CHAPTER

9

Solving Systems of Linear Equations Algebraically

Analysing linear relations is a skill that is required in many professions. Air traffic controllers monitor the speed and altitude of airplanes. Electronic engineers determine how to speed up computers. Chemical engineers analyse the flow rates of mixtures. Business owners analyse market data. Consumers, like you, make decisions about the things they buy. What is the cost? What is the environmental impact of their purchases?

Sometimes, a graphical approach to solving a linear system takes too long. At other times, it may not be accurate enough. An algebraic approach may provide a faster and more precise solution.

Big Ideas

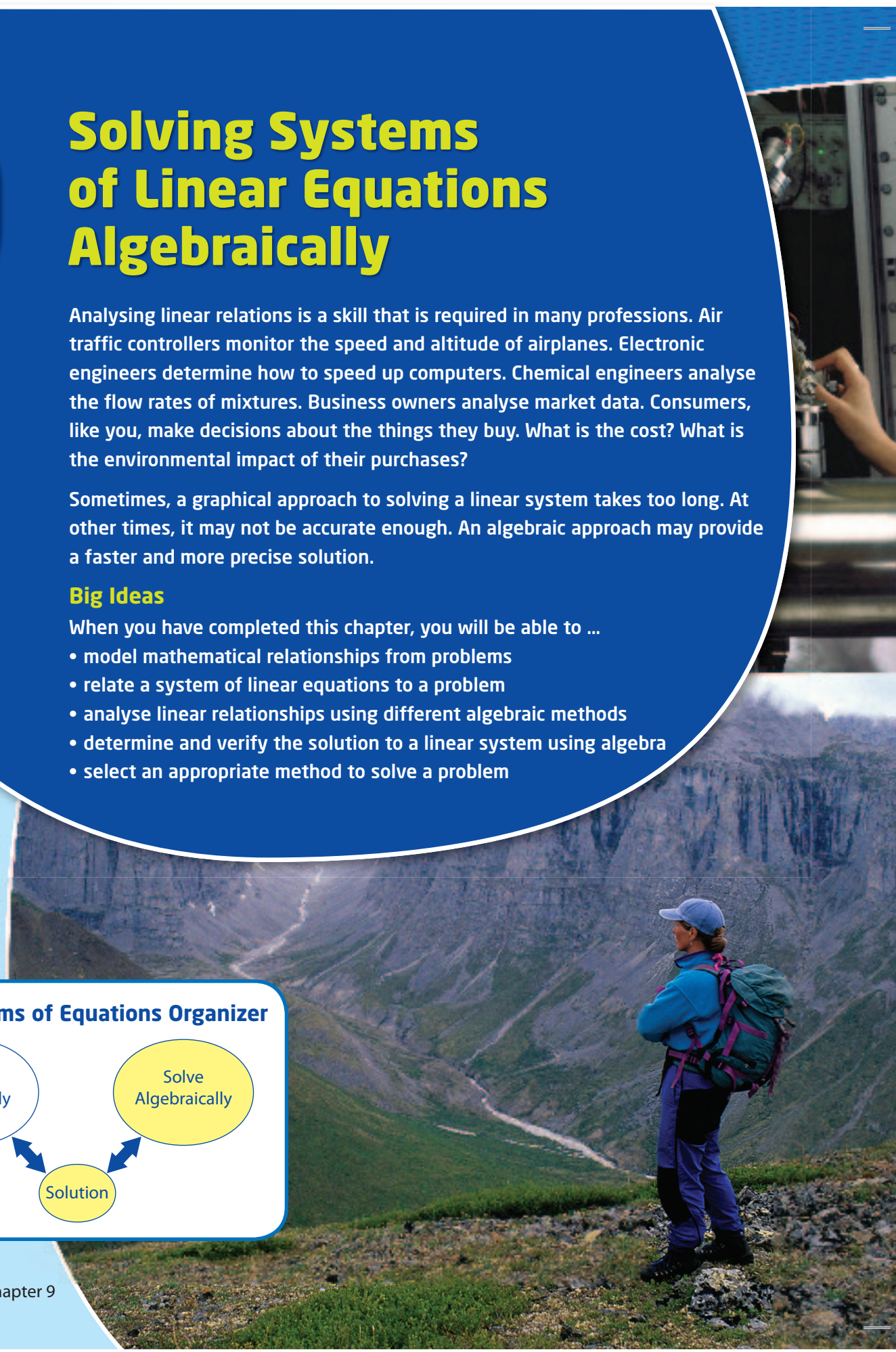
When you have completed this chapter, you will be able to ...

- model mathematical relationships from problems
- relate a system of linear equations to a problem
- analyse linear relationships using different algebraic methods
- determine and verify the solution to a linear system using algebra
- select an appropriate method to solve a problem

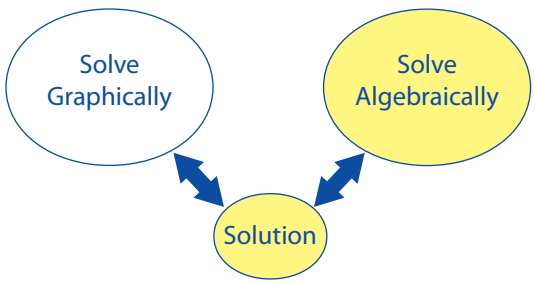
Key Terms

substitution method

elimination method



Your Systems of Equations Organizer



Air Traffic Controller

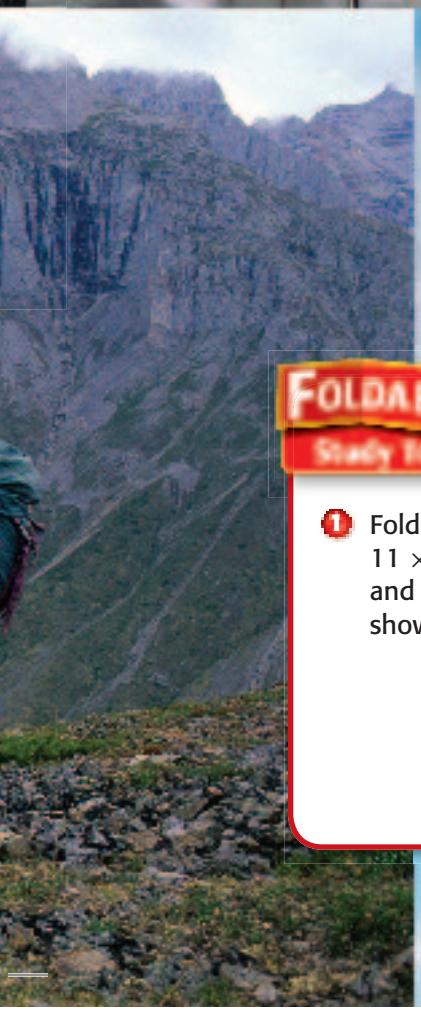
Air traffic controllers (ATCs) are responsible for directing air traffic in the sky and on the ground. They consider such variables as changing weather patterns, airplane types, and time of day. They may work visually from a control tower. They can also work electronically on a computer screen. ATCs analyse numerical relationships to do with wind speeds, airplane specifications, traffic, and rate of speed.

WWW Web Link

To learn more about becoming an air traffic controller, go to www.mhrmath10.ca and follow the links.

Did You Know?

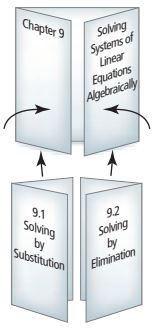
Many planes fly over the Arctic on their way to Europe. For this reason, there are aircraft monitoring sites in small Arctic airports.



FOLDABLES Study Tool

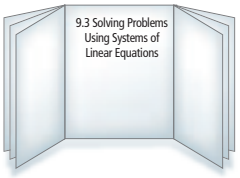
Make the following Foldable™ to take notes on what you will learn in Chapter 9.

1. Fold a sheet of 11 × 17 paper and label as shown.



2. Fold two sheets of 8.5 × 11 paper in half. Label as shown. Attach one sheet inside the left flap and one inside the right flap.

3. Label the inside centre as shown. Label the back What I Need to Work On, and Project Ideas and Questions.



9.1 Solving Systems of Linear Equations by Substitution

Focus on ...

- solving systems of linear equations algebraically using substitution

This year, the environmental club is having a fundraiser. Members are selling compact fluorescent light bulbs and 100% organic T-shirts with natural dyes. The price of one T-shirt is three times the price of one light bulb. You purchase two shirts and one light bulb for $42. If you lost your receipt, how could you determine the unit price for a light bulb and for a T-shirt?

Investigate Solving Systems of Linear Equations by Substitution

In the following balance diagrams, each block is identical in mass. Each cone is identical in mass.

Diagram 1

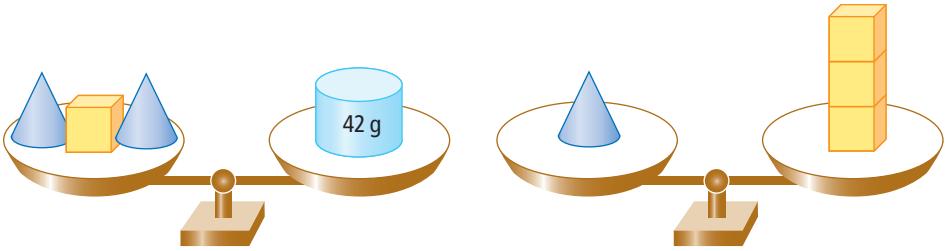
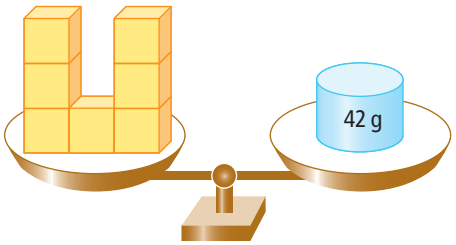


Diagram 2



1. Describe how Diagram 2 relates to Diagram 1.

2. Describe how you could determine the mass of one block from Diagram 2. What is the mass of one block?

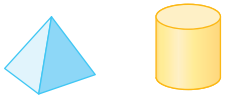
3. What is the mass of one cone? How did you determine your answer?

4. Write an equation for each balance scale in Diagram 1. Remember to state what your variables represent.

5. Write an equation for Diagram 2.

6. Suppose the mass of a block represents the cost of one light bulb, the mass of a cone represents the cost of one T-shirt, and each gram represents one dollar. Use algebra to show how you can determine the cost of one light bulb and the cost of one T-shirt for the scenario on page 468.

