UNIT

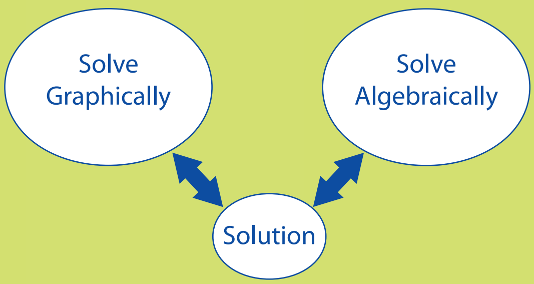
4 Systems of Equations

Some day, you and your parents may need to make decisions about buying a new or used car. Should you purchase or lease the car? Should you buy an electric, a hybrid, or a gasoline-powered vehicle? How much money should you borrow at a specific interest rate? Decisions often affect several important aspects of your life.

Many decisions involve two relations where each can be modelled using a linear equation. Analysing a system of two linear equations allows consumers and businesses to make well-informed decisions. In this unit, you will learn how to identify the solution to a system of linear equations shown on a graph. You will learn several algebraic methods for solving systems of linear equations. You will also learn strategies to help you determine when to use each method.

Your Systems of Equations Organizer

You can use this organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in the chapter will be highlighted.



Looking Ahead

In this unit, you will solve problems involving …

- the point of intersection of a system of linear equations
- the number of solutions to a linear system
- strategies for solving systems of linear equations graphically and algebraically

Unit 4 Project

Water Conservation

Water is one of the most precious resources in our ecosystem. Many of us take the water we use every day for granted. How do our actions affect the environment we live in? What about the plants and animals in our ecosystem? Human survival depends on our ability to live in harmony with the environment. What can we do to improve some habits and conserve water?

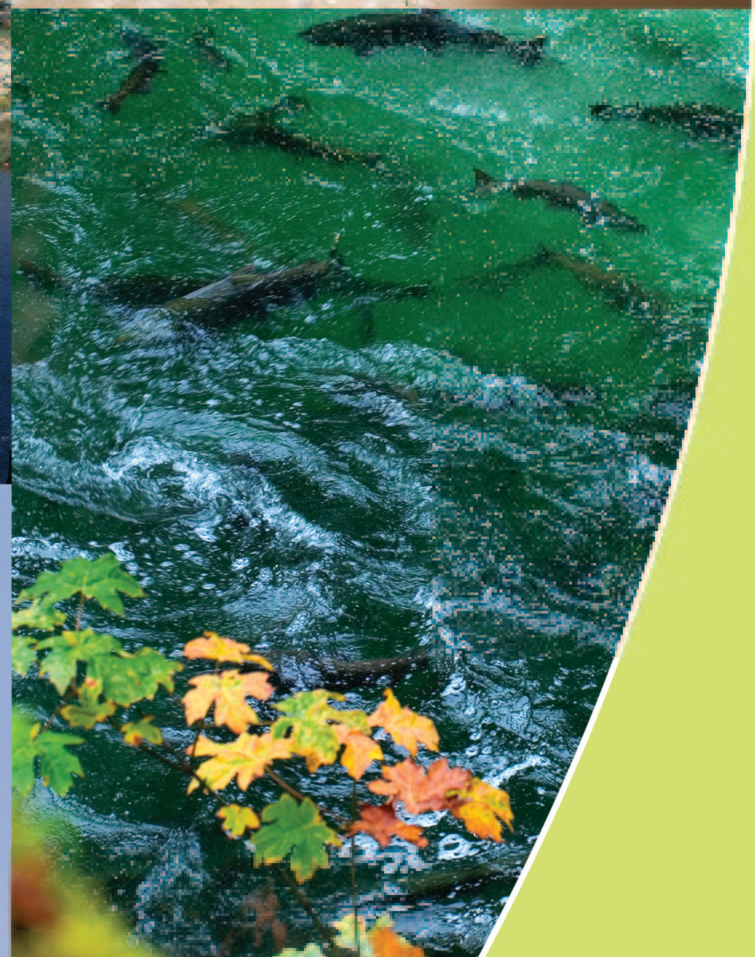
In the Unit 4 project, you will collect and analyse information related to water use, wildlife, and retrofitting. The term *retrofit* refers to changing or replacing fixtures in a house with ones that are better for the environment. You will then prepare a presentation that convinces people to make changes to help reduce water use.

Unit Project questions and activities are included in Chapters 8 and 9. As you move through Chapter 8, you will use graphs to compare information about wildlife and pricing options for retrofitting fixtures in a home. As you move through Chapter 9, you will use several strategies to solve linear systems involving water use and conservation.

While completing your project, you will ...

- compare data involving the effect of water contamination on populations of wildlife (Chapter 8)
- use systems of linear equations to analyse the costs of retrofitting (Chapter 8)
- compare the money saved using various water-saving methods in a home (Chapters 8 and 9)
- represent and analyse linear systems involving wildlife that live at a lake, where the lake is slowly being depleted (Chapter 9)






CHAPTER

8 Solving Systems of Linear Equations Graphically

The ability to quantify situations and compare options is very important. This skill could be used, for example, when choosing a cell phone plan or determining travel time for various modes of transportation. It could also be used to predict whether a business will make a profit. Many relationships found in common situations and career contexts are linear. Graphing several linear relations on the same grid may provide insight that will help you analyse situations, solve problems, and make informed decisions.

Big Ideas

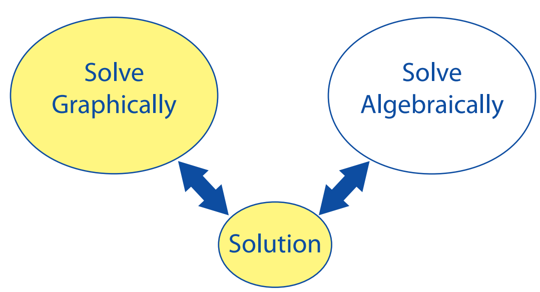
When you have completed this chapter, you will be able to ...

- generate systems of linear equations and create graphs to model situations
- solve two-variable systems of linear equations graphically
- verify solutions to two-variable systems of linear equations
- explain why systems of linear equations may have zero, one, or an infinite number of solutions

Key Terms

point of intersection
system of linear equations
solution (to a system of linear equations)
coincident lines

Your Systems of Equations Organizer



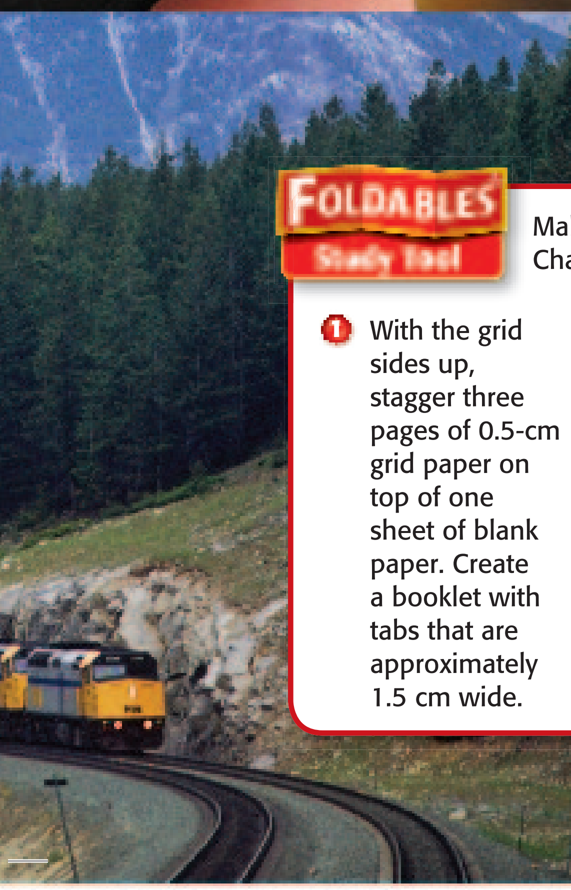
Biologist

Biologists study living things and their environments. For hundreds of years, biologists have used mathematics in many areas. Due to recent advances in technology, biologists have tremendous computing power. The capability to examine vast amounts of data enables biologists to study predator–prey relationships, analyse genetic sequences, and understand animal body structure and locomotion.

Biologist removing satellite collar.

Web Link

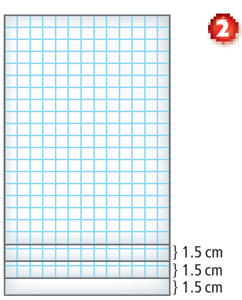
To learn more about biologists, go to www.mhrmath10.ca and follow the links.



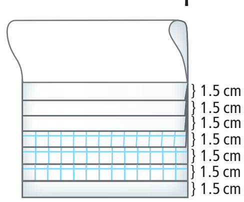
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 8.

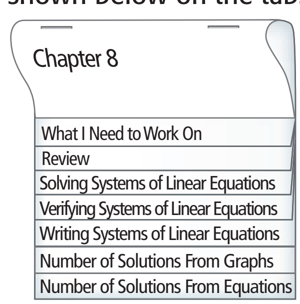
1. With the grid sides up, stagger three pages of 0.5-cm grid paper on top of one sheet of blank paper. Create a booklet with tabs that are approximately 1.5 cm wide.



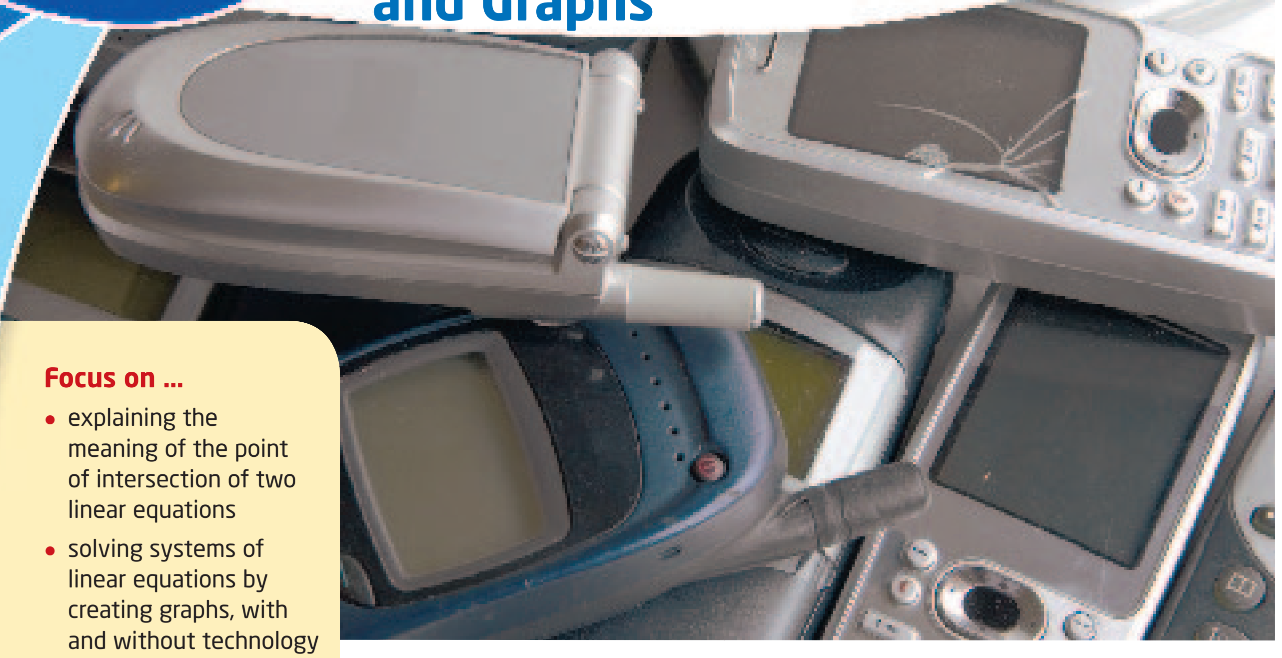
2. Hold the booklet firmly. Fold the top toward you and line up the tabs. Make a firm crease at the top.



3. Staple the top of the booklet. Write the headings shown below on the tabs.



8.1 Systems of Linear Equations and Graphs



Focus on ...

- explaining the meaning of the point of intersection of two linear equations
- solving systems of linear equations by creating graphs, with and without technology
- verifying solutions to systems of linear equations using substitution

The average cellular phone in North America is used for one and a half years and then replaced. Only 5% of discarded cell phones are recycled. That creates large amounts of waste.

Cell phone communication has increased dramatically in recent years. Many people need to decide whether to buy a cell phone, which type of plan is most beneficial to them, and what to do with a cell phone that they no longer need.

Materials

- 0.5-cm grid paper
- ruler
- coloured pencils

Investigate Ways to Represent Linear Systems

How can you compare and analyse cell phone plan options?

- Plan A costs 30¢ per minute.
- Plan B costs $15 one time plus 10¢ per minute.

1. Create tables of values to show the cost of each option for up to 100 min. Use intervals of 10 min.
2. On the same sheet of 0.5-cm grid paper, graph the data from both tables of values.

Remember to include a scale, labels on the axes, and a title on your graph.

3. **Reflect and Respond** From the graph, explain the cost of each plan as the number of minutes increases.

4. What is the significance of the **point of intersection** of the lines? Explain the connection between this point on the graph and the tables of values you created.

5. Which cell phone plan do you think is a better option? Justify your choice.

point of intersection

- a point at which two lines touch or cross

Link the Ideas

Relations can be represented numerically using a table of values. They can be represented graphically, and verified algebraically. A **system of linear equations** is often referred to as a linear system. It can be represented graphically in order to make comparisons or solve problems. The point of intersection of two lines on a graph represents the **solution** to the system of linear equations.

system of linear equations

- two or more linear equations involving common variables

solution (to a system of linear equations)

- a point of intersection of the lines on a graph
- an ordered pair that satisfies both equations
- a pair of values occurring in the tables of values of both equations

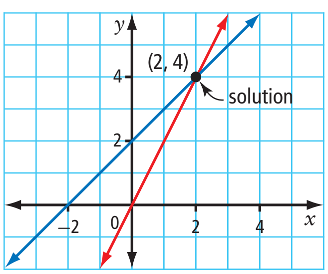
Numerically

x	y
0	0
1	2
2	4
3	6
4	8

x	y
0	2
1	3
2	4
3	5
4	6

There are an infinite number of pairs of values that could be written in each table. The solution, (2, 4), is the only pair that can be written in *both* tables.

Graphically



There are an infinite number of points on each line. The solution point, (2, 4), is the only point that lies on *both* lines.

Algebraic Verification

$y = 2x$
$4 = 2(2)$
$4 = 4$

$y = x + 2$
$4 = 2 + 2$
$4 = 4$

There are an infinite number of ordered pairs that satisfy each equation. Only one ordered pair, (2, 4), satisfies *both* equations.

Example 1 Represent Systems of Linear Equations

Nadia has saved $16, and her sister Lucia has saved $34. They have just started part-time jobs together. Each day that they work, Nadia adds $5 to her savings, while Lucia adds $2. The girls want to know if they will ever have the same amount of money. If so, what will the amount be and on what day?

Solution

The girls use a linear system to model their savings. They represent it numerically and graphically.

Method 1: Use a Table of Values

The amount of money each girl saves is a function of the number of days worked. They each create a table of values to show how their savings will grow:

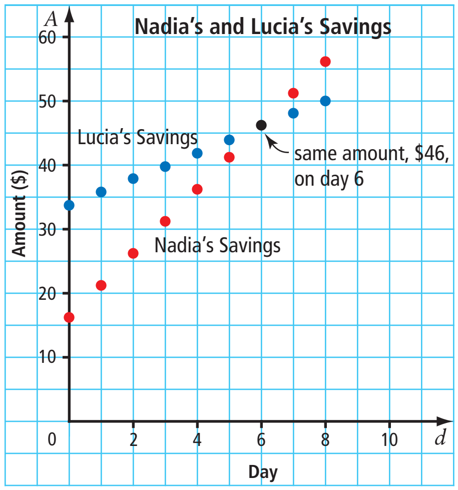
Nadia's Savings	
Day	Amount ($)
0	16
1	21
2	26
3	31
4	36
5	41
6	46
7	51
8	56

Lucia's Savings	
Day	Amount ($)
0	34
1	36
2	38
3	40
4	42
5	44
6	46
7	48
8	50

The tables of values show that both girls will have $46 on day 6. The pair of values, 6 and 46, is the only pair found in *both* tables of values. It represents the only day when the girls will have the same amount of money.

Method 2: Use a Graph

The girls draw graphs on the same grid. This enables them to compare the linear relationships for their savings.

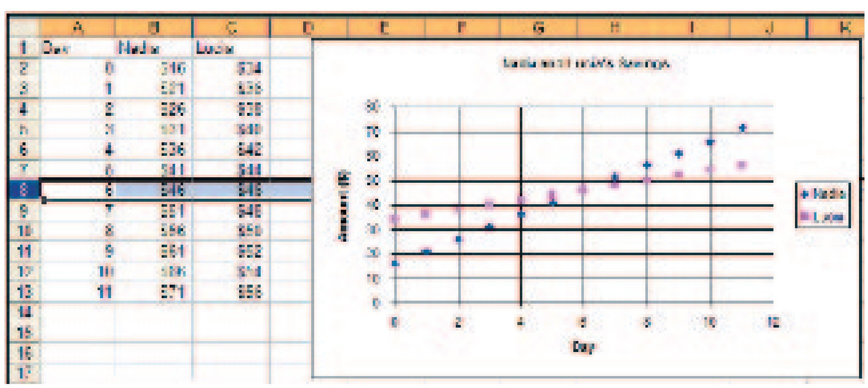


Why are the points not joined on this graph?

The intersection point of the two relationships is (6, 46). This means the girls will have the same amount of money, $46, on day 6.

Method 3: Use a Spreadsheet

In a spreadsheet, the girls enter the headings and amounts shown. Then, they use the spreadsheet's graphing features.



The table of values shows that the girls will have the same amount of money on day 6. The point of intersection on the graph is (6, 46). They will both have $46 on day 6.

Your Turn

Davidee earns $40 plus $10 per hour. Carmen earns $50 plus $8 per hour.

a) Represent the linear system relating the earnings numerically and graphically.

b) Identify the solution to the linear system and explain what it represents.

Example 2 Solve a Linear System Graphically

a) Consider the system of linear equations $2x + y = 2$ and $x - y = 7$. Identify the point of intersection of the lines by graphing.

b) Verify the solution.

Solution

How will the form of each equation help you choose your method of graphing?

a) Graph the equations together.

Method 1: Use Slope-Intercept Form

Rearrange each equation into slope-intercept form by isolating y. Identify the y-intercept and slope to draw the graph.

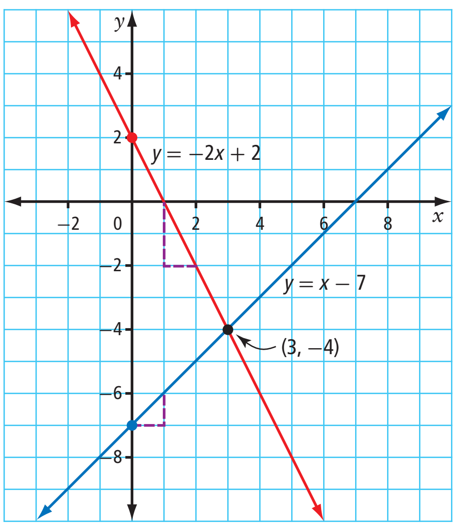
$$\begin{aligned} 2x + y &= 2 \\ 2x + y - 2x &= 2 - 2x \\ y &= 2 - 2x \\ y &= -2x + 2 \end{aligned}$$

$$\begin{aligned} x - y &= 7 \\ x - y + y &= 7 + y \\ x &= 7 + y \\ x - 7 &= 7 + y - 7 \\ x - 7 &= y \\ y &= x - 7 \end{aligned}$$

The y-intercept is 2.
The slope is -2.

The y-intercept is -7.
The slope is 1.

The solution is shown on the graph. It is the point of intersection, $(3, -4)$.



Method 2: Use *x*-Intercepts and *y*-Intercepts

Determine the x-intercept and y-intercept of the line $2x + y = 2$.

If given the equation of a line, how can you determine its intercepts?

x-intercept: $y = 0$	y-intercept: $x = 0$
$2x + y = 2$	$2x + y = 2$
$2x + 0 = 2$	$2(0) + y = 2$
$2x = 2$	$y = 2$
$x = 1$	

For the equation $2x + y = 2$, the x-intercept is 1 and the y-intercept is 2.

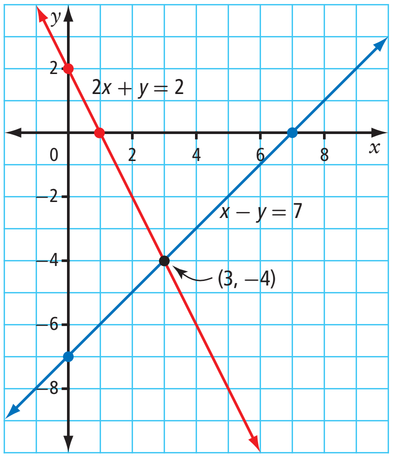
Determine the intercepts of the line $x - y = 7$.

x-intercept: $y = 0$	y-intercept: $x = 0$
$x - y = 7$	$x - y = 7$
$x - 0 = 7$	$0 - y = 7$
$x = 7$	$y = -7$

For the equation $x - y = 7$, the x-intercept is 7 and the y-intercept is -7.

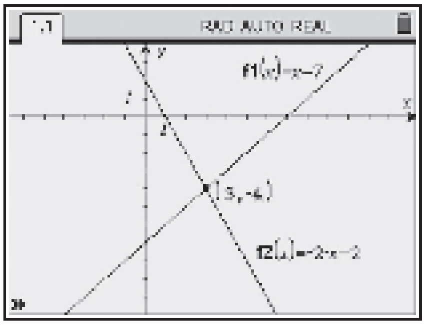
For each equation, plot the x-intercept and y-intercept; then, join the points.

The lines intersect at the point $(3, -4)$. So, the solution to the linear system is $(3, -4)$.



Method 3: Use Technology

Graph each equation using technology. Adjust the dimensions of the graph until you see both intercepts of each line, as well as the point of intersection. Then, use the intersection feature to find the solution.



The intersection point $(3, -4)$ is the solution to the linear system.

b) To verify that (3, −4) is the solution to the linear system $2x + y = 2$ and $x - y = 7$, use a different representation than the method of solving.

Method 1: Substitute Using Paper and Pencil

Verify the solution (3, −4) by substituting the values of x and y into each equation.

In $2x + y = 2$:

Left Side	Right Side
$2x + y$	2
$= 2(3) + (-4)$	
$= 6 - 4$	
$= 2$	

Left Side = Right Side

In $x - y = 7$:

Left Side	Right Side
$x - y$	7
$= 3 - (-4)$	
$= 3 + 4$	
$= 7$	

Left Side = Right Side

Since the ordered pair (3, −4) satisfies both equations, it is the solution to the linear system.

Method 2: Create a Table of Values Using Technology

Enter the equations of the lines and generate a table of values.

x	f1(x):= 2·x+2	f2(x):= x 7
0	2	7
1	0	6
2	-2	-5
3	-4	-4
4	6	3
5	8	2

When $x = 3$, both equations have the same value for y of −4. So, the point (3, −4) is the solution to the linear system.

Your Turn

Verify by graphing and one other way that (3, −2) is the solution to the system of linear equations $x - 3y = 9$ and $2x + y = 4$.

Example 3 Connect a Solution and a Graph

Guy solved the linear system $x - 2y = 12$ and $3x - 2y = 4$. His solution is (2, −5). Verify whether Guy's solution is correct. Explain how Guy's results can be illustrated on a graph.

Solution

Substitute $x = 2$ and $y = -5$ into each equation. Evaluate each side to determine whether the values satisfy both equations.

In $x - 2y = 12$:

Left Side	Right Side
$x - 2y$	12
$= 2 - 2(-5)$	
$= 2 + 10$	
$= 12$	

Left Side = Right Side

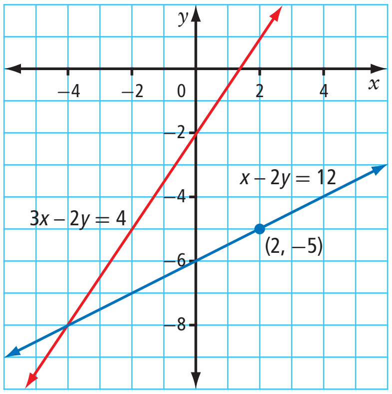
In $3x - 2y = 4$:

Left Side	Right Side
$3x - 2y$	4
$= 3(2) - 2(-5)$	
$= 6 + 10$	
$= 16$	

Left Side ≠ Right Side

The given values satisfy the first equation but not the second equation. Since both equations do not result in true statements, the point (2, −5) is not the solution to this linear system.

The point (2, −5) is not a solution to the given linear system. So, a graph of this system will not have a point of intersection at (2, −5). The point (2, −5) is on one of the lines, not *both* lines.



Your Turn

For each system of linear equations, verify whether the given point is a solution. Explain what the results would show on a graph.

a) $3x - y = 2$
$x + 4y = 32$
(2, 5)

b) $2x + 3y = -12$
$4x - 3y = -6$
(−3, −2)

Example 4 Solve a Problem Involving a Linear System

The Skyride is a red aerial tram that carries passengers up Grouse Mountain in Vancouver, BC. The Skyride travels from an altitude of about 300 m to an altitude of 1100 m. The tram can make the trip up or down in 5 min and can carry 100 passengers.

There is also a blue tram that can carry 45 passengers. This tram takes approximately 8 min to travel up or down the mountain. Each tram travels at a constant speed.

a) Create a graph to represent the altitudes of the trams if the red tram starts at the top and the blue tram starts at the base.
b) Explain the meaning of the point of intersection.

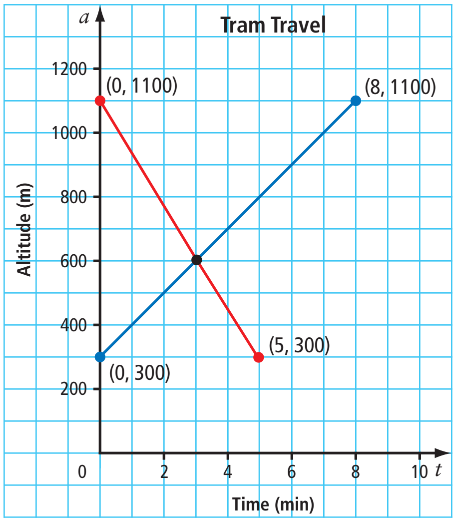
Solution

a) Organize the information before graphing.

Tram	Start		End		Representation on a Graph
	Time	Altitude	Time	Altitude	
Red	0 min	1100 m	5 min	300 m	Line segment joining the points (0, 1100) and (5, 300)
Blue	0 min	300 m	8 min	1100 m	Line segment joining the points (0, 300) and (8, 1100)

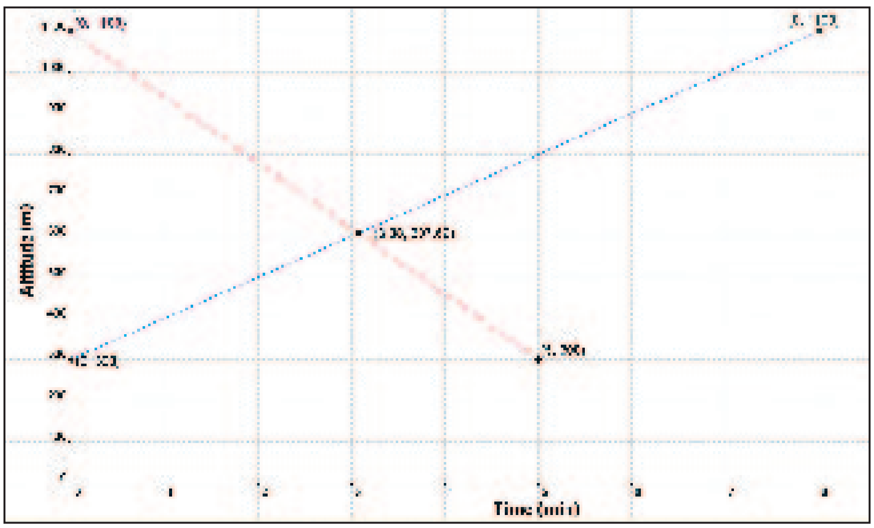
Method 1: Use Paper and Pencil

Label time from 0 min to 10 min on the horizontal axis. Label altitude on the vertical axis, up to 1200 m. Graph a line segment for each tram using the start and end points.



Method 2: Use Technology

Plot the start and end points for the travel of each tram. Use the "segment between two points" or equivalent feature to connect the points to show the continuous travel for each tram.



Use the intersection feature to determine the solution to the linear system.

b) At the point of intersection, the two trams will have the same altitude at the same time. The lines appear to intersect at approximately (3, 600). Therefore, after about 3 min, the two trams will pass each other at about 600 m in altitude.

The red tram travels faster, so it will travel farther than the blue tram in the same time interval. The red tram starts at the top, so the expected solution will be closer to the base of the mountain.

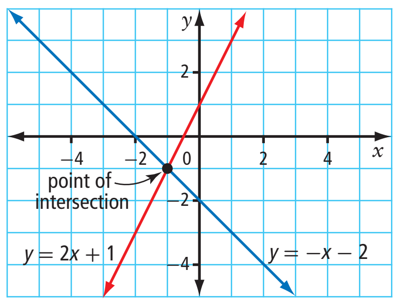
Your Turn

Eric works on the 23rd floor of a building. It takes Eric 90 s to walk down the stairs to the 14th floor. Nathan works on the 14th floor and needs to go up to the 30th floor. He knows it will take 40 s by elevator if the elevator makes no other stops.

Suppose both men leave their offices at the same time. Create a graph to model their travel. What does the point of intersection represent?

Key Ideas

- Systems of linear equations can be modelled numerically, graphically, or algebraically.
- The solution to a linear system is a pair of values that occurs in each table of values, an intersection point of the lines on a graph, or an ordered pair that satisfies each equation.
- One way to solve a system of linear equations is to graph the lines and identify the point of intersection on the graph.

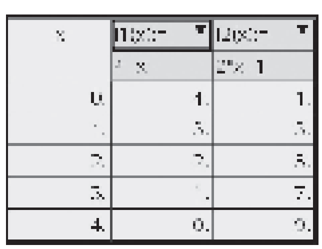


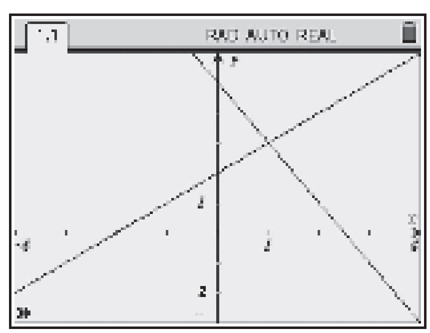
- A solution to a system of linear equations can be verified using several methods:
  - Substitute the value for each variable and evaluate the equations.
  - Create a graph and identify the point of intersection.
  - Create tables of values and identify the pair of values that occurs in each table.

Check Your Understanding

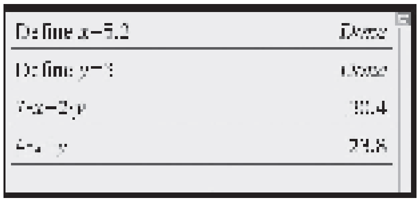
Practise

1. One linear system is shown in the table of values, and another in the graph. Do the two systems have the same solution? Justify your answer.





2. John uses technology to check whether (5.2, 3) is the solution to the linear system $7x - 2y = 30.4$ and $4x + y = 25.1$. The results are shown on the screen. Is John's solution correct? Explain.



3. a) Represent the system of linear equations $y = 2x + 6$ and $y = 3x$ using a table of values and a graph.
   b) Explain how you can identify the solution to the linear system from your table of values and your graph.
   c) Verify by substitution that the solution you found is correct.

4. Consider the system of linear equations $y = -x + 4$ and $y = \frac{1}{2}x - 5$.
   a) Show how a table of values can be used to solve the linear system.
   b) Show how the linear system can be solved using a graphical representation.
   c) Explain how the solution is related to the original equations.

5. Is each given point a solution to the system of linear equations? Explain.
   a) $y = 3x - 5$
   $y = 11 - x$
   (4, 7)
   b) $4x + 3y = 5$
   $x + 4y = 13$
   (−1, 3)
   c) $2x - 3y = 18$
   $x + 2y = -26$
   (−6, −10)
   d) $12x - 3y = 7$
   $y = 4.5x - 3$
   (1.2, 2.4)

6. On grid paper, graph each system of linear equations. What is the solution for each linear system?
   a) $y = -2x + 5$
   $y = x - 4$
   b) $4x - y = -8$
   $2x + 3y = -18$

7. Solve each system of linear equations graphically.
   a) $y = 2x - 10$
   $y = -3x + 8$
   b) $y = \frac{1}{2}x - 5$
   $y = -\frac{4}{3}x + 1$
   c) $2x + y = 24$
   $2x + 5y = 50$
   d) $x - 2y = -18$
   $3x + 4y = -12$

8. Solve each linear system graphically. Then, verify your solution.

a) $y = 0.5x + 4$
$y = 0.8x + 1$

b) $2x - 5y = 40$
$-6x + 5y = -60$

9. Is each given point a solution to the system of linear equations? Explain what the results would show on a graph of the linear system.

a) $3x - y = 2$
$x + 4y = 22$
$(2, 5)$

b) $2x + 3y = -12$
$4x - 3y = -6$
$(-3, -2)$

10. Brad and Sharon are collecting money from family and friends for a local charity. Brad has $35 and plans to add $5 each day. Sharon does not have any money yet. She plans to collect $12 each day.

a) Represent the donations Brad and Sharon are collecting using a table of values and a graph.

b) What is the solution of the linear system? What does it represent?

Apply

11. Maya makes bead necklaces for a craft fair. It costs her $2.50 to make each necklace. She needs to pay $49 for a table at the craft fair. She plans to sell each necklace for $6. Maya models her costs and revenue with the following equations:
Costs: $y = 2.5x + 49$
Revenue: $y = 6x$
In the equations, x represents the number of necklaces and y represents the amount, in dollars.

a) Create a graph of the linear system.

b) How many necklaces must Maya sell in order to break even?

c) Explain how to use the graph to determine the profit Maya will make if she sells 20 necklaces.

12. Consider the system of linear equations $y = \frac{1}{2}x - 5$ and $y = -\frac{1}{3}x + 11$.

a) Sketch a graph of the linear system on grid paper. Estimate the solution.

b) Use technology to solve the linear system by graphing.

c) Discuss the advantages and disadvantages of each method of solving systems of linear equations.

13. Can you solve the linear system involving $f_1(x) = 41 - 2x$ and $f_2(x) = 14 + 4x$ using the table of values shown? Explain.

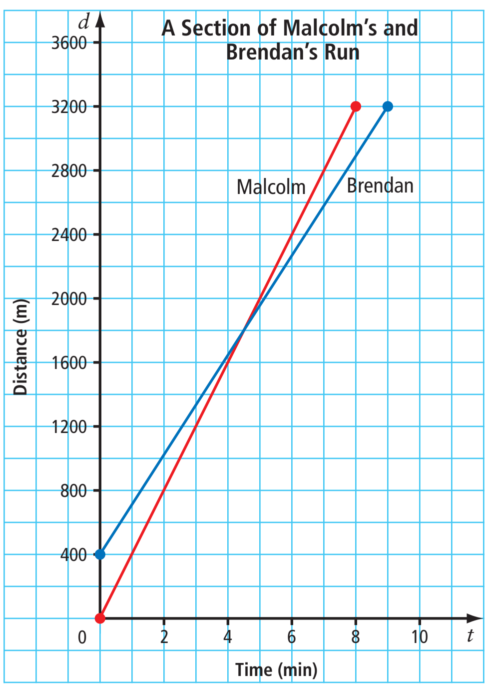
x	$f_1(x) =$ $41 - 2x$	$f_2(x) =$ $14 + 4x$
-1.	43.	10.
0.	41.	14.
1.	39.	18.
2.	37.	22.
3.	35.	26.
4.	33.	30.
5.	31.	34.
6.	29.	38.

14. Kayla and Sam sketch a graph to solve the system of linear equations $3x - 2y = -8$ and $4x - 3y = -9$. Sam completes his graph first and tells Kayla, "You need to graph extra carefully to solve this system." Solve the linear system by graphing and verify your solution. Why might Sam be justified in making his comment to Kayla?

15. The Calgary Marathon has been an annual event since 1971. It attracts participants from Canada and the United States. Malcolm and Brendan are training for the marathon. The graph shows a section of one of their long-distance runs.

a) Describe the part of their run represented in the graph.

b) Why can a system of linear equations represent this part of their run?



16. **Unit Project** Two groups of ducks are leaving a field and heading for a water source 50 km away. The green-winged teals leave 25 min before the canvasback ducks. Green-winged teals fly at a speed of 48 km/h.

a) How far do the green-winged teals fly during the 25 min?

b) Canvasback ducks fly at a speed of 115 km/h. The distance, d, in kilometres, travelled by each species is related to time, t, in hours, by the following equations:

Green-winged teals: $d = 48t + 20$
Canvasback ducks: $d = 115t$

What does time, $t = 0$ represent? Justify your answer. Then, sketch a graph of the system of linear equations.

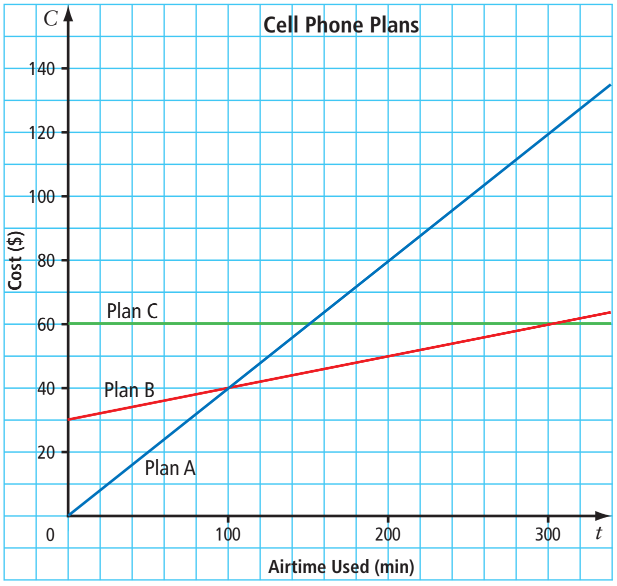
c) Use the graph to describe the trip to the water source for the two groups of ducks. Explain your reasoning.

17. One hot-air balloon is 100 m above ground. It rises at a constant rate and reaches a height of 1000 m in 16 min. Another hot-air balloon is 1600 m above ground. It descends at a constant rate to ground level in 20 min. Create a graph to represent the travel of the balloons. What does the point of intersection represent?

18. Sarah and George are both using the road from Qamani'tauq to the bridge over the Prince River in Nunavut. The road is 35 km long. Sarah drives her ATV at a constant speed from Qamani'tauq to the bridge. She takes 30 min. George drives his snowmobile at a constant speed from the bridge to Qamani'tauq. He takes 23 min. Create a graph to represent each person's distance from the bridge. What does the point of intersection represent?

Extend

19. The graph shows the costs for three different cell phone plans. Describe a situation in which a user would benefit most from each plan as compared with the other plans.



20. A truck travels along a highway. The truck's speed can be modelled with the equation $s = 1.5t + 5$. In the equation, s represents speed, in metres per second, and t represents time, in seconds. As the truck reaches a parked car, the car begins to move ahead. The car's speed can be modelled with the equation $s = 2.3t$. How many seconds does the car travel until its speed is the same as the truck's speed?

Create Connections

21. Describe a situation in your life that could be represented with a system of linear equations. Sketch a graph of the linear system. Explain what the point of intersection would represent.

22. How does *solving* a system of linear equations differ from *verifying* a solution to a system of linear equations? Provide an example.

23. Consider the system of linear equations $Ax + By + C = 0$ and $Dx + Ey + F = 0$. For each set of criteria, describe the lines. Justify your answers.

a) $A = D, B = E, C \neq F$

b) $A = D^{-1}, B = -E^{-1}, C = F$

c) $A \neq D, B = E, C = F$

d) $A = D, B = E, C = F$

8.2 Modelling and Solving Linear Systems

Focus on ...

- translating word problems into systems of linear equations
- interpreting information from the graph of a linear system
- solving problems involving systems of linear equations

A big part of travelling involves choosing the mode of transportation. Different modes of transportation usually take different lengths of time, have different costs, and may have different effects on the environment. *The Canadian* is a passenger train that travels across Canada, between Vancouver and Toronto. How might taking *The Canadian* compare with other modes of travel? In what ways could you represent your comparison?

Materials

- map of Canada
- ruler
- grid paper or computer with graphing software

Investigate Creating a System of Linear Equations

1. Work with a partner. Select two cities in Canada that are in different provinces. Make sure each city has passenger rail service. Research the distance between the cities.

2. Suppose both of you need to travel from city A to city B. One person will drive and the other person will take a train. Assign the modes of travel. Then, determine the travel time for your particular mode. Assume the average speed of the car is 90 km/h and the average speed of the train is 70 km/h.

3. Assign a variable to represent each measurement.

a) the distance from city A

b) the length of time that the car has been travelling

4. a) Write an equation to model the travel of the car.

b) Suppose the train leaves 1 h before the car. Write an equation that represents the travel of the train.

5. Graph the system of linear equations you developed. What is the solution to your linear system? What does the solution represent? Discuss your answers with your partner.

6. Suppose the car leaves at 7:30 a.m. and the train leaves at 9:00 a.m.
   a) Write a system of linear equations representing the travel from city A to city B. Compare your linear system with your partner's system.
   b) Solve your linear system by graphing. What does the solution represent?

7. **Reflect and Respond**
   a) Which system of equations did you find easier to write? Explain why. Share your strategies with a classmate.
   b) How are the two equations in each linear system similar? How do they differ? Explain.
   c) How are the information given in a problem, the equations, and the graph connected? Provide examples to support your explanation.

8. Why can this situation be represented with a system of linear equations? Explain.

Link the Ideas

The ability to translate words or phrases into the language of mathematics is an important skill for solving problems in context. There are a limited number of mathematical operations, but there are many phrases that can be used to describe the operations.

For example, the following situations can all be represented by the expression $7x + 3$.
- 7 km/h for a length of time and then three more kilometres
- $7 per person plus $3
- 7 times as many years increased by 3

When it comes to subtraction, the translation requires greater consideration.

$x - 3 \neq 3 - x$ when $x \neq 3$

For example, 3 less than a number means $x - 3$ whereas 3 decreased by a number means $3 - x$.

Situations involving quantities that change at constant rates can be represented algebraically with a system of linear equations.

Example 1 Model a Linear System Algebraically and Graphically

People can rent ski and snowboard equipment from two places at Winterland Resort.
Option A charges a one-time \$30 fee and then \$8 per hour.
Option B charges \$14 per hour.

a) Create a system of linear equations to model the rental charges.
b) Solve the linear system graphically. What does the solution represent?

Solution

a) Both rental options involve a constant rate per hour, so they represent linear relations.
Identify the unknown values and assign variables.
Let C represent the cost, in dollars.
Let t represent the length of time, in hours, of the rental.

Write an equation to model the cost for each rental option.

Option A: The cost is \$30 plus \$8 per hour.
$C = 30 + 8t$

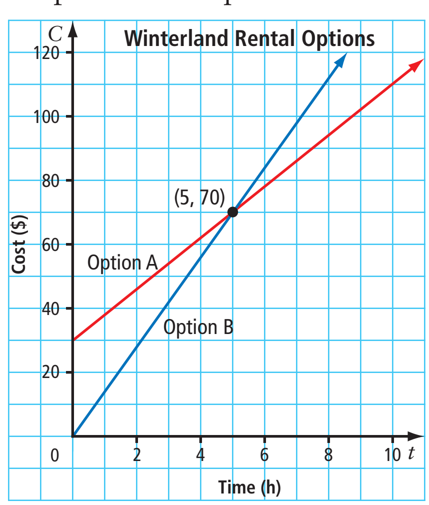
How is this equation related to the slope-intercept form?

Option B: The initial value is \$0 and the rate per hour is \$14.
The cost is \$14 per hour.
$C = 14t$
The equations $C = 30 + 8t$ and $C = 14t$ form a linear system.

b) To solve the linear system $C = 30 + 8t$ and $C = 14t$, graph the equations together and identify the point of intersection.

Method 1: Use Paper and Pencil

Graph the two equations.



What is the length of rental when both options have the same charge? What is that amount? How would you decide which option is better for you?

From the graph, the point of intersection is (5, 70). This is the solution to the linear system. It represents the length of rental when both options have the same charge.

The solution (5, 70) can be verified by substitution.

Option A: $C = 30 + 8t$

Left Side	Right Side
C	$30 + 8t$
$= 70$	$= 30 + 8(5)$
	$= 30 + 40$
	$= 70$

Left Side = Right Side

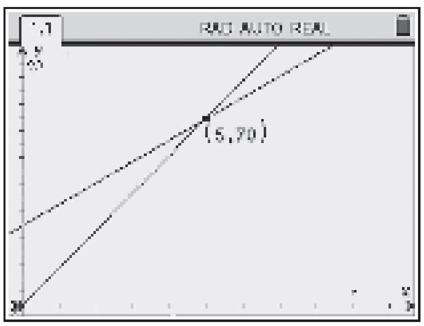
Option B: $C = 14t$

Left Side	Right Side
C	$14t$
$= 70$	$= 14(5)$
	$= 70$

Left Side = Right Side

Method 2: Use Technology

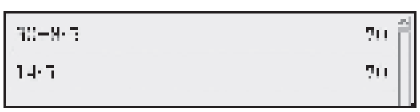
Graph the equations $C = 30 + 8t$ and $C = 14t$. You will likely need to use the equations $y = 30 + 8x$ and $y = 14x$.



How will you determine what range of values to plot?

Use the intersect feature to find the point of intersection, (5, 70). For a 5-h rental, both options cost \$70.

The solution can be verified by substituting using technology.



Your Turn

During a performance by a theatre company, the main act was on stage for 3 min less than twice the time of the opening act. Together, the two acts performed for 132 min.

a) Write a system of linear equations to represent the length of time each act performed.

b) What is the solution to this linear system? What does the solution represent?

Did You Know?

La Troupe du Jour is the only professional francophone theatre company in Saskatchewan. It was founded in 1985 and develops French-language theatre for the community.

Example 2 Interpret Information From the Graph of a Linear System

Two hopper-bottom grain bins are being emptied starting at the same time.

- The larger bin holds 45 m^3 of grain. It is emptied at a rate of 1 m^3 per minute.
- The smaller bin stores 30 m^3 of grain. This bin is emptied at a rate of 0.5 m^3 per minute.

a) Model the volume of grain remaining as a function of time using a system of linear equations.

b) Represent the linear system graphically. Describe how the information shown in the graph relates to the grain bins.

Solution

a) Define the variables.
Let V represent the volume of grain remaining in each bin, in cubic metres.
Let t represent time, in minutes.

Organize the information using a table.

Bin	Starting Volume (m^3)	Volume of Grain Removed (m^3)	Volume of Grain Remaining in Bin, V (m^3)
Larger	45	t	$45 - t$
Smaller	30	$0.5t$	$30 - 0.5t$

The larger bin starts with 45 m^3. It is being emptied at a rate of 1 m^3/min.

$V = 45 - t$

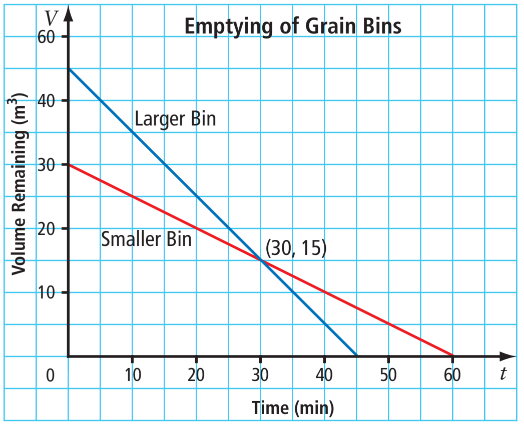
Why is the rate of change a negative value?

The smaller bin starts with 30 m^3. It is being emptied at a rate of 0.5 m^3/min.

$V = 30 - 0.5t$

A system of linear equations that models this situation is $V = 45 - t$ and $V = 30 - 0.5t$.

b) On the same grid, graph the system of linear equations, $V = 45 - t$ (larger bin) and $V = 30 - 0.5t$ (smaller bin).



The graph shows that the amount of grain remaining in each bin decreases over time. Both lines stop at the horizontal axis because the volume of grain left inside is 0 m^3.

The two lines intersect at (30, 15). This is the only point when the two bins contain the same amount of grain at the same time. At exactly 30 min, each bin has 15 m^3 of grain remaining.

Before 30 min, the line representing the larger bin is *above* the line for the smaller bin. This means that before 30 min, the larger bin has more grain inside. After 30 min, the line for the larger bin is *below* the line for the smaller bin. So, after 30 min, the smaller bin has more grain inside.

Your Turn

Two pools start draining at the same time. The larger pool contains 54 675 L of water and drains at a rate of 25 L/min. The smaller pool contains 35 400 L of water and drains at a rate of 10 L/min.

a) Model the draining of the pools algebraically using a system of linear equations.

b) Represent the linear system graphically. Describe how the information shown in the graph relates to the pools.

Example 3 Model and Solve a Problem Involving a Linear System

A movie theatre charges \$11 for an adult ticket and \$8 for children's or seniors' tickets. Suppose 240 people attended the early movie and ticket sales totalled \$2370.

a) The box office manager wants to know how many adults attended the early movie. What system of linear equations could help the manager determine the answer?

b) How many adults attended the early movie?

Solution

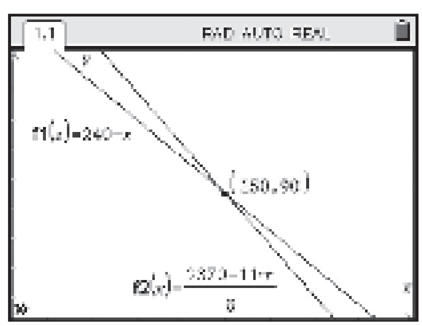
a) Define the variables.
Let a represent the number of adult tickets sold.
Let c represent the number of children's or seniors' tickets sold.

Write an equation to model the number of people at the early movie.
$a + c = 240$

Write an equation to model the ticket sales.
$11a + 8c = 2370$

The manager could use the linear system $a + c = 240$ and $11a + 8c = 2370$ to help determine the number of adults at the movie.

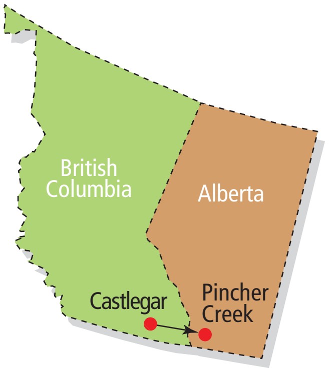
b) Graph both equations and identify the point of intersection.



The coordinates of the point of intersection are (150, 90). This is the only point that satisfies both equations. So, (150, 90) is the solution to the system of linear equations. There were 150 adults and 90 children and seniors at the early movie.

Your Turn

Jamie is travelling with her family from Castlegar, BC, to Pincher Creek, AB. Her dad and cousin do all of the driving. The 440-km trip takes 5.25 h, excluding stops. Jamie's father drives at an average speed of 90 km/h. Her cousin drives at 80 km/h. What system of linear equations could help Jamie determine the length of time each person drove? How many hours does each person drive?



Key Ideas

- When modelling word problems, assign variables that are meaningful to the context of the problem.
- To assist in visualizing or organizing a word problem, you can use a diagram and/or a table of values.
- If a situation involves quantities that change at constant rates, you can represent it using a system of linear equations.

  Two tanks are being filled at constant rates:
  One tank contains 100 L and is filling at 20 L/min.
  The other tank is empty and filling at 25 L/min.
  The situation can be represented by the system
  $V = 100 + 20t$ and $V = 25t$.
  In the equations, V represents volume, in litres, and
  t represents time, in minutes.

- If you know the initial values and rates, you can write the equations directly in slope-intercept form because the initial value is the y-intercept and the rate is the slope. Otherwise, you can determine the rate of change using start and end values.

  An electronics store charges a fee of \$36 plus \$6 per hour to fix a repair.
  The initial value is \$36, so $b = 36$.
  The rate is \$6 per hour, so $m = 6$.
  The equation is $C = 6t + 36$, where C represents cost, in dollars, and t represents time, in hours.

Check Your Understanding

Practise

1. Model each situation using a system of linear equations.

a) One music download option costs $0.99 per song. Another option costs $11 plus $0.79 per song.

b) A helicopter is 800 m above ground and descending at 55 m/min. An airplane is taking off and rising 80 m/min.

c) A recycling plant sorts material at a rate of 20 t per hour. It has sorted 100 t of material so far today. A new plant just opened. It sorts material at a rate of 40 t per hour.

Did You Know?

In 2009, Calgary began the Blue Cart recycling program. In this program, waste is sorted at a local plant owned by Metro Waste Paper Recovery. The plant is the most automated waste recovery facility in North America. It uses high-tech equipment and sorts 40 t of material every hour. In one year, the plant sorts and processes about 120 000 t of waste.

2. Write a system of linear equations to represent each situation.

a) Jamal is three times as old as Maria. In seven years, he will be twice as old as she will be.

b) One day, the temperature in one city drops at a constant rate from 2 °C to −6 °C in 4 h. Meanwhile in another city, the temperature rises at a constant rate from −8 °C to 4 °C in 3 h.

3. Molly has a total of 32 points in her hockey league. One point is earned for an assist or a goal. If she has three times as many assists as goals, write a system of linear equations to represent how many goals and assists she has.

4. A collection of 50 coins contains only dimes and quarters. The value of the collection is $6.80.

a) Use the table to write a system of linear equations relating the number of dimes to the number of quarters.

Type of Coin	Value of One Coin ($)	Number of Coins	Value of Coins ($)
Dime	0.10	d	$0.10d$
Quarter	0.25	q	$0.25q$

b) Rewrite the table, expressing the value of each type of coin in cents. Then, write a system of equations to model this relationship.

Apply

5. Two full water tanks drain at the same time. One tank holds 800 L of water. It drains at a rate of 30 L/min. The other tank holds 300 L of water. It drains at a rate of 12 L/min.

a) Express the draining of the water tanks algebraically using a system of linear equations.

b) Graph the linear system.

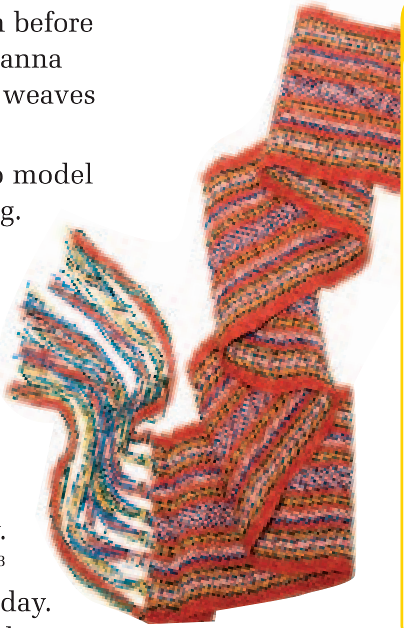
c) Explain how the information shown in the graph relates to the water tanks. What does the point of intersection represent?

6. Kianna weaves 45 cm of her Métis sash before Naomi starts weaving her own sash. Kianna weaves about 15 cm each hour. Naomi weaves about 25 cm every hour.

a) Write a system of linear equations to model the progress of the two girls' weaving.

b) Create a graph of the system of linear equations.

c) Describe how the information shown in the graph relates to the Métis sashes that the girls are weaving.



Did You Know?

Métis sashes are traditionally made with Aboriginal finger-weaving techniques. The weavers use European materials. In the past, the sash had many uses, including as a rope, washcloth, dog harness, baby carrier, and belt. Today, the sash is a symbol of pride. It is common for Prairie Métis organizations to bestow the Order of the Sash. There is no higher honour in the Métis community than receiving a sash as a gift.

7. One oil well has produced 2100 m^3. It produces oil at a rate of 7 m^3 per day. Another oil well has produced 1500 m^3 and produces oil at a rate of 15 m^3 per day. Suppose the two wells continue to produce oil at their current rates. When will both oil wells have produced the same amount? How much oil will they have produced at that time?

8. Megan considers two different car rental options. One option costs \$19 per day plus \$0.12 per kilometre. Another option costs \$42.50 per day for unlimited kilometres.

Compare the two options for a one-day rental. Which option should Megan choose? Why?

Web Link

To learn more about low-flow shower heads and other water-saving fixtures, go to www.mhrmath10.ca and follow the links.

Did You Know?

All Arctic communities except the very largest are on trucked water systems. So, most showers in the North use low-flow shower heads.

Did You Know?

The Test of Metal is a mountain bike race that takes place in Squamish, BC, every summer. The 67-km course includes many long, steep climbs and lots of technical off-road riding. More than 800 competitors of all ability levels compete. Finish times range from under 3 h to over 6 h.

9. **Unit Project** The Benoit family is deciding whether to replace their conventional shower head with a low-flow model. Their current shower head uses 170 L of water per 10-min shower. A typical low-flow shower head costs \$25 to purchase. It uses 85 L per 10-min shower. Heating the water with electricity costs approximately \$0.002 per litre.

a) If n represents the number of 10-min showers, write an expression for the cost of n showers using their current shower head.

b) Write a system of linear equations to represent the cost of showering using each type of shower head.

c) Graph the system of linear equations.

d) What is the solution to your linear system? What does it represent?

e) How would your solution change if each shower was reduced to 8 min? Justify your answer with a graphical analysis.

10. Amaruk and Mary are both travelling by boat between Igloolik and Hall Beach, NU. Amaruk is 35 km away from Hall Beach. He is travelling south to Hall Beach at a speed of 40 km/h. Mary is 15 km away from Hall Beach. She is travelling north at a speed of 25 km/h. When they pass each other, how far will they be from Hall Beach?

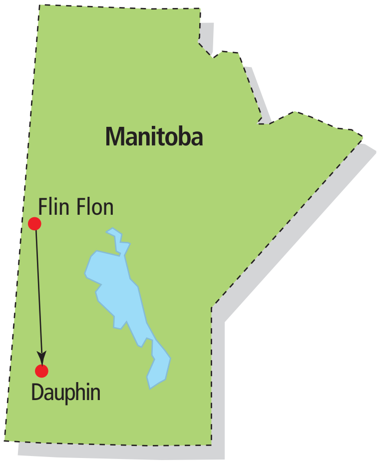
11. Caleb and Mitch compete in the Test of Metal mountain bike race. For a 2-km-long section, Caleb is 400 m up the hill and rides at a rate of 7.5 km/h. Mitch is at the base of the hill and rides at 10 km/h. Use a linear system to model this section of Caleb's and Mitch's race. Will Mitch catch Caleb before they reach the top of the hill? Explain why.

12. **Unit Project** A nearby wetland is estimated to have 100 ducks and 300 fish. A source of pollution seems to have contaminated the water. A local environmental group realizes that the number of ducks is decreasing at an average rate of 5 per year. The number of fish is decreasing at an average rate of 20 per year. Suppose the situation is considered critical if the number of fish equals the number of ducks in the area.

a) Write a system of linear equations to represent the numbers of fish and ducks in the wetland. Create a graph of your system.

b) Will the decreasing rates of fish and ducks become critical? If so, when? Justify your answer.

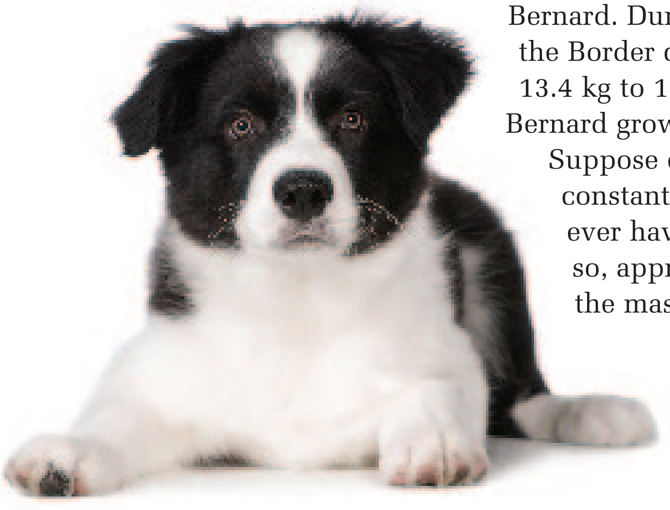
13. While driving from Flin Flon to Dauphin, MB, Kevin and his family had a flat tire. Before the flat tire, Kevin's parents drove at an average speed of 90 km/h. Once the flat tire was replaced with the spare tire, they travelled at an average speed of 75 km/h for the remainder of the trip. The total distance between the two cities is 538 km. The total driving time was 6 h. Write a linear system to model the family's travel. How far did they travel before the flat tire occurred? Include a labelled diagram.



14. Chris paints from one end of a 120-ft-long fence. He paints at a rate of 9 ft/h. Robert paints from the other end at a rate of 12 ft/h. Use a system of linear equations to determine when and where they will finish painting the fence.

15. Trevor is doing a project on tree growth rates. He measures the heights of two trees in early spring and again 20 days later. The younger tree grows from 120 cm to 130 cm tall. The older tree grows from 140 cm to 144 cm tall. Assuming each tree grows at a constant rate, when will the trees be the same height? What will this height be?

16. For part of the first year of a dog's life, its growth can be approximated using a linear function. Emilie has two puppies: a Border collie and a younger Saint Bernard. During a 4-week period, the Border collie grows from 13.4 kg to 17 kg, while the Saint Bernard grows from 6 kg to 12.4 kg. Suppose each dog grows at a constant rate. Will the dogs ever have the same mass? If so, approximately what is the mass?



17. A parachutist descends from a height of 500 m to 300 m above ground in 50 s. During the same time, a balloonist rises from 200 m to 450 m.

a) Write a system of linear equations to model their heights.

b) When are the two people at the same height? What is that height?

18. Andrea has three times as many grapes as Hunter. If she gives Hunter six grapes, she will have twice as many as he has now. Write a system of linear equations to model the number of grapes each person has. How many grapes does each person have before the exchange?

Extend

19. Jesse asks his math teacher her age and her husband's age. Jesse's teacher responds that it is not an appropriate question, but that she would use the opportunity to challenge Jesse with a riddle. She says, "One third of my age is ten less than one half of my husband's age. The sum of our ages is 105." How old is Jesse's math teacher?

20. A man swims 200 m against the current of a stream in 3 min. He swims with the same effort downstream for 150 m in 45 s. Create a system of linear equations to determine the man's swimming speed and the speed of the current of the water, in metres per minute.

21. An alloy is a mixture of a metal with a cheaper metal. A jeweller wishes to make pendants using a 94% silver alloy of sterling silver and pure silver. Sterling silver is 92.5% pure silver. The jeweller wishes to make 100 g of the silver alloy. What linear system could be used to determine how many grams of sterling silver and pure silver must be mixed?

22. Eunji wants to spend time at a local amusement park this summer. She is deciding which option will cost less.

Option A: A season's pass costs \$22, but she will have to pay \$6 for parking each visit.

Option B: A two-visit pass costs \$16.50 and includes parking. She can buy as many passes as she wants.

Represent Eunji's options using linear equations. What decision will she need to make before she can choose an option? Why?

Create Connections

23. Gavin leaves Calgary at 1:30 p.m. and drives to Edmonton. He travels at a speed of 110 km/h. At 2:15 p.m., James leaves Calgary in a helicopter travelling along the same route as Gavin. The helicopter travels at 240 km/h. The distance between Calgary and Edmonton is 300 km.

a) Draw a diagram of their travel or use a table to organize the information.

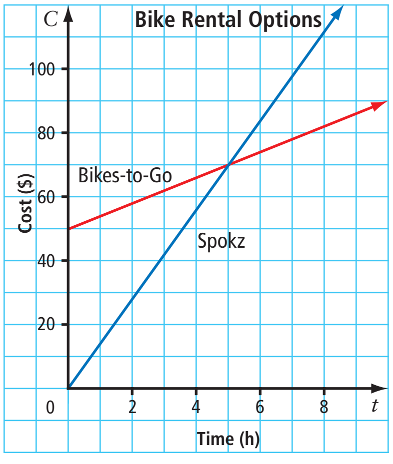
b) Describe how to determine whether James will catch up to Gavin.

24. Two bike stores, Bikes-to-Go and Spokz, have different rental options. Spokz charges an hourly rate only. For a 5-h-long rental, both stores charge the same price.

a) How could you use the graph to determine the hourly rate charged by Spokz?

b) Write a system of linear equations to model the cost of renting from each store.

c) How could the graph help you decide which option to choose?



8.3 Number of Solutions for Systems of Linear Equations

Focus on …

- explaining why systems of linear equations can have different numbers of solutions
- identifying how many solutions a system of linear equations has
- solving problems involving linear systems with different numbers of solutions

At the Arctic Winter Games, Northern youth share cultural experiences and compete in various events. One of the events is snowshoe racing. Situations involving time and distance, such as a race, can be represented with a system of linear equations. If you solve a linear system involving a race, will you always expect a single solution?

Web Link

To learn more about the Arctic Winter Games, go to www.mhrmath10.ca and follow the links.

Materials

- stopwatch or other timer or clock showing seconds
- measuring tape
- grid paper and coloured pencils, or computer with graphing software

Investigate Number of Solutions for Systems of Linear Equations

Work in groups of four to act out different race scenarios. Each group needs to assign two people to race, one person to measure time, and one person to record data.

1. As a group, design a data table to record start and end times and distances for each racer.

2. Complete several different race scenarios. Each student should move at a constant speed for the entire race. For each race, record the data. Consider the starting line distance zero.

3. Create a distance-time graph to represent each race. You may wish to use a different colour for each line.

4. Write a system of linear equations to represent each race.

5. **Reflect and Respond** Solve each system of linear equations you graphed. How many solutions were there for each linear system? Explain how each solution relates to details of the race.

6. How could you have predicted the number of solutions for each linear system just by knowing the starting points and speeds of the racers?

Link the Ideas

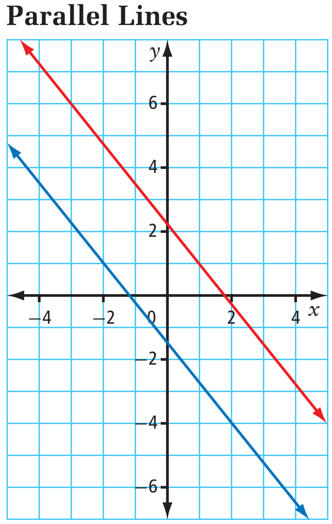
When two lines are graphed on the same grid, they do not always have exactly one point of intersection as seen in sections 8.1 and 8.2. Parallel lines do not intersect at all. So, a system of parallel lines has no solution.

Coincident lines have an infinite number of solutions because the lines are equivalent. They overlap.

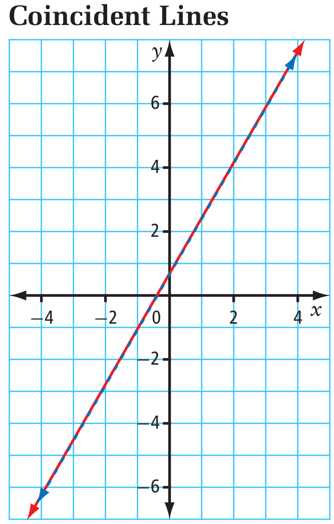
coincident lines

- lines that occupy the same position
- in a graph of two coincident lines, any point of either line lies on the other line

Parallel Lines



Coincident Lines



Reducing the equations to lowest terms may help you identify whether the equations are equivalent. If they are equivalent, then they must have an infinite number of solutions.

For the linear system $x - 2y + 5 = 0$ and $3x - 6y + 15 = 0$, the first equation is in lowest terms, but the second equation is not. Dividing each side of the equation $3x - 6y + 15 = 0$ by 3 gives $x - 2y + 5 = 0$, which is equivalent to the first equation. Therefore, the linear system has an infinite number of solutions.

Example 1 Connect the Number of Solutions to the Situation

A particular dog-mushing race is 13 km long. The distances and speeds for several competitors at a certain time during the race are shown in the table of values.

	Current Distance Travelled (km)	Current Speed (km/h)
Competitor A	6.0	24
Competitor B	5.0	32
Competitor C	4.0	24
Competitor D	4.0	24

Assume the racers continue at their current speeds. For each pair of competitors below,
- write a system of linear equations representing their travel from this point forward
- graph each system of linear equations
- identify and interpret the solution to each linear system

a) competitors A and B
b) competitors A and C
c) competitors C and D

Did You Know?

Many breeds of dogs are used for competitive racing. Teams of Canadian Eskimo Dogs have helped people survive in the harsh climate of northern Canada for centuries. Canadian Eskimo Dogs help move people and materials throughout regions that are covered in snow during most of the winter. The Canadian Eskimo Dog is Nunavut's official mammal in honour of its vital role.

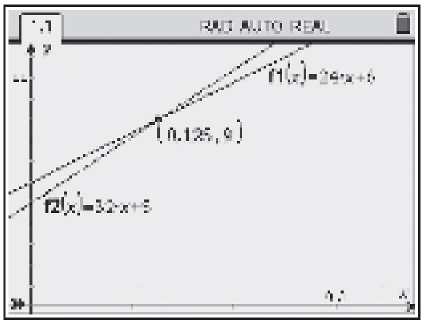
Solution

Let d represent distance from the start of the race. Let t represent time from this point on in the race.

a) Competitor A is travelling at 24 km/h and has travelled 6.0 km. Competitor B is travelling at 32 km/h and has travelled 5.0 km. Their travel can be represented by the following equations:

$d = 24t + 6$

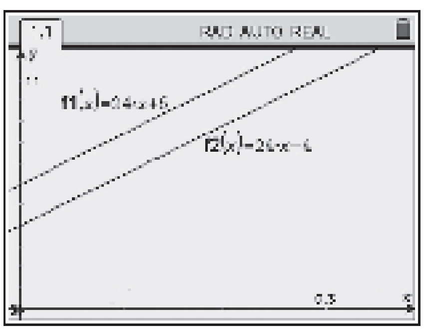
$d = 32t + 5$



The lines intersect at one point. So, the linear system has one solution.

Competitor B will catch up to competitor A after 0.125 h, or 7.5 min, and then proceed past. The point (0.125, 9) is the only point that lies on both lines.

b) Competitors A and C are both travelling at 24 km/h. Competitor A has travelled 6.0 km. Competitor C has travelled 4.0 km. Their travel can be represented by the equations $d = 24t + 6$ and $d = 24t + 4$.

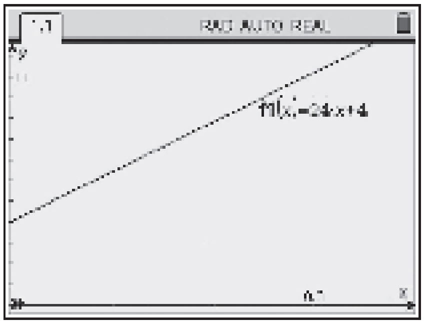


The lines have the same slope but different *y*-intercepts.

The lines are parallel. They have no points in common, so the linear system has no solutions.

Competitors A and C travel at the same speed, but at different distances from the start line. Competitor C will never catch up to competitor A. There is no point where they are at the same distance at the same time.

c) Competitor C is travelling at 24 km/h and has travelled 4.0 km. Competitor D is also travelling at 24 km/h and has also travelled 4 km. They are currently at the same distance and travelling at the same speed. Their travel can be represented by the equations $d = 24t + 4$ and $d = 24t + 4$.



The lines have the same slope and the same *y*-intercept.

The lines are coincident, so they share all the same points. The linear system has an infinite number of solutions.

Competitors C and D are side by side on the course. They will continue this way because they are travelling at the same speed.

Your Turn

Four vehicles travel on a long, straight stretch of the Trans-Canada Highway. Their current distances and speeds are shown in the table of values.

	Current Distance (km)	Current Speed (km/h)
Car	40	90
Minivan	25	90
Truck	30	110
RV	40	90

For each pair of vehicles, represent the distance-time relationship using a system of linear equations. Suppose the vehicles continue at their current speeds. Identify and interpret the solution to each linear system.

a) the car and the minivan
b) the car and the RV
c) the truck and the RV

Example 2 Predict and Confirm the Number of Solutions

Predict the number of solutions for each system of linear equations. Explain your reasoning, and then confirm each answer by graphing the linear system.

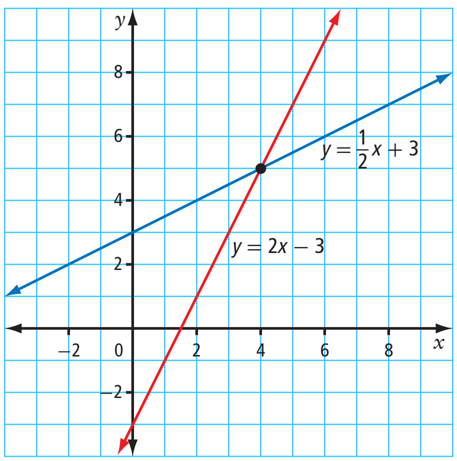
a) $y = 2x - 3$
$y = \frac{1}{2}x + 3$

b) $4x + 10y = 30$
$2x + 5y = 35$

c) $10x - 6y = -12$
$21y = 42 + 35x$

Solution

a) The slope of $y = 2x - 3$ is 2. The slope of $y = \frac{1}{2}x + 3$ is $\frac{1}{2}$.

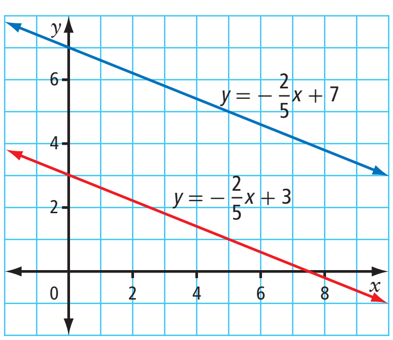


The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

b) Rearrange each equation to slope-intercept form by isolating y.

$$4x + 10y = 30 \qquad\qquad 2x + 5y = 35$$
$$4x + 10y - 4x = 30 - 4x \qquad\qquad 2x + 5y - 2x = 35 - 2x$$
$$10y = -4x + 30 \qquad\qquad 5y = -2x + 35$$
$$y = \frac{-2}{5}x + 3 \qquad\qquad y = \frac{-2}{5}x + 7$$

Since the lines have the same slope and different y-intercepts, the graph will result in parallel lines. The lines will never intersect. Therefore, this linear system has no solutions.



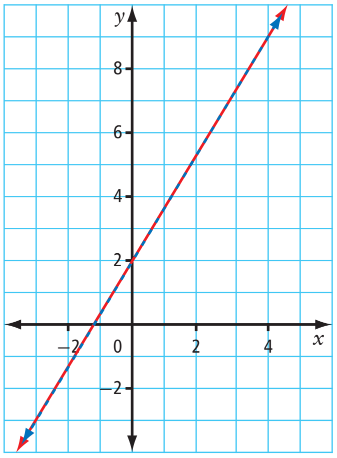
c) For the linear system $10x - 6y = -12$ and $21y = 42 + 35x$, isolate y in each equation to compare the slopes and y-intercepts.

$$10x - 6y = -12$$
$$10x - 6y + 6y + 12 = -12 + 6y + 12$$
$$10x + 12 = 6y$$
$$\frac{5}{3}x + 2 = y$$
$$y = \frac{5}{3}x + 2$$

$$21y = 42 + 35x$$
$$\frac{21y}{21} = \frac{42}{21} + \frac{35x}{21}$$
$$y = 2 + \frac{5}{3}x$$
$$y = \frac{5}{3}x + 2$$

Both equations have a slope of $\frac{5}{3}$ and a y-intercept of 2.

The graph will result in coincident lines. Therefore, this linear system has an infinite number of solutions.



Your Turn

Predict the number of solutions for each system of linear equations. Justify your answers using a graph.

a) $x + 2y = 4$
$y = -\frac{1}{2}x + 4$

b) $6y - 4x = 6$
$y = \frac{2}{3}x + 1$

c) $y = 3x - 1$
$y = 2x - 1$

Example 3 Identify Zero and Infinite Solutions by Comparing Coefficients

Sabrina's teacher gives her the following systems of linear equations and tells her that each system has either no solution or an infinite number of solutions. How can Sabrina determine each answer by inspecting the equations?

a) $2x + 3y = 12$
$2x + 3y = 20$

b) $2x + 3y = 12$
$4x + 6y = 24$

Solution

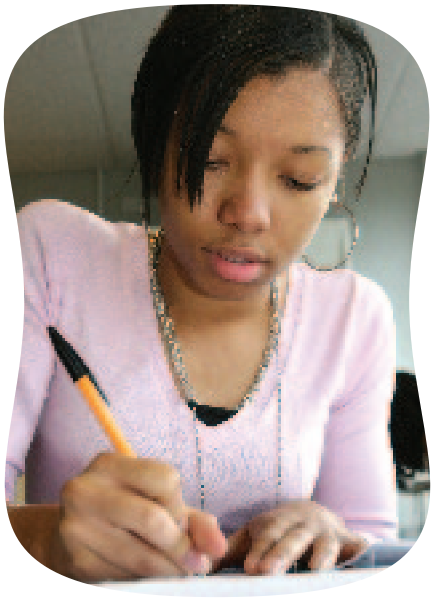
a) Sabrina notices that the left sides of the equations are identical. So, any ordered pair she substitutes will result in the same value on the left side of each equation. However, the right sides are not equal. There are no ordered pairs that can satisfy both equations, so the lines never intersect. Sabrina concludes that the linear system has no solutions.

How else could you confirm that the lines are parallel?

b) Sabrina notices that the second equation, $4x + 6y = 24$, is not in lowest terms. She divides each term by 2. This results in an equation that is identical to the first equation, $2x + 3y = 12$. Therefore, the equations are equivalent and the graph will be a pair of coincident lines. The linear system has an infinite number of solutions.

$$\frac{4x}{2} + \frac{6y}{2} = \frac{24}{2}$$
$$2x + 3y = 12$$

ME



Your Turn

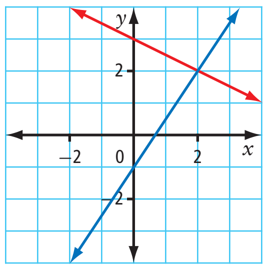
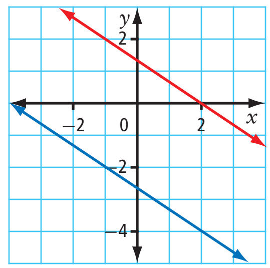
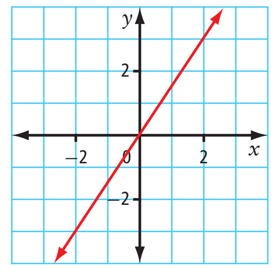
Determine, by inspection, whether each linear system has an infinite number of solutions or no solution. Explain your reasoning.

a) $2x + 10y - 16 = 0$
$x + 5y - 8 = 0$

b) $x + 2y + 4 = 0$
$x + 2y - 6 = 0$

Key Ideas

- A system of linear equations can have one solution, no solution, or an infinite number of solutions.
- Before solving, you can predict the number of solutions for a linear system by comparing the slopes and *y*-intercepts of the equations.

Intersecting Lines	Parallel Lines	Coincident Lines
one solution	no solution	an infinite number of solutions
		
different slopes	same slope	same slope
y-intercepts can be the same or different.	different *y*-intercepts	same *y*-intercept

- For some linear systems, reducing the equations to lowest terms and comparing the coefficients of the *x*-terms, *y*-terms, and constants may help you predict the number of solutions.

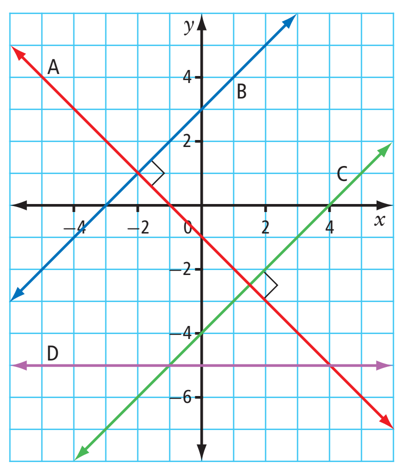
Check Your Understanding

Practise

1. Which pair(s) of lines in the graph form a linear system that has

a) exactly one solution?

b) no solution?



2. Predict the number of solutions for each system of linear equations. Justify your answers.

a) $y = x + 2