \text{ ft}} = \frac{1 \text{ m}}{3.2808 \text{ ft}}$$ Use the conversion factor 1 m = 3.2808 feet to convert the length of the planter to metres.

$$4\left(\frac{x}{4}\right) = 4\left(\frac{1}{3.2808}\right)$$

$$x = 1.22 \text{ m}$$

$\frac{x \text{ m}}{2 \text{ ft}} = \frac{1 \text{ m}}{3.2808 \text{ ft}}$ Convert the depth to metres.

$2\left(\frac{x}{2}\right) = 2\left(\frac{1}{3.2808}\right)$

$x = 0.61 \text{ m}$

$\left(\frac{x \text{ m}}{1.5 \text{ ft}}\right) = \left(\frac{1 \text{ m}}{3.2808 \text{ ft}}\right)$ Convert the width to metres.

$1.5\left(\frac{x}{1.5}\right) = 1.5\left(\frac{1}{3.2808}\right)$

$x = 0.46 \text{ m}$

$V = 1.22 \times 0.61 \times 0.46$ Use the formula for volume to calculate the cubic metres of soil needed for one planter.

$V = 0.34 \text{ m}^3$ One planter requires 0.34 m^3 of soil.

$24 \times 0.34 = 8.16$ Multiply by 24 to find the cubic metres of soil needed to fill all the planters.

Reshma would need 8.16 m^3 of potting soil.

$8.16 \times \$21.50 = \$175.44$ The soil from Bob's Best Buy would cost $175.44.

Reshma should buy her potting soil from Bob's Best Buy.

ACTIVITY 3.10
DRIVEWAY CONSTRUCTION

Maynard has decided to pave his driveway, which measures 74 ft × 18 ft. He has narrowed his choices to concrete or paving stones.

EXCAVATION AND PAVING COSTS

Item	*Cost*
Soil excavation and removal	$\$75.00/\text{yd}^3$
Gravel	$\$12.00/\text{yd}^3$
Sand	$\$30.00/\text{yd}^3$
Concrete	$\$135.00/\text{yd}^3$
Crushed limestone	$\$35.00/\text{yd}^3$
Paving stone	$\$6.50/\text{yd}^3$

a) If the driveway is paved with concrete, he will need 4 inches of concrete on top of 8 inches of gravel.

b) If he uses paving stones that are $2\frac{1}{2}''$ thick, they will need a foundation of $3\frac{1}{2}$ inches of sand and 12 inches of crushed limestone. He will also need to add 10% to his paving stone order to allow for breakage and cutting.

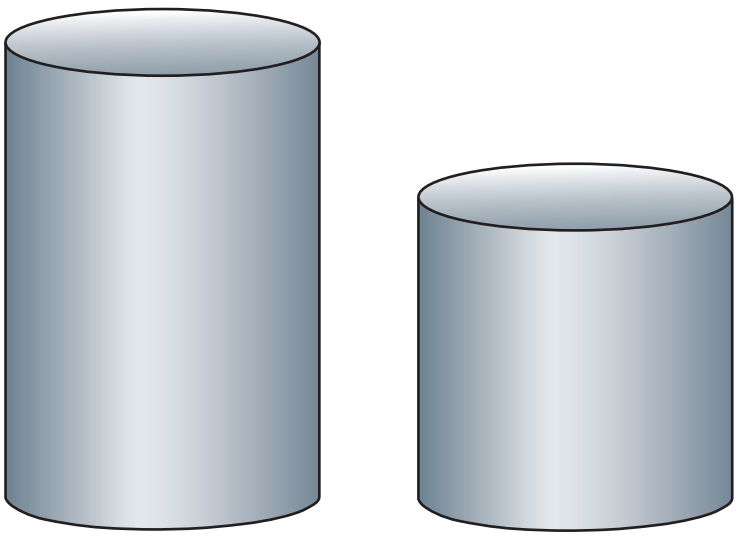
The costs of excavation and materials are shown on the previous page. Working in your small group, compare the cost of paving the driveway with concrete and with paving stones. Write a report with an itemized cost analysis for both options.

PUZZLE IT OUT

THE DECANTING PUZZLE

You have two empty containers. One has a capacity of 5 units and the other has a capacity of 3 units. Neither container has any unit markings. Your job is to fill one of the containers with exactly 4 units of water. You can fill a container, empty a container, and pour water from one container to the other, without spilling over.

Find the fewest number of pours needed to reach your goal.



5 units 3 units

BUILD YOUR SKILLS

1. A baker needs to double a recipe that requires 3 cups of milk, but he only has SI measuring utensils. How many mL of milk will he need?

2. A convenience store in the US sells 4 different sizes of slushy drinks: 12 US fl oz, 16 US fl oz, 28 US fl oz, and 40 US fl oz. What would be the equivalent sizes in millilitres? Do the millilitre sizes correspond to the drink sizes in your neighbourhood?

3. Anne-Laure is a toy manufacturer. She imported a supply of boxes from the US with the dimensions 12″ × 6″ × 8″. She needs a box for a game that measures 20 cm × 11 cm × 16 cm. Will the game fit in the boxes she imported?

4. An American tourist has crossed the Canadian border. His vehicle has a 15-gallon capacity and he has $\frac{1}{8}$ tank of gas left. If the price of gas is $1.10 a litre, how much will it cost him to fill his tank?

5. Everett owns and operates an auto repair shop. He wants to pour a concrete pad 24′ × 22′ × 4″ in front of the garage. J & L Concrete sells concrete for $145.00/yd^3 and M & W Concrete sells concrete for $165.00/m^3. How much will each company charge? Which company should Everett buy his concrete from?

6. Elann is a bridgeworker who needs to calculate the amount of concrete required to repair concrete spalls on a bridge deck. Spalls are shallow holes where fragments of concrete have broken off due to freezing, use of road salt, or wear. There are three concrete spalls on the bridge deck. The spalls are rectangular and each is $2\frac{1}{2}$ in deep. The first spall measures $6\frac{1}{2}$ ft by 3 ft. The second is 16 ft by 6 ft. The third is $15\frac{1}{2}$ ft by 9 ft. She needs to order enough concrete to fill the spalls, plus an extra $\frac{1}{2}$ yd^3 to compensate for any waste. How many cubic yards of concrete should she order?

Driving a fuel-efficient car is economical and environmentally responsible.

Extend your thinking

7. Jamal wants to buy a fuel-efficient vehicle. He knows that in countries that use the SI, fuel efficiency is measured by the number of litres of fuel the vehicle uses in 100 km. He can use litres per 100 km to compare fuel efficiency. The vehicle that uses the least fuel per 100 km is the most fuel efficient. Jamal saw an ad for a car that averages 45 miles to 1 US gallon and an ad for a minivan that uses 10 litres per 100 km. Which vehicle is more fuel efficient?

MAKE A PRESENTATION

You are now ready to present your ice-fishing shelter. Start by planning your presentation. What information will you include? How can you best communicate the information you have gathered and present your shelter? Presentations are often made using illustrated posters or handouts, or you could use presentation software. Be sure to include the following items in your presentation:

- a description of your materials, quantities, and costs;
- an accurate two-dimensional floor plan; and
- a three-dimensional scale model.

Include pictures of your materials and the amenities you have found for your shelter. You may also want to explain why you have chosen certain materials or amenities.

Share ideas with your classmates about the best way to present your project.

REFLECT ON YOUR LEARNING

LENGTH, AREA, AND VOLUME

Now that you have finished this chapter, you should be able to

- ❒ understand the relationships between units in the SI and imperial system;
- ❒ convert a given measurement from SI units to imperial units;
- ❒ convert a given measurement from imperial units to SI units;
- ❒ estimate measurements using a referent;
- ❒ calculate perimeter, circumference, and area in imperial units;
- ❒ calculate the surface area and volume of a three-dimensional object in imperial units.

In addition, you have completed a project that applied your new skills in a practical context.

PRACTISE YOUR NEW SKILLS

1. A walking trail in Stanley Park in Vancouver is 10.8 km long. The Parks Board wants to install benches every 600 m along the trail. The benches cost $350.00 each, and labour will cost $1500.00. How much will it cost to install the benches?

2. Brenda was hired to replace the wood case moulding on a window frame that measures 90 inches by 48 inches. If the moulding costs $3.25 a linear foot and her labour charge is $8.50 a linear foot, how much will it cost to replace the mouldings?

3. A family room is 21 ft × 12 ft with four 8-foot walls. It has one door to the deck outside that is 30 in wide and 7 ft high and two windows that each measure 5 ft × 3 ft. The interior walls are to be painted. The painter charges $\$6.95/ft^2$. How much will the painter charge to do this job?

4. Antoine needs to buy metal primer to rust proof a cylindrical fuel storage tank. The tank has a diameter of 15′6″ and a height of 18′. A 3.8 L can of metal primer costs $47.13 and covers 40 m^2. What will be the total cost of the metal primer? (Assume that the bottom of the storage tank will be painted.)

5. A snack bar advertises ice cream by displaying an oversized ice-cream cone on its roof. The cone is made of aluminum and has a 12-inch diameter, a height of 36 inches, and a slant height of 36.50 inches. How many square feet of aluminum were used to make the cone?

Putting mulch around blueberry plants is an environmentally friendly method of encouraging their growth.

6. An American tourist crossing the border into Canada notices a sign that states, "227 km to the next service station." There is a service station across the road from the sign. Her car's 18-gallon fuel tank is $\frac{1}{2}$ full, and she knows her vehicle gets 28 miles to the gallon. Should she stop for gas now or fill up at the next service station? Show the evidence for her decision.

7. Blueberry farmers use peat moss as mulch around blueberry bushes to conserve moisture and prevent weeds. A farmer has a blueberry field that is 30 ft long and 18 ft wide. He wants to lay 6 in of mulch on the field. He can buy a 1 yd^3 bale of peat moss for $39.00 or a 3.8 ft^3 bale for $12.49. Which size bale would give the farmer the best total price?

8. Dawai is creating a flower bed alongside his house. The bed will be 15 ft by 3 ft, and he needs to add soil to a depth of 12 in. Dirt for Less sells garden soil for \$15.99/yd^3, and Rocks and Soils sells soil for \$18.99/m^3.

   a) If he can buy fractions of a cubic yard or metre, where should he buy the soil?

   b) If he must buy whole cubic yards or metres, where should he buy the soil?

If Dawai chooses to plant tulips, he should plant each bulb 8 inches deep.

9. Steve is building an 8-inch-thick concrete retaining wall that is 75 feet long and $2\frac{1}{2}$ feet high. Concrete is made by mixing one part cement, two parts sand, and four parts gravel with water.

   a) How many cubic yards of the following ingredients will Steve need to build the wall?

      i) cement

      ii) sand

      iii) gravel

   b) Sand costs \$18.00/yd^3, gravel costs \$8.99/yd^3, cement costs \$65.00/yd^3, and Steve is charging \$1500.00 for labour. How much will the job cost?