7. **Reflect and Respond** Use diagrams to explain how to determine the mass of a single pyramid and the mass of a single cylinder for the following scenario.
   - Five pyramids and three cylinders have a mass of 44 g.
   - Two pyramids have the same mass as one cylinder.



8. Use algebra to determine the mass of one pyramid, p, and the mass of one cylinder, c.

9. Describe a situation where using a diagram is less effective than using algebra.

WWW Web Link

To practise the algebraic method with a virtual scale, go to www.mhrmath10.ca and follow the links.

Link the Ideas

substitution method
- an algebraic method of solving a system of equations
- Solve one equation for one variable, substitute that value into the other equation, and solve for the other variable.

The skill of substituting algebraic expressions is used regularly in math and science. The **substitution method** can provide a quick solution to a linear system.

Solve the following linear system.
$4x + 5y = 26$
$3x = y - 9$

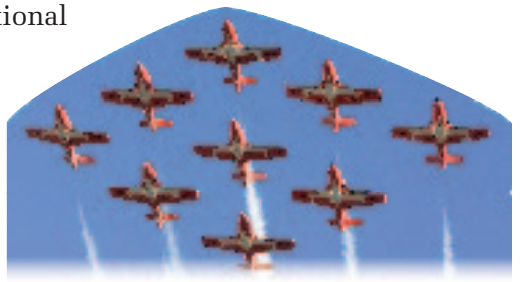
First, solve for y in $3x = y - 9$.
$3x + 9 = y - 9 + 9$
$3x + 9 = y$

Substitute $3x + 9$ for y in $4x + 5y = 26$.
$4x + 5(3x + 9) = 26$
$4x + 15x + 45 = 26$
$19x + 45 = 26$
$19x + 45 - 45 = 26 - 45$
$19x = -19$
$\frac{19x}{19} = \frac{-19}{19}$
$x = -1$

Substitute -1 for x in $3x = y - 9$.
$3(-1) = y - 9$
$-3 = y - 9$
$-3 + 9 = y - 9 + 9$
$6 = y$

Example 1 Solve a System of Linear Equations by Substitution

Admission to the 2009 Abbotsford International Airshow cost $80 for a car with two adults and three children. Admission for a car with two adults cost $50. Use algebra to determine the cost for one child and the cost for one adult. There was no charge for the vehicle or parking.



Did You Know?

The Abbotsford Airshow is held every August in Abbotsford, BC. It is one of the largest events of its kind in the world.

Solution

Let C represent the cost for one child, in dollars.
Let A represent the cost for one adult, in dollars.

We often use variables that are capital letters to represent values of money.

For the first car, $2A + 3C = 80$.
For the second car, $2A = 50$.

How do the equations represent the cost of admission for the first and second cars?

Determine the admission prices.

Method 1: Solve for *A* First

The second equation has only one variable. So, determine the cost for one adult first.

$2A = 50$

$A = 25$

Solve for C by replacing A with 25.

$2A + 3C = 80$ — You can also replace $2A$ with 50.

$2(25) + 3C = 80$

$50 + 3C = 80$

$3C = 30$ — If $50 + 3C = 80$, how do you know that $3C = 30$?

$C = 10$ — What does the value 10 represent?

Method 2: Solve for *C* First

Use substitution.

$50 + 3C = 80$ — Replace $2A$ with 50 in the equation.

Solve for C.

$50 + 3C = 80$

$3C = 30$

$C = 10$

Solve for A by replacing C with 10.

$2A + 3(10) = 80$

$2A + 30 = 80$

$2A = 50$ — If $2A = 50$, how do you know that $A = 25$?

$A = 25$ — What does the value 25 represent?

Check:

Substitute into the original equations, $2A + 3C = 80$ and $2A = 50$.

Left Side	Right Side
$2A + 3C$	80
$= 2(25) + 3(10)$	
$= 50 + 30$	
$= 80$	

You can also check your solution by graphing.

Left Side = Right Side

Left Side	Right Side
$2A$	50
$= 2(25)$	
$= 50$	

Left Side = Right Side

The admission price is \$10 for a child and \$25 for an adult.

Your Turn

Solve the following linear system algebraically using substitution.

$3x + 5y = 27$

$4x = 16$

Example 2 Isolate a Variable Before Solving by Substitution

At a dance recital, there were 220 people. Tickets cost \$9 for an adult and \$6 for a child. The dance school collected \$1614 in ticket sales. How many adults and how many children attended the recital?

Solution

Let a be the number of adults at the recital.
Let c be the number of children at the recital.
Write an equation that represents the total number of adults and children.

$a + c = 220$ ①

You can number the equations to make it easier to refer to them throughout the solution.

Write an equation that represents the amount collected by the dance school.

$9a + 6c = 1614$ ②

Isolate a variable in one of the equations.

Method 1: Isolate the Variable *c* in ①

$a + c = 220$

$c = 220 - a$

You can isolate the variable c in ① easily because the coefficient of the variable is 1.

Substitute for c in ②.

$9a + 6(220 - a) = 1614$
$9a + 1320 - 6a = 1614$
$3a + 1320 = 1614$
$3a = 294$
$a = 98$

What does the value 98 represent?

Substitute the number of adults into ① to finish solving the system.

$98 + c = 220$
$c = 122$

What does the value 122 represent?

Method 2: Isolate the Variable c in ②

$$9a + 6c = 1614$$
$$6c = 1614 - 9a$$
$$c = 269 - \frac{9}{6}a$$

Compare isolating c in Method 2 with isolating c in Method 1. Why does Method 2 take more steps?

Substitute for c in ①.

$$a + \left(269 - \frac{9}{6}a\right) = 220$$
$$-\frac{3}{6}a + 269 = 220$$
$$-\frac{1}{2}a + 269 = 220$$
$$-\frac{1}{2}a = -49$$
$$-\frac{1}{2}a(-2) = -49(-2)$$
$$a = 98$$

Substitute a in ① to finish solving the system.

$$98 + c = 220$$
$$c = 122$$

Check:
Substitute into ① and ②.

Left Side	Right Side
$a + c$	220
$= 98 + 122$	
$= 220$	

Left Side = Right Side

Left Side	Right Side
$9a + 6c$	1614
$= 9(98) + 6(122)$	
$= 882 + 732$	
$= 1614$	

Left Side = Right Side

At the dance recital, there were 98 adults and 122 children in attendance.

Which method do you prefer? Why? How might the solution be different if you isolated the variable a instead of c?

Your Turn

Solve the following linear system algebraically using substitution. Check your solution.

$$2x + y = 13$$
$$x - 0.4y = -16$$

Key Ideas

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations.
  - Where possible, choose a variable with a coefficient of 1.

    Solve the linear system.

    $3x + 2y = -11$ ①

    $-2x + y = 12$ ②

    Isolate the variable y in ② since its coefficient is 1.

    $y = 12 + 2x$

    Substitute the expression for y in ①.

    $3x + 2(12 + 2x) = -11$

    $3x + 24 + 4x = -11$

    $7x + 24 = -11$

    $7x = -35$

    $x = -5$
  - Substitute the solution for the first variable into one of the original equations. Solve for the remaining variable.

    $-2(-5) + y = 12$

    $10 + y = 12$

    $y = 2$
  - Check your answer by substituting into both original equations.

Check Your Understanding

Practise

1. Solve the following systems of linear equations by first substituting for y.

a) $y = 3x + 2$
$x + y = 14$

b) $y = -3x$
$y - x = 24$

c) $y = x - 7$
$x + y = 17$

2. Solve the following linear systems by substitution.

a) $2x - 3y = 10$
$x + y = 0$

b) $m = 8j$
$-m + 2 = -7j$

c) $2k = 6n + 9$
$n - 2k = -4$

3. Solve each linear system two ways. First, solve by isolating x. Then, solve by isolating y. For each linear system, explain which method you prefer and why.

a) $y = 0.3x - 5$
$1.7x + y = 9$

b) $y = 10 - 2.2x$
$5x + y = 70$

c) $\frac{x}{2} = 5 - y$
$x + y = 7$

4. Solve the following systems of linear equations. Check your answers.

a) $y = \frac{1}{3}x - 5$
$x - \frac{y}{5} = 13$

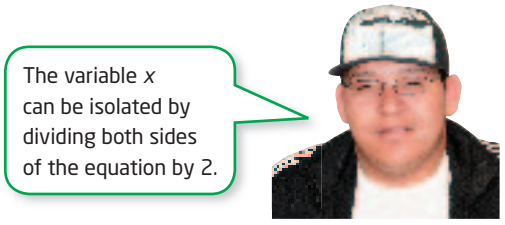
b) $\frac{y - x}{2} = 5$
$x + \frac{3}{4}y = 4$

c) $3y = \frac{1}{3} - \frac{2x}{3}$
$x + \frac{3y}{2} = 12$

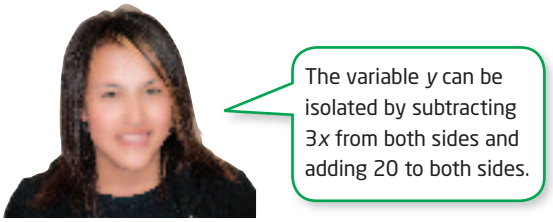
5. Jaret and Helen are going to solve the following system of equations.

$2x = 3y + 6$
$3x + y - 20 = 0$

As an intial step, Jaret decides to isolate x in the first equation.



As an intial step, Helen decides to isolate y in the second equation.



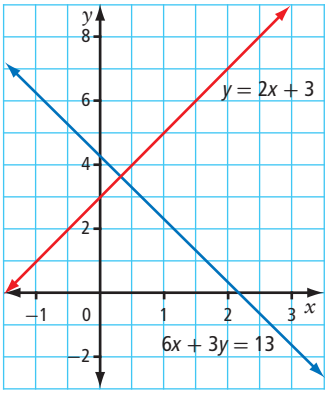
a) Solve this system using Jaret's method. Then, solve it using Helen's method.

b) Explain which method you prefer.

6. The sum of two numbers is 20. Twice one number is four more than four times the other. Write a system of linear equations and determine both numbers.

7. The graph represents the solution to the following linear system.

$y = 2x + 3$
$6x + 3y = 13$



a) What are the coordinates of the point of intersection?

b) Solve the linear system using the substitution method.

c) Compare your answers for parts a) and b). What is the advantage of the algebraic approach?

8. a) Solve the following system of linear equations by substitution.
$0.1y = 0.3x - 1.5$
$x - 0.2y = 5.6$

b) Multiply both sides of each equation by 10 first. Then, solve by substitution. How does the multiplication by 10 help you to solve?

c) Solve the system graphically.

d) Which of the three methods do you prefer? Explain.

Apply

Solve problems 9 to 20 using the substitution method. Check your answers.

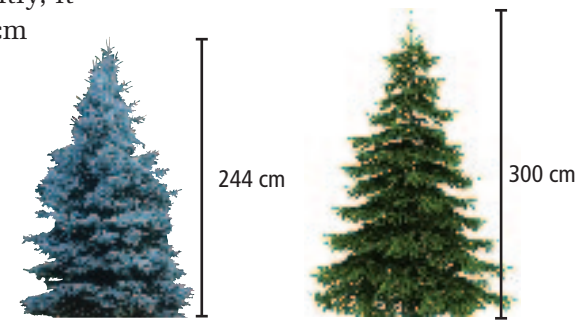
9. An 82-m cable is cut into two pieces. One piece is 18 m longer than the other. What is the length of each piece?

10. Whitehorse, YT, has three times as much snowfall each year as Vancouver, BC. The total combined snowfall for these two cities is approximately 192 cm. What is the snowfall in each city?

11. Alaina has \$72 and earns \$6 each day. Joel has \$48 and earns \$8 each day. In how many days will Joel have as much money as Alaina?

12. In Manitoba, teenagers watch approximately 11 fewer hours of TV each week than adults do. The sum of the hours watched per week for an adult and a teenager is about 37 h. Approximately how many hours per week do teenagers watch?

13. A young Colorado blue spruce tree is growing at a rate of 20 cm per year. Currently, it is 244 cm tall. A 300-cm tall white spruce tree is growing at a rate of 12 cm per year. In how many years will the two trees be the same height?



14. Rory's grandmother is 58 years older than Rory. In 5 years, they plan to have a party to celebrate that their ages have a sum of 100. How old are they now?

15. **Unit Project** A section of a local habitat was damaged during a storm. A local company wishes to preserve the wetland and ensure water quality. The organizers decide to replace some of the bushes and trees. They place two orders with a nursery.

- One order is for 40 bushes and 12 trees. It totals $1484.
- The other order is for 25 bushes and 18 trees. It totals $1421.

Create and solve a system of linear equations to determine the cost of one bush and the cost of one tree.

16. In Amir's coin collection, the number of dimes is one more than three times the number of nickels. The total number of nickels and dimes is 69.

a) Amir also has 40 quarters in his collection. How many more dimes than quarters does he have? How many more quarters than nickels does he have?

b) Explain why you can solve this problem without knowing the value of a nickel, dime, or quarter.

17. Students from two schools went on a trip to the Wascana Waterfowl Park in Regina, SK. They learned about the value of conserving natural resources. School A rented and filled 8 vans and 8 buses with 400 people. School B rented and filled 4 vans and 1 bus with 68 people. Every van had the same number of students in it, as did every bus. Determine the number of students in each van and in each bus.

WWW Web Link

For more information about Wascana Waterfowl Park, go to www.mhrmath10.ca and follow the links.

Did You Know?

Making white bread requires 45% more water than making whole-wheat bread. It takes more flour to make white bread. Also, the flour requires extra processing to remove the brown colour.

18. Water is used during each step of bread manufacturing, from processing the wheat into flour to making the bread itself. Less water is used to produce a slice of whole-wheat bread than a slice of white bread.

- To produce 60 slices of whole-wheat bread and 10 slices of white bread, 2080 L of water are used.
- To produce 20 slices of whole-wheat bread and 50 slices of white bread, 2560 L of water are used.

How many litres of water are used to produce one slice of whole-wheat bread? How many litres of water are used to produce one slice of white bread?

19. Andrew has a collection of 132 coins that consists of quarters and loonies. The value of the collection is $77.25. He wants to determine the number of quarters and the number of loonies he has without counting.

$q + n = 132$ and $0.25q + 1.00n = 77.25$, where q is the number of quarters and n is the number of loonies.

$$0.25q + 1.00(132 - q) = 77.25 \quad \text{Step 1}$$
$$100(0.25q) + (1.00)(132 - q) = (100)77.25 \quad \text{Step 2}$$
$$25q + 1.00(132 - q) = 7725 \quad \text{Step 3}$$
$$25q + 132 - q = 7725 \quad \text{Step 4}$$
$$24q = 7593 \quad \text{Step 5}$$
$$q = \frac{7593}{24} \quad \text{Step 6}$$

Andrew stopped when he realized he had made a mistake.

a) How did Andrew know $q = \frac{7593}{24}$ could not be correct?

b) Identify where Andrew made the first error in his work.

c) Solve the linear system correctly.

20. Solve the following linear systems. Express your answers as fractions.

a) $y = \frac{1}{7}x - 2$
$5x = 3y + 1$

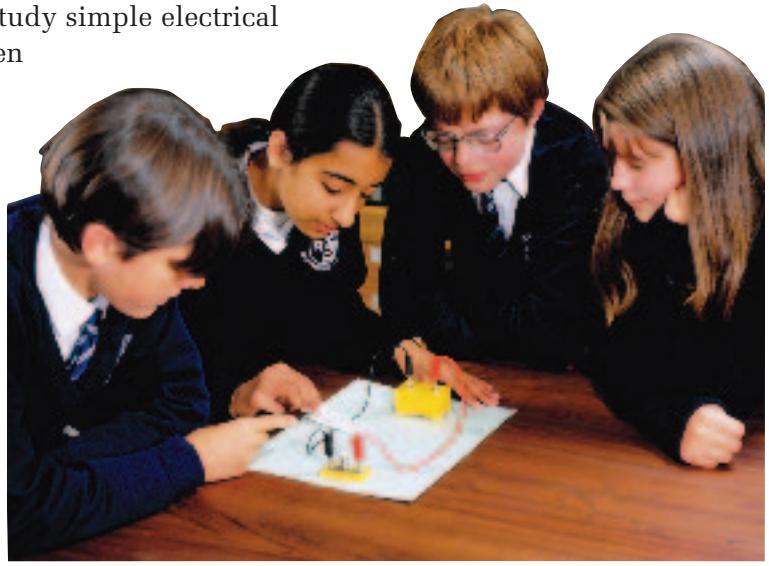
b) $\frac{1}{3}x + 4y = \frac{47}{15}$
$5x + 2y = 3.5$

Extend

21. On a graph, a line with the equation $y = mx + b$ passes through the points (2, 7) and (5, 1). Solve a linear system algebraically to determine the values of m and b. Include a diagram of the coordinate plane with your solution.

22. Arman walks to the train station at 5 km/h. He misses his train by 1 min. If he had run at 10 km/h, he would have had 2 min to spare. How far is it to the station?

23. In science, students frequently study simple electrical circuits. The relationship between the resistance of a circuit, R, the current, I, and the voltage, V, is $V = RI$. The relationship between the power, P, the voltage, and the current is $P = VI$. Use substitution to write a formula that determines the power from the resistance and the current. Show your work.



24. Use substitution to show that the linear system $y = 2x + 5$ and $2y - 4x = -15$ has no solution. How do you know there is no solution?

Create Connections

25. Compare solving a linear system by substitution to solving graphically.

a) How are the methods similar?

b) How are the methods different?

26. Choose a question from section 9.1 that has a system of linear equations.

a) Solve the system using a graphical approach.

b) Compare the graphical solution to the solution using substitution. Which method do you prefer for solving this system of equations? Explain.

9.2 Solving Systems of Linear Equations by Elimination

Focus on ...

- writing equivalent equations to eliminate a variable
- solving systems of linear equations algebraically using elimination

Did You Know?

Fair trade products are goods from developing countries. They are priced to ensure that the producers receive fair payment.

Environmental clubs promote many activities and products that are environmentally friendly. They promote chemical-free foods such as organic coffee. They also promote products that minimize waste. These include reusable shopping bags, and food and drink containers. What other similar products can you think of?

For their fundraiser, an environmental club is selling reusable shopping bags and organic fair-trade coffee. Two recent sales were:

- three bags and two packets of coffee for $17
- one bag and one packet of coffee for $7

How could you determine the unit price of one bag and one packet of coffee?

Investigate Solving Systems of Equations by Elimination

In the following balance diagrams, each block is identical in mass. Each cone is identical in mass.

Diagram 1

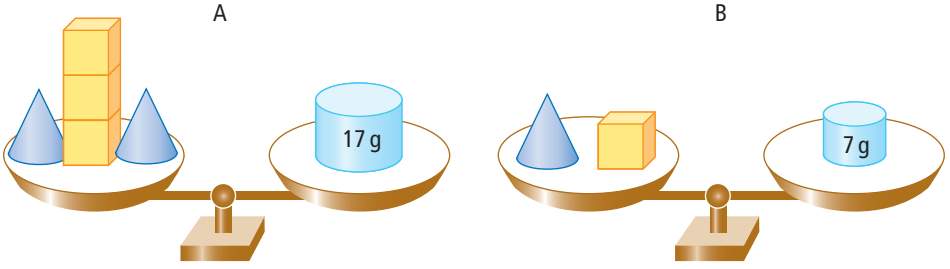
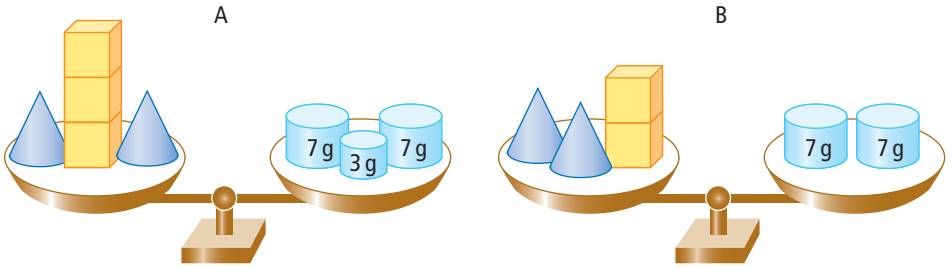


Diagram 2



1. Explain why scale B in Diagram 2 is balanced.
2. Draw a diagram of a scale balance to show how to determine the mass of one block.
3. Explain how you can determine the mass of the cone given the mass of the block.
4. Write equations for each scale in Diagram 1.
5. Use algebra to show how you would change scale B in Diagram 1 in order to write the equation for scale B in Diagram 2.
6. Use algebra to show how you can write the equation for the scale in your diagram from the two equations in Diagram 2.
7. Suppose the mass of a block represents the cost of one bag, the mass of a cone represents the cost of one packet of coffee, and each gram represents one dollar. How can you determine the cost of one bag and the cost of one coffee packet using algebra for the scenario on page 480?
8. **Reflect and Respond** Use diagrams to explain how to determine the mass of a single pyramid and the mass of a single cylinder for the following scenario.
   - Four pyramids and three cylinders have a mass of 23 g.
   - Two pyramids and five cylinders have a mass of 29 g.

9. Determine algebraically the mass of one pyramid, p, and the mass of one cylinder, c.
10. Could you use the substitution method to solve this Investigate? Explain.

WWW Web Link

To practise your algebraic skills using a virtual balance, go to www.mhrmath10.ca and follow the links.

elimination method

- an algebraic method of solving a system of equations
- Add or subtract the equations to eliminate one variable and solve for the other variable.

Link the Ideas

You can solve a system of linear equations using the **elimination method**. To do this, a variable in both equations must have the same or opposite coefficient. It is often necessary to multiply one or both equations by a constant.

For example, solve the following linear system:

$$6a + 5b = 24$$
$$4a + 3b = 12$$

In order to eliminate variable a, you need to multiply the first equation by 2. Multiply the second equation by 3. Now, both equations will contain the term $12a$.

$$2(6a + 5b) = 2(24)$$
$$3(4a + 3b) = 3(12)$$

Why should you choose a constant with the smallest possible value?

Example 1 Solve a System of Linear Equations by Elimination

Connor downloaded two orders of games and songs. The first order consisted of five games and four songs for $26. The second order consisted of three games and two songs for $15. All games cost the same amount, and all songs cost the same amount. Write a system of linear equations. Then, determine the cost of one song and the cost of one game.

Solution

Let S represent the cost of one downloaded song, in dollars.
Let G represent the cost of one downloaded game, in dollars.

Write two linear equations. Write an equation to represent the first order.

$5G + 4S = 26$ ①

How does ① represent the cost of five games and four songs?

Write an equation to represent the second order.

$3G + 2S = 15$ ②

Determine which variable to eliminate. One strategy is to examine each variable in both equations. Look for a least common multiple for the coefficients of the G terms or the S terms.

$5G + 4S = 26$
$3G + 2S = 15$

The coefficients of the term $4S$ in ① and the term $2S$ in ② have a least common multiple of 4.

Multiply ② by -2 so that there is an opposite S term to $-4S$ in ①. Then, you can add the equations to eliminate the S term.

You can also multiply by 2. Then, you can subtract the equations.

$$-2(3G + 2S) = -2(15)$$
$$-6G - 4S = -30 \quad ③$$

Add ③ and ① to eliminate the S terms.

$$\begin{array}{rcr} -6G - 4S & = & -30 \\ +\ (5G + 4S & = & 26) \\ \hline -G & = & -4 \\ G & = & 4 \end{array}$$

Solve for *G*.

What does the value 4 represent?

Solve for the remaining variable, S, by substituting 4 for G in ① or ②.

$$5(4) + 4S = 26$$
$$20 + 4S = 26$$
$$4S = 6$$
$$S = 1.5$$

What does the value 1.5 represent?

Check:

Substitute into ① and ②.

Left Side	Right Side
$5G + 4S$	26
$= 5(4) + 4(1.5)$	
$= 20 + 6$	
$= 26$	

Left Side = Right Side

Left Side	Right Side
$3G + 2S$	15
$= 3(4) + 2(1.5)$	
$= 12 + 3$	
$= 15$	

Left Side = Right Side

The cost of one game is \$4.00, and the cost of one song is \$1.50.

Your Turn

A group of people bought tickets for a University of Alberta basketball playoff game. Two student tickets and six adult tickets cost \$102. Eight student tickets and three adult tickets cost \$114. What was the price for a single adult ticket? What was the price for a single student ticket?

Did You Know?

A carbon sink is the term used for trees and plants that absorb carbon atoms into their roots and leaves. Carbon sinks reduce the amount of carbon dioxide in the atmosphere. Why might this be important?

Example 2 Solve a System of Linear Equations Using a Table and Elimination

A crop farmer has contracted with the Pacific Carbon Trust (PCT) to convert some of her cropland into woodland. This will create a carbon sink that is used to offset the production of carbon resulting from her farm activities. The farmer has 500 ha of cropland. She earns approximately \$220/ha from the crops. The PCT will pay her \$60 for every hectare of cropland that she converts. She would like a minimum revenue of \$90 800 that year. Using the elimination method, determine the number of hectares that she needs to convert to woodland. How many hectares of cropland would be left?

Solution

Let c represent the number of hectares of cropland.
Let w represent the number of hectares of woodland.
Organize the information in a table.

Type of Land	Revenue Generated Per Hectare (\$)	Number of Hectares	Revenue Generated (\$)
Cropland	220	c	$220c$
Woodland	60	w	$60w$
Total		500	90 800

Write an equation to show the total number of hectares.

$c + w = 500$ ①

Write an equation to determine the revenue created.

$220c + 60w = 90\ 800$ ②

The farmer wants a minimum revenue of \$90 800.

Determine which variable to eliminate. One strategy is to examine each variable in both equations. Then, identify the coefficient, other than 1, that is closest to zero.

$c + w = 500$ ①
$220c + 60w = 90\ 800$ ②

The term $60w$ in ② has the coefficient closest to zero.

Multiply ① by -60 so that there is an opposite w term.

$-60(c + w) = -60(500)$
$-60c - 60w = -30\ 000$ ③

What do you multiply ① by if you want to subtract the equations?

Add ② and ③ to eliminate the w terms.

$$\begin{array}{rrr} 220c + 60w = & 90\ 800 \\ +\ (-60c - 60w = & -30\ 000) \\ \hline 160c & = 60\ 800 \\ c & = 380 \end{array}$$

What does the value 380 represent?

Solve for the remaining variable, w, by substitution.

$380 + w = 500$
$w = 120$

Why is it more efficient to use ① instead of ② to solve for w?

What does the value 120 represent?

Check:
Substitute into ① and ②.

Left Side	Right Side
$c + w$	500
$= 380 + 120$	
$= 500$	
Left Side = Right Side	

Left Side	Right Side
$220c + 60w$	90 800
$= 220(380) + 60(120)$	
$= 83\ 600 + 7200$	
$= 90\ 800$	
Left Side = Right Side	

To generate a revenue of $90 800, the farmer could convert up to 120 ha to woodland. This would leave 380 ha for cropland.

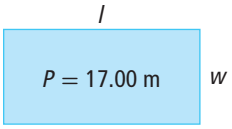
Your Turn

During lunch, the cafeteria sold a total of 160 muffins and individual yogurts. The price of each muffin is $1.50. Each container of yogurt is $2.00. The cafeteria collected $273.50. Set up and solve a linear system in order to determine the number of muffins and the number of yogurts sold.

Example 3 Solve a System of Linear Equations in Different Forms by Elimination

The perimeter of a rectangular garden is 17.00 m. Triple the length is 2.46 m longer than five times the width. Sketch and label a diagram. Create a system of linear equations to determine the dimensions of the rectangle. Solve the system using elimination.

Solution



Let w represent the width of the rectangle, in metres.
Let l represent the length of the rectangle, in metres.

Write an equation to represent the perimeter.
$2w + 2l = 17.00$ ①

Write an equation to represent the difference in the dimensions.
$3l = 5w + 2.46$ ②

Rearrange ② so that it is in the form $ax + by = c$, similar to ①.
$3l - 5w = 5w + 2.46 - 5w$
$-5w + 3l = 2.46$ ③

Decide whether you need to multiply one or both equations by a constant to eliminate a variable. Multiply ① by 3 and multiply ③ by -2 so that there are opposite l terms.
$3(2w + 2l) = 3(17.00)$
$6w + 6l = 51.00$ ④

$-2(-5w + 3l) = -2(2.46)$
$10w - 6l = -4.92$ ⑤

Add ④ and ⑤ to eliminate l.

$$\begin{array}{rl} 10w - 6l = & -4.92 \\ +\ (6w + 6l = & 51.00) \\ \hline 16w = & 46.08 \\ w = & 2.88 \end{array}$$

Solve for w.

Solve for l by substituting into ②.

$3l = 5w + 2.46$
$3l = 5(2.88) + 2.46$
$3l = 14.40 + 2.46$
$3l = 16.86$
$l = 5.62$

Check:
Substitute into ① and ②.

Left Side	Right Side
$2w + 2l$	17.00
$= 2(2.88) + 2(5.62)$	
$= 5.76 + 11.24$	
$= 17.00$	

Left Side = Right Side

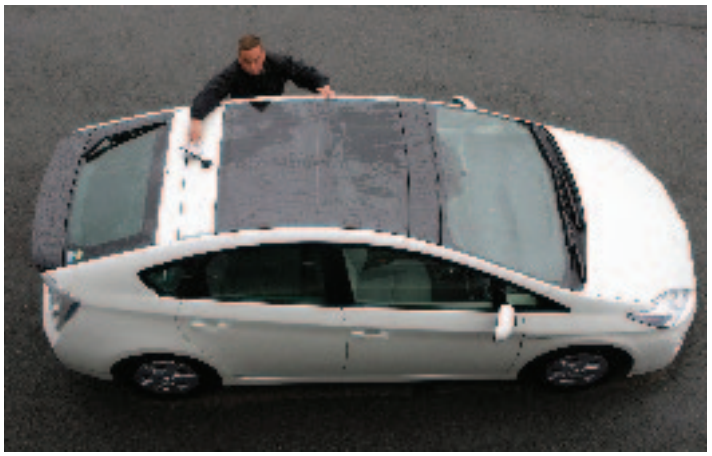
Left Side	Right Side
$3l$	$5w + 2.46$
$= 3(5.62)$	$= 5(2.88) + 2.46$
$= 16.86$	$= 14.40 + 2.46$
	$= 16.86$

Left Side = Right Side

The garden has a width of 2.88 m. Its length is 5.62 m.

Your Turn

A rectangular parking pad for a car has a perimeter of 12.2 m. The width is 0.7 m shorter than the length. Use a linear system to determine the dimensions of the pad.



Key Ideas

- A table can help you organize information in a problem. This can help you to determine the equations in a linear system.
- You can solve a linear system by elimination.

  $3x + 2y + 6 = 0$
  $7y = 5x + 41$

  - If necessary, rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$.

    $3x + 2y + 6 = 0$ $\qquad$ $7y = 5x + 41$
    $3x + 2y = -6$ ① $\qquad$ $-5x + 7y = 41$ ②
  - Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

    Multiply ① by 5 and ② by 3 so that the coefficients of the terms involving x add to zero.

    $5(3x + 2y) = 5(-6)$ $\qquad$ $3(-5x + 7y) = 3(41)$
    $15x + 10y = -30$ ③ $\qquad$ $-15x + 21y = 123$ ④

    Add ③ and ④ to eliminate x.

    $$\begin{array}{r} 15x + 10y = -30 \\ +\ (-15x + 21y = 123) \\ \hline 31y = 93 \end{array}$$

    You can also multiply by −3 or by −5 and then subtract.
  - Solve for the remaining variable.

    $31y = 93$
    $y = 3$
  - Solve for the second variable by substituting the value for the first variable into one of the original equations.

    $7(3) = 5x + 41$
    $21 = 5x + 41$
    $-20 = 5x$
    $-4 = x$
- Check your solution by substituting each value into both original equations.

Check Your Understanding

Practise

1. Solve using elimination.

a) $x + y = 10$
$x - y = 4$

b) $x + 2y = 13$
$x - y = 8$

c) $y - 2x = -4$
$y + 3x = 16$

2. Rearrange the equations so that the variables are ordered in the same way for both equations.

a) $y - 3x = 11$
$x - y = -5$

b) $x + 7 = y$
$2x + y = -8$

c) $4 - 3y = x$
$x - y = 16$

3. Use the table to set up a linear system. Use the elimination method to determine the number of tickets sold to:

a) adults **b)** students

Ticket Type	Price Per Ticket ($)	Number of Tickets Sold by Type	Revenue Collected by Theatre ($)
Students	10	s	$10s$
Adults	13	a	$13a$
Total		430	4804

4. Solve the following systems of linear equations by elimination. Check your answers.

a) $3x + 2y = 7$
$4x + 5y = 14$

b) $7x - 6y = 27$
$2x + 9y = -3$

c) $4y + 29 = 3x$
$8x + 7 = 3y$

5. Solve using the elimination method. Leave your answers in fraction form.

a) $3x + 2y = 10$
$2x - y = 4$

b) $\frac{x}{3} - y = \frac{3}{5}$
$x + 6y = 4$

c) $2 - \frac{y}{2} = \frac{x}{3}$
$\frac{2}{3}(2x - 3y) = 4$

6. Solve the following system. Explain the result.

$3x + 2y = 7$
$9x + 6y = 16$

Apply

Solve problems 7 to 14 using the elimination method. Check your answers.

7. A preschool playground has both bicycles and tricycles. There is a total of 30 seats and 70 wheels. How many bicycles are there? How many tricycles are there?

8. Students at Evergreen High School want to help the community with the Communities in Bloom project. They decide to sell flower bulbs to raise money. Nancy sells 10 bags of tulip bulbs and 12 bags of iris bulbs for $380. Shawn sells 6 bags of tulip bulbs and 8 bags of iris bulbs for $244. What is the cost of one bag of tulip bulbs? What is the cost of one bag of iris bulbs?

Did You Know?

Communities in Bloom is a national non-profit organization dedicated to the creation and maintenance of green spaces in urban settings. On September 20, 2008, Lethbridge, AB, hosted the 14th Communities in Bloom National Awards Ceremony.

9. At the snack bar, five toasted bagels and three cans of juice cost $12.50. Three toasted bagels and six cans of juice cost $12.75. What is the price for one bagel? What is the price for one juice?

10. A total of 430 dogs and people attended the Woof Walk fundraiser. Altogether, 1210 legs participated in the walk. How many dogs were there?

11. A ferry is carrying 600 vehicles, including trucks and passenger vehicles. The fees collected total $29 200. The charge per truck is $100. The charge per passenger vehicle is $45. How many trucks and how many passenger vehicles is the ferry carrying?

12. An avalanche rescue team travels 8.55 km along a snow-covered trail. For the first section, the trail is flat. The team averages a speed of 2.7 km/h. Then, the terrain becomes mountainous and their average speed is only 1.2 km/h. The one-way trip takes the team 4.0 h. Determine the distance that the team travels on each type of terrain.

Did You Know?

A Joule (J) is the energy involved when a force of 1 N (newton) acts to move an object through a distance of 1 m.

13. Soo Jin had basketball practice after school. Then, she cycled home. Playing basketball, she expends energy at a rate of 25 kJ per minute. Cycling home, she burns energy at a rate of 21 kJ per minute. She spent a total of 90 min doing both forms of exercise. During this time, she expended a total of 2178 kJ of energy. How much time did she spend doing each activity?

14. **Unit Project** Sharon estimates that she saves 260 L of water per week by washing her car with a bucket and sponge. Her sister Bev washes her car with a hose, which uses more water. Sharon's washing machine uses 225 L of water per load. Bev has upgraded to a washing machine that uses only 95 L of water per load. Both sisters wash the same number of loads of laundry per week. Both wash their car once a week.

a) Develop a system of equations representing their water usage in one week.

b) When their water usage is the same, how many loads of laundry does each sister do in one week?

c) If each sister does eight loads of laundry per week, who uses more water weekly? Explain.

Extend

15. Simplify. Then, solve the following linear systems using elimination.

a) $3(x + 2) + 7y = 11$
$-5(3 - x) + 9y = -12$

b) $5x - 2(y + 4) = y - 3x$
$2(x + 8y) - 4y = 9x$

16. Brittany invested a total of $3000 in two different investments. The safer investment earned 3.5% interest by the end of the year. The riskier investment earned 5.2% interest by the end of the year. Her total interest earned was $126.25. How much did she invest in the safer investment?

17. Milk that has 3.25% milk fat (MF) is mixed with milk that has 1% MF. What volume of each is needed to obtain 60 L of milk that has 3% MF? Express your answers to the nearest tenth of a litre.

Did You Know?

Milk fat, or butterfat, is the fatty portion of milk. Milk and cream are sold according to the amount of milk fat they contain.

18. Determine the value of k so that $6x + 4y = 7$ and $kx + 8y = 7$ do not have a common solution.

19. The solution to the system $10x + 12y = -18$ and $5x + 4y = b$ is $(9, a)$. What can b and a be? Is there more than one possibility? Explain.

Create Connections

20. a) Choose a question from section 9.2 and solve it using the substitution method.

b) Explain why you selected the question you did.

c) Are there any questions in section 9.2 that do not suit being solved using the substitution method? Explain.

21. What do you need to consider when choosing whether to use the substitution method or the elimination method to solve a system of linear equations? Provide examples to clarify your explanation.

9.3 Solving Problems Using Systems of Linear Equations

Focus on ...

- choosing a strategy to solve a problem that involves a system of linear equations

Did You Know?

Dog teams are making a comeback in the Arctic. Part of the reason is that dog teams do not need gas or expensive parts.

Many people drive hybrid vehicles because these cars consume less gas. Some people drive hybrids to reduce their ecological footprint. Others like the savings in the cost of gas. How can you determine if a hybrid is cost efficient?

Investigate Solving a Problem Involving a System of Linear Equations

Materials

- graphing calculator or computer with spreadsheet software

A sample price for a hybrid car is $28 000. The price of a similar car powered by gas is $21 500. The hybrid vehicle costs $0.18 per kilometre to operate. The non-hybrid vehicle costs $0.22 per kilometre to operate.

1. Write a system of linear equations that models the total cost for each vehicle in relation to the distance travelled.

2. Solve the linear system using a graphing calculator or spreadsheet software. After how many kilometres will the hybrid car be more cost efficient?

3. **a)** Solve the same system of equations from step 1 algebraically. Use either the substitution method or the elimination method.

   b) Explain why you chose the method you did.

4. **Reflect and Respond** Compare and contrast the following three methods of solving a system of linear equations:
   - graphically
   - algebraically by substitution
   - algebraically by elimination

   Include examples.

5. Compare your response to step 4 with a classmate's. Note any insights that your classmate provides.

Link the Ideas

You can use graphical or algebraic methods to solve systems of linear equations. Each method has its advantages and disadvantages.

Method	Advantages	Disadvantages
Graphical	• provides a visual that can show how two variables relate • can be done with or without a graphing calculator • can result in an accurate and quick solution when using a graphing calculator	• can be time-consuming • may not provide an exact solution
Algebraic	• allows for an exact solution relatively quickly • can be done using more than one method (substitution and elimination)	• does not provide any visual insight into how the two variables relate • can result in an incorrect answer due to a minor arithmetic error

Example 1 Compare Methods of Solving

Jeremy and Shilan participated in their school's Plant-a-thon fundraiser. Jeremy started planting seedlings at 10:00 a.m. He planted at a steady rate of one tree per minute. Shilan started planting at 11:30 a.m. Her planting rate was three trees every 2 min.

a) At what time had they planted the same number of trees? Use a system of linear equations to find out. Solve the system using a graphing calculator and algebraically.

b) Which method do you prefer? Why?

Solution

a) Let n represent the number of trees that were planted after 11:30 a.m. Let m represent the number of minutes that have passed since 11:30 a.m. Determine equations for the number of trees planted by Jeremy and Shilan.

For Jeremy: $n = 90 + 1m$

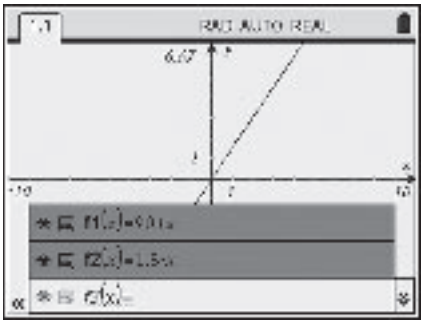
Jeremy started at 10:00 a.m. He planted at a steady rate of one tree per minute. What does 90 represent in the equation?

For Shilan: $n = 1.5m$

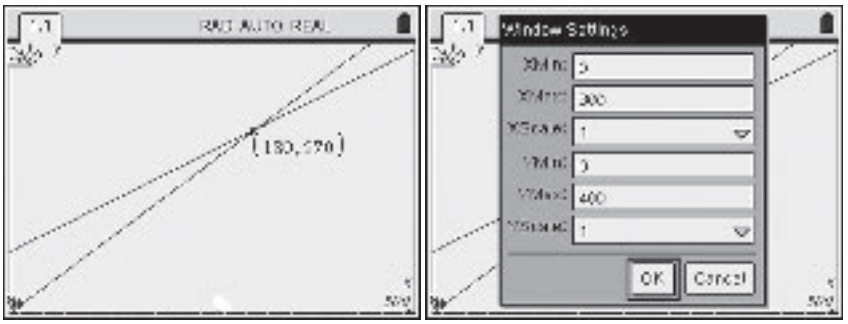
Shilan planted three trees every 2 min. What does 1.5 represent in the equation?

Method 1: Solve Using a Graphing Calculator

Graph the equations using a calculator.



Determine an appropriate window and graph it.



The point of intersection is (180, 270).

Check:
Substitute the intersection point into the original equations.

For Jeremy:		For Shilan:	
Left Side	Right Side	Left Side	Right Side
$n = 270$	$90 + 1m$	$n = 270$	$1.5m$
	$= 90 + 180$		$= 1.5(180)$
	$= 270$		$= 270$
Left Side = Right Side		Left Side = Right Side	

Method 2: Solve Algebraically

In both equations, n is isolated. Therefore, the substitution method is an appropriate algebraic method.

Write Shilan's planting rate to be equal to Jeremy's rate.

$1.5m = 90 + 1m$
$0.5m = 90$ Solve for m.
$m = 180$

Solve for n by substituting into one of the initial equations.

$n = 1.5m$
$n = 1.5(180)$
$n = 270$

Check:
Substitute into the original equations.

For Jeremy:		For Shilan:	
Left Side	Right Side	Left Side	Right Side
$n = 270$	$90 + 1m$	$n = 270$	$1.5m$
	$= 90 + 180$		$= 1.5(180)$
	$= 270$		$= 270$
Left Side = Right Side		Left Side = Right Side	

Both Jeremy and Shilan had planted 270 trees 180 min after Shilan started. 180 min = 3 h and 11:30 a.m. + 3 h = 2:30 p.m. They had planted the same number of trees at 2:30 p.m.

b) The algebraic method is preferable. For the graphing method, the equations were already in the form $y = mx + b$, so it was easy to graph. However, it took some time to find a reasonable window to determine the intersection point. For the algebraic method, the substitution method was easy to use, since both equations were in the form $y = mx + b$.

Your Turn

Solve the following system of linear equations algebraically and graphically. Which method do you prefer? Explain.

$y = 0.25x - 200$
$3x + 2y = 160$

Did You Know?

There are numerous multicultural festivals across western Canada. People celebrate and learn about the culture, food, and entertainment of various ethnic and indigenous groups. Every August, the Folklorama Festival in Winnipeg and Folkfest in Saskatoon have a Métis pavilion.

Example 2 Compare Algebraic Methods

Folkfest, Saskatoon

At the Métis People Pavilion, visitors can enjoy bannock and buffalo stew. A recent sale of three orders of stew and two orders of bannock cost $13.50. A second sale of four orders of stew and five orders of bannock cost $21.50.

a) Use a system of linear equations to determine the price of one order of bannock and the price of one order of stew. Solve the system algebraically using two methods.

b) Compare the two methods.

Solution

a) Let B represent the cost for one order of bannock, in dollars.
Let S represent the cost for one order of buffalo stew, in dollars.
Write an equation to represent the first sale.

$3S + 2B = 13.50$ ①

Write an equation to represent the second sale.

$4S + 5B = 21.50$ ②

Method 1: Use Substitution

Isolate the variable B in ① since it has the smallest coefficient, 2.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 2B &= 13.50 - 3S \\ B &= 6.75 - 1.5S \end{aligned}$$

You can try isolating the variable S in ① instead.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 3S &= 13.50 - 2B \\ S &= 4.5 - \frac{2}{3}B \end{aligned}$$

Why might it be better to isolate B?

Substitute the expression for B into ②.

$$\begin{aligned} 4S + 5(6.75 - 1.5S) &= 21.50 \\ 4S + 33.75 - 7.5S &= 21.50 \\ -3.5S + 33.75 &= 21.50 \\ -3.5S &= -12.25 \\ S &= 3.50 \end{aligned}$$

What does 3.50 represent?

Substitute the value of S into ① or ② to solve for B.

$$\begin{aligned} 3(3.50) + 2B &= 13.50 \\ 10.50 + 2B &= 13.50 \\ 2B &= 3.00 \\ B &= 1.50 \end{aligned}$$

What does 1.50 represent?

Method 2: Use Elimination

Multiply ① by 5. Multiply ② by -2. Then, eliminate the variable B by addition.

$5(3S + 2B) = 5(13.50)$ $\quad -2(4S + 5B) = -2(21.50)$

$15S + 10B = 67.50$ ③ $\quad -8S - 10B = -43.00$ ④

Add ③ and ④ to eliminate the variable B.

$$\begin{array}{rrl} 15S + 10B = & 67.50 & \\ +\ (-8S - 10B = & -43.00) & \\ \hline 7S & = 24.50 & \text{Solve for } S. \\ S & = 3.50 & \end{array}$$

Substitute the value of S into ① or ② to solve for B.

$3(3.50) + 2B = 13.50$

$10.50 + 2B = 13.50$

$2B = 3.00$

$B = 1.50$

Check:

Substitute into ① and ②.

Left Side	Right Side
$3S + 2B$	13.50
$= 3(3.50) + 2(1.50)$	
$= 10.50 + 3.00$	
$= 13.50$	

Left Side = Right Side

Left Side	Right Side
$4S + 5B$	21.50
$= 4(3.50) + 5(1.50)$	
$= 14.00 + 7.50$	
$= 21.50$	

Left Side = Right Side

The price of one order of stew is \$3.50. The price of one order of bannock is \$1.50.

b) For this question, it took more steps to isolate the first variable using the substitution method than the elimination method. Also, if the variable S had been isolated in ①, the result for S would have made it complicated to solve for B.

Your Turn

Solve the linear system twice, using both algebraic methods. Compare the two methods.

$3x - 4y = 17$

$4x + 5y = 48.5$

Key Ideas

- Systems of linear equations can be solved
  - graphically
  - algebraically by substitution or by elimination
- It may be better to use a graphical approach to solve linear equations when you wish to see how the two variables relate, such as for cost analysis and speed problems.
- It may be better to use an algebraic approach to solve linear equations when
  - you need only the solution (intersection point)
  - it is unclear where to locate the solution on a coordinate plane

Check Your Understanding

Practise

1. Solve each system of linear equations using a method of your choice. Check your answer graphically.

a) $2x - 5y = 12$
$-7x + 5y = 48$

b) $3y = 6 - x$
$5x + 6y = -6$

c) $n = 3k - 2$
$2n - 6k = -4$

2. Solve each system of linear equations using your preferred method.

a) $0.2y + x = 0.7$
$2y + 12x = 11$

b) $\frac{m}{7} + \frac{n}{2} = 7$
$2m + 6 = 3n$

c) $4x - 7y = 6$
$5x = 2y + 3$

Apply

3. In January, the average high temperature for Calgary is 9.9 °C greater than Winnipeg's average high temperature. The sum of these two temperatures is −15.5 °C. What is the average high temperature in January for each of these two cities?

4. In Canada, the percent of workers who drive themselves to work is approximately 11.3 times the percent of workers who walk. The combined percent of Canadian workers who drive themselves and walk is about 78.7%. Approximately what percent of Canadian workers walk to work? Express your answer to the nearest tenth of a percent.

5. A school's multicultural club is selling muffins for a fundraiser. The club spends \$16.00 on advertising. The cost of ingredients for each muffin is \$0.30. The club decides to sell the muffins for \$0.75 each. The following equations model this situation: $C = 0.3m + 16$ and $C = 0.75m$.

a) Describe in words what each equation represents.

b) Determine the minimum number of muffins the club will have to sell to cover their total costs.

6. An incandescent 60-W light bulb sells for approximately \$0.75. It costs \$0.0072 to operate per hour. An equivalent compact fluorescent bulb costs \$4.00. It uses 15 W of power and costs \$0.0018 per hour to operate. The following equations model the cost: $C = 0.75 + 0.0072h$ and $C = 4 + 0.0018h$. In these equations, C is the total cost, in dollars, and h is the number of hours. How many hours will it take for the compact fluorescent bulb to be less expensive?

Did You Know?

A compact fluorescent bulb has a lifespan of approximately 10 000 h versus 1000 h for an incandescent bulb.

7. A circus recently had a sold-out performance. There were varying admission prices. The admission for premium seating was \$250 for adults and \$175 for students. The total revenue for premium seating was \$29 125. The receipts showed that 130 premium seats were sold. Determine how many adults and how many students were in premium seats.

Cirque du Soleil

Did You Know?

Cirque du Soleil is a dramatic mix of circus arts and street entertainment. Based in Montréal, Québec, it was founded in 1984. The founders were two former street performers, Guy Laliberté and Daniel Gauthier. Cirque expanded rapidly. It grew from one show to multiple shows around the world.

8. Jason is renting a car for one week. Speed-E-Car Rental offers a compact car for \$379 plus \$0.10 per kilometre. Easy 4 U Auto offers a compact car for \$249 plus \$0.35 per kilometre. Use a system of linear equations to determine when each company would be the better choice for Jason.

9. **Unit Project** The water level in a lake is decreasing. Wildlife biologists are concerned about the effect on the fish population. They decide to track the number of fish in the lake. The osprey is a fish-hunting bird. As part of their study, the biologists need to estimate the number of fish eaten by osprey.

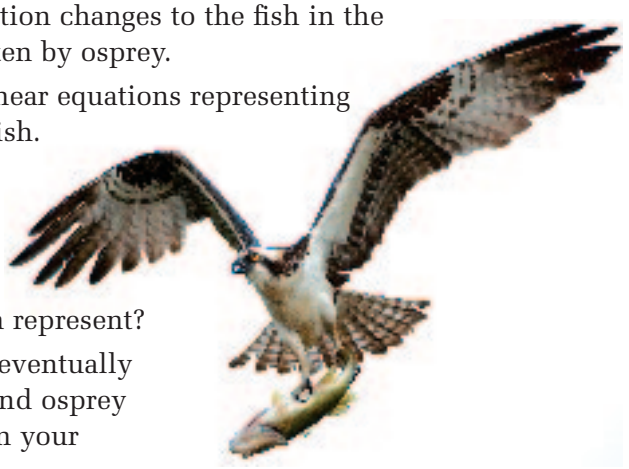
Year	Fish in Lake	Fish Eaten by Osprey
1	10 000	700
2	9 000	900
3	8 000	1100
4	7 000	1300

a) Describe the population changes to the fish in the lake and the fish eaten by osprey.

b) Write a system of linear equations representing the populations of fish.

c) Solve the system of linear equations graphically. What does the point of intersection represent?

d) Predict what might eventually happen to the fish and osprey populations. Explain your thinking.



Did You Know?

Scuba divers can suffer from a potentially lethal condition called *decompression sickness*, or *the bends*. This occurs if they rise too quickly to the surface of the water. At greater depths, there is extra pressure on the diver. The extra pressure causes nitrogen gas to dissolve in the diver's blood. As the diver slowly rises back to the surface, the nitrogen gas comes out of solution and forms bubbles in the blood. If the diver ascends too quickly, the nitrogen remains dissolved in the blood. The result is the bends, a painful condition that may be fatal.

10. Scuba divers can spend only a limited amount of time at depths between 60 m and 90 m. This amount of time can be represented by a linear relation. A diver can remain for 60 min at a depth of 60 m, and 30 min at a depth of 90 m. Write a system of linear equations to determine the slope intercept form, $y = mx + b$, for this linear relation.

11. Last Saturday, Juan went cross-country skiing in the morning. In the afternoon, he played squash. Cross-country skiers expend 50 kJ of energy per minute. Squash players burn 42 kJ per minute. In total, Juan exercised for 100 min. He used 4850 kJ in energy. How much time did he spend doing each activity?

Extend

12. The lines that enclose a triangle can be represented by graphs of the equations $y = 3$, $y = -x + 7$, and $y = 2x + 16$. Use a system of linear equations to determine the area of the triangle.

13. Answer the following questions using the two systems of linear equations shown.

$$\begin{aligned} 39x + 49y &= 2283 \\ 43x + 54y &= 2516 \end{aligned} \quad \text{and} \quad \begin{aligned} 39x + 49y &= 2283 \\ 43x + 54y &= 2517 \end{aligned}$$

a) What difference do you see between the two systems of linear equations just by looking at them?

b) Solve both systems algebraically.

c) Solve both systems graphically.

d) Explain why these linear systems are difficult to solve.

Create Connections

14. a) Create a system of linear equations. Solve your system using a method of your choice.

b) How do you decide on a strategy for solving a system of linear equations? What do you consider? Why?

15. a) Create a system of linear equations with a solution involving a rational number that cannot be expressed exactly on a graphing calculator. Solve your system graphically and algebraically.

b) Describe your results using the two methods.

9 Review

9.1 Solving Systems of Linear Equations by Substitution, pages 468–479

1. Solve by substitution.

a) $y = 3x - 1$
$x + y = 11$

b) $x - 2y = 4$
$x - 3y = 6$

c) $2 + y = 3x$
$6x - 5y = 8$

2. Determine the intersection point of the two lines $y = 3x - 4$ and $4x + y = 13$. Solve graphically, then algebraically. Which method is a better choice? Why?

3. The table shows data about two vehicles. What distance will both vehicles need to travel for the cost to be the same?

Type of Car	Initial Cost ($)	Cost Per Kilometre ($)
Compact hybrid	31 300	0.27
Compact non-hybrid	26 500	0.42

4. On a web site, the cost to download a game is three times the cost to download a song. The cost for five songs and two games is $15.40. What is the cost of one song and the cost of one game?

9.2 Solving Systems of Linear Equations by Elimination, pages 480–491

Vancouver, BC

Yellowknife, NT

5. Solve using elimination.

a) $x - y = 17$
$x + y = -9$

b) $3x + 2y = 10$
$2x - y = 4$

c) $\frac{y}{2} = 2x - 3$
$3x + 2y = \frac{9}{2}$

6. The number of wet days in a year for Vancouver, BC, is 47 days greater than the number for Yellowknife, NT. The sum of the numbers of wet days for one year in these two cities is 285. How many wet days occur in each city?

7. In an isosceles triangle, the two base angles have a sum that is 6° more than the third angle. Sketch and label a diagram. What is the measure of each of the three angles?

8. The percent of carbohydrates by weight in grapes is 15%. The percent of carbohydrates in an orange is 7%. Danika consumed a total of 325 g of grapes and oranges. The percent of carbohydrates in the mixture she ate was 10%. How many grams of grapes did she eat? How many grams of oranges?

9.3 Solving Problems Using Systems of Linear Equations, pages 492–501

9. The operators of a national park want to be more water efficient. They decide to start with the park's comfort stations. First, they purchase one front-load washing machine and four shower heads for $900. Then, they purchase ten washers and eight shower heads for $8200. What is the cost of each washing machine? What is the cost of each shower head?

10. Michelle rented a car from the same company twice last month. The cost of the first rental was $116.70 for three days. She drove a distance of 98 km. The cost of the second rental was $78.80 for two days of driving a distance of 72 km.

a) What is the daily rental cost? What is the charge per kilometre?

b) What method did you use to solve the problem? Why?

11. A local nature club wants to convert 57 acres of land to campgrounds. Basic campgrounds have a density of 1.5 sites per acre. Developed campgrounds have 4 sites per acre. The amount of land used for basic sites is twice the amount of land used for developed sites.

a) Determine the number of acres to be used for each type of campground if all 57 acres of land are used.

b) How many campsites of each type will there be?

12. Yesterday, Keegan read a book that had 220 pages. He started the book before dinner and read at a speed of 50 pages per hour. At bedtime, he read at a rate of 41 pages per hour and finished the book. He spent a total of 5 h reading. How much time did he spend reading before dinner? Express your answer in hours and minutes.

13. a) Solve the following system of linear equations graphically and algebraically: $y = 4 - x$ and $x + y = 6$.

b) Use the graphs of these two lines to explain your results.

9 Practice Test

Multiple Choice

For #1 to #4, choose the best answer.

1. The ordered pair $(3, -2)$ is the solution for which linear system?

A $5x - y = 7$, $x + y = 12$ **B** $y = 4x + 11$, $x - y = 1$ **C** $2x - 7y = 1$, $x + y = -4$ **D** $x - y = 5$, $3x + 2y = 5$

2. Which of the following is the solution to the system of equations $y = 8 - x$ and $2x + 3y = 14$?

A $(-10, 18)$ **B** $(3, 5)$ **C** $(10, -2)$ **D** $(19, -11)$

Did You Know?

Recycling in the Arctic is a challenge because of the expense of shipping the materials south to be recycled. Some Northern schools run can-recycling programs. These programs are possible thanks to local airlines. The airlines transport the crushed and packaged cans south to a recycler. They will often do this at a reduced rate or even no charge.

3. A recycling company sells its recycling bins for \$20 each. The fixed expenses to manufacture the bins total \$4000. In addition, there is an expense of \$12 per bin. Let x represent the number of bins. Which linear system allows you to determine when the company's expenses equal their sales?

A $y = 4000$, $y = 12x + 20$ **B** $y = 4000 - x$, $y = 20x + 12$

C $y = 4000 + 12x$, $y = 20x$ **D** $y = 4000 + 20x$, $y = 12x$

4. The cost of three compact fluorescent light bulbs (CFBs) and five incandescent light bulbs (ILBs) is \$15.00. The cost of a CFB is five times the cost of an ILB. What is the cost of one CFB? What is the cost of one ILB?

A CFB = \$0.40, ILB = \$2.00 **B** CFB = \$1.25, ILB = \$0.25

C CFB = \$3.75, ILB = \$0.75 **D** CFB = \$7.00, ILB = \$1.40

Short Answer

5. Solve each linear system algebraically. Show your work.

a) $3x - y = 7$, $x + y = 10$ **b)** $y = 7 - 9x$, $17 - 2y = 16x$ **c)** $\frac{x}{3} - y = 5$, $5x + 3y = 12$

6. The length of a rectangle is 3 m less than five times the width. The perimeter is 10.8 m. What are its dimensions?

7. At the bulk store, peanuts cost \$1.20 per 100 g and almonds cost \$2.00 per 100 g. They are mixed together to create a 300-g bag of mixed nuts. This mix sells for \$1.50 per 100 g. What amount of each type of nut is used?

8. Use the information in the table to determine the number of nickels and the number of quarters in the coin collection.

Coin Type	Value of One Coin (¢)	Number of Coins	Total Value of Coins (¢)
Nickel	5	n	$5n$
Quarter	25	q	$25q$
Total:		49	885

9. A golf club charges an annual fee. It also charges a green fee for each game played. Tegan played 38 games and paid a total of $986. Cassandra played 15 games and paid $480. How much are the annual fee and the green fee?

10. Francofièvre, or French Fever, is an event celebrating francophone culture. Last year, a total of 696 students and teachers from one high school went to the event. On average, every teacher brought 23 students. How many students and how many teachers from the high school attended the event that year?

Did You Know?

Francofièvre is the largest francophone youth rally in western Canada. It is held annually in Saskatchewan each spring. High school students visit to celebrate French music, dance, and artists.

Extended Response

11. Mallory drove 805 km from Edmonton to Regina. From Edmonton to Saskatoon, her average speed was 88 km/h. From Saskatoon to Regina, her average speed was 71 km/h due to road construction. She drove for 9.85 h. Express your answers to the following questions to two decimal places.

a) How long did each part of her trip take?

b) What is the distance between Edmonton and Saskatoon? Explain the method you used.

12. The three lines that enclose a triangle can be represented by graphs of the equations $x = 3$, $x + y = 10$, and $5x - 2y = 15$.

a) Graph the three lines.

b) Determine the intersection points between each pair of lines. Solve algebraically and graphically.

4 Unit Connections

Unit 4 Project

WWW Web Link

For information and suggestions about reducing water use in the home, go to www.mhrmath10.ca and follow the links.

For this project, you will do an analysis of the effect of our water use on wildlife, reducing water use in homes, and retrofitting plumbing fixtures. Use your answers to the unit project questions throughout Chapters 8 and 9, as well as your own research. Your analysis should include the following:

- data involving the effect of our water use on populations of wildlife
- information about costs and flow rates of various low-flow plumbing fixtures
- a comparison using linear systems (represented multiple ways) of the cost of keeping conventional fixtures and the cost of retrofitting

To complete your project, prepare a presentation that outlines the environmental and economic benefits of retrofitting and reducing water use. Keep in mind that your presentation is intended for community residents or local governments. Address the following questions in your presentation:

- What impact does reducing water use have on the environment?
- In what ways can water use in homes be reduced?
- What should be considered when making decisions about water use in homes?
- What costs and savings are associated with retrofitting?
- Do your local governments offer any incentives to encourage people to retrofit their houses or reduce water use? If so, what are they? If not, what suggestions do you have?

Unit Review

Chapter 8 Solving Systems of Linear Equations Graphically

1. Sketch and label a graph of two linear equations that could be used to represent each scenario.

a) The cost of two different cell phone plans over a number of months

b) The height of two hot-air balloons descending at the same rate from different initial heights

c) The constant speed of a racehorse running around a track and the constant speed of a racehorse after it has crossed the finish line

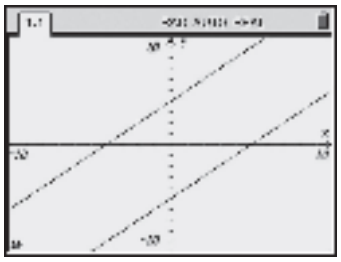
2. Match each system of equations to the correct graph.

A $y = -x + 4$
$y = x - 5$

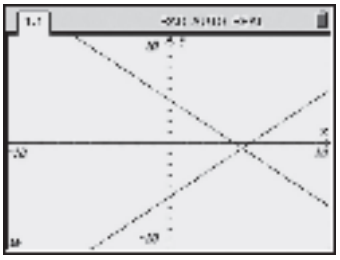
B $y = x + 4$
$y = x - 5$

C $y = -x + 4$
$y = -x - 5$

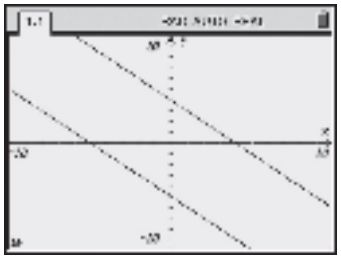
i)



ii)



iii)



3. Explain the meaning of the given point in relation to the system of linear equations.

a) $(1, -1)$, $y = -3x + 2$ and $y = x - 2$

b) $(0, 3)$, $y = \frac{3}{2}x + 2$ and $y = -x + 3$

4. Solve each of the linear systems graphically. Verify the solution.

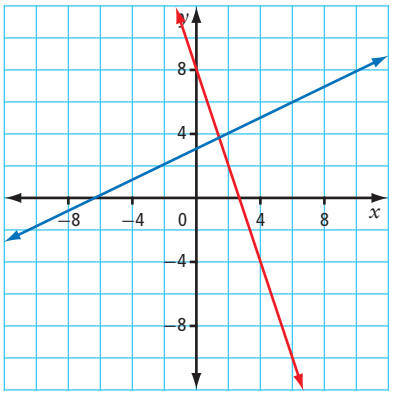
a) $y = -5x - 3$
$y = 4x - 3$

b) $y = -3x - 6$
$y = x + 2$

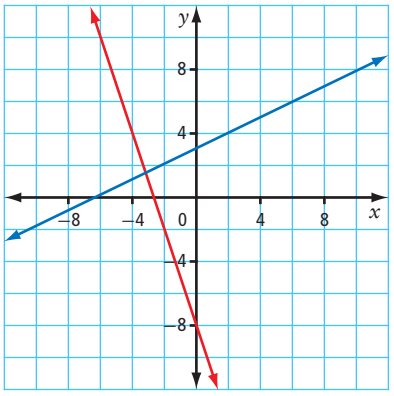
c) $y = 2x - 3$
$y = -x - 9$

5. Which of the following graphs shows the solution to the linear system of equations $2x + 4y - 12 = 0$ and $3x + y - 8 = 0$?

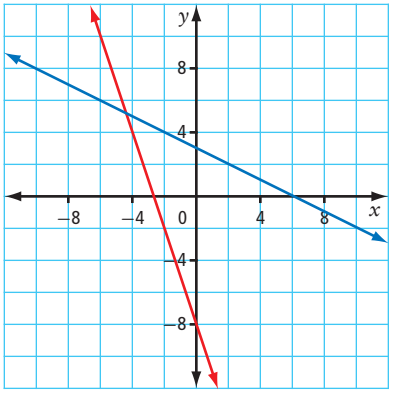
A



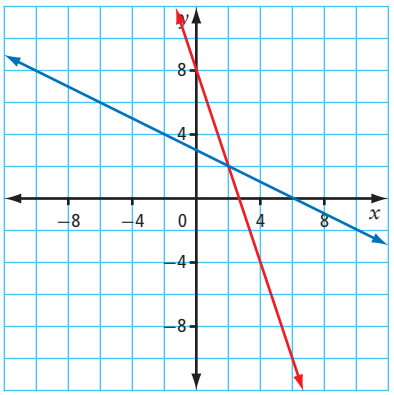
B



C



D



6. Use technology to solve each linear system graphically. Express solutions to the nearest tenth, if necessary.

a) $y = -5x + 2$
$y = 6x - 9$

b) $y = 7x - 2$
$y = x + 5$

c) $y = 4x - 5$
$y = -5x + 1$

7. Mei compared the cost of renting a car from two car rental companies. She graphed the number of days versus cost for each. She discovered that the point of intersection was (7, 580). Explain the meaning of this point.

8. Daniel had 20 coins in his pocket, consisting of dimes and quarters. The total value of the coins was $2.75.

a) Model the situation using a system of linear equations.

b) What would be the domain and range of each function?

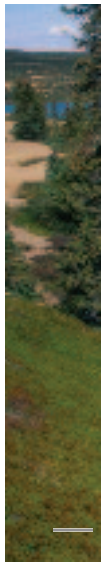
c) Solve graphically to determine the number of each type of coin Daniel has.

9. Determine whether each linear system has no solution, one solution, or an infinite number of solutions.

a) $y = \frac{5}{3}x - 4$
$y = \frac{3}{5}x - 4$

b) $3x + y - 11 = 0$
$3x + y + 11 = 0$

c) $2x - 6y + 5 = 0$
$4x - 12y + 10 = 0$



10. Colin's team is part of a lacrosse association in Manitoba. One point is awarded for an assist and two points for a goal. Colin has 23 points so far this season. Twice the number of goals is one more than the number of assists Colin has. Write a system of linear equations to model this situation.

11. Identify which method you would use to solve each linear system algebraically: the substitution method or the elimination method. Explain each choice.

a) $y = -x + 5$
$2x + 3y - 7 = 0$

b) $5y - 3x = 9$
$2y + 3x = 12$

c) $3x - 4y + 6 = 0$
$2x + 8y - 5 = 0$

d) $x + 8y = 20$
$3x + 6y = 24$

12. Solve each linear system. Verify your solutions.

a) $3x + y = 2$
$2x + 5y = 23$

b) $3x + 2y = 8$
$x - 12y = -10$

c) $3x + 5y = 1$
$7x + 9y = 5$

13. Solve each linear system. For each one, explain why you chose the method you did.

a) $x + 2y = 11$
$3x - 2y = 9$

b) $2x - 6y - 12 = 0$
$3x - 2y - 4 = 0$

c) $8x + 5y = -11$
$3x + 2y = -4$

14. The Thelon River stretches across 900 km of Northern Canada. Cheng and Tammy took a ten-day canoe trip along part of the river. They left camp going with the current of the river. After 20 min, they discovered they forgot something and had to return. It took 36 min to paddle the 3 km back to camp against the current.

a) Write a system of linear equations to represent the situation.

b) What was their paddling speed? What was the speed of the current?

4 Unit Test

Multiple Choice

For #1 to #4, choose the best answer.

1. The interpretive centre at Batoche, SK, depicts Métis life between 1860 and 1900. The site charges $7.80 for each adult and $3.90 for each youth. Imagine that on one day the total for adult and youth admissions was $214.50. There were five fewer youths than adults. Which linear system could be used to solve for the number of adults, a, and the number of youths, y?

A $7.80a + 3.90y = 214.50$
$a - y = 5$

B $7.80a + 3.90y = 214.50$
$y + a = 5$

C $7.80a + 3.90y = 214.50$
$a = y - 5$

D $7.80a + 3.90y = 214.50$
$y = 5 - a$

2. James graphs two lines. Under what condition(s) must there be a point of intersection?

i) Both lines have the same y-intercept and the same slope.

ii) Both lines have the same y-intercept but different slopes.

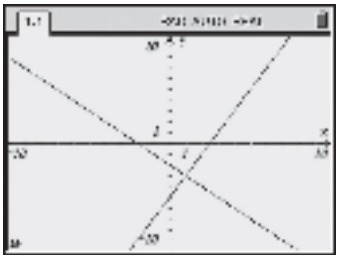
iii) Both lines have the same slopes and the same x-intercept.

iv) Both lines have different slopes but the same x-intercept.

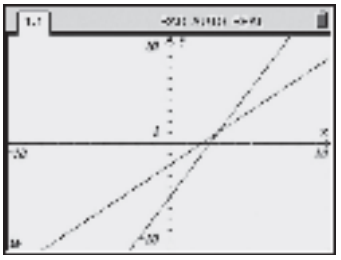
A II only **B** I and III **C** II and IV **D** IV only

3. Which of the following graphs could represent the following linear system: $x + y = -2$ and $2x - y = 5$?

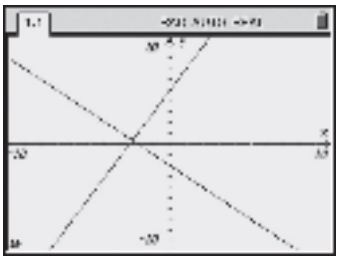
A



B



C



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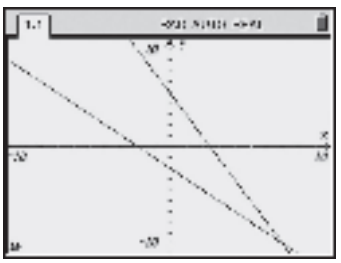

4. Carla correctly described the number of solutions to the system of equations $7x - 5y = 2$ and $7x - 5y = 4$ as

A one solution
B two solutions
C an infinite number of solutions
D no solution

Numerical Response

Complete the statements in #5 and #6.

5. You solve the system of equations $x + 3y = 0$ and $x + 6y - 30 = 0$ graphically. The y-coordinate of the solution is ■.

6. Mary Kuutsiq Mariq of Baker Lake, NU, created the wall hanging shown. It has a perimeter of 120 in. Its length is 6 in. shorter than twice its width. The length of the wall hanging is ■ in.

Hungry for Wildlife by Inuk artist Mary Kuutsiq Mariq

Written Response

7. Solve the following system of linear equations: $\frac{1}{2}x - \frac{2}{3}y = 6$ and $\frac{1}{4}x + \frac{1}{3}y = -1$. Verify the solution. Then, explain why you chose the method you did.

8. Dakota is going to make a bead necklace. The cost of 25 red beads and 15 green beads is \$2.75. The cost of 7 red beads and 13 green beads is \$1.65.

a) Write a linear system of equations to represent the situation.

b) Determine the cost of a red bead and the cost of a green bead.

c) How much would ten red beads and five green beads cost in total?

9. American bison and North American elk are surprisingly fast. The sum of their top speeds is 104 km/h. The difference is 8 km/h. If the bison is the faster animal, what is its top speed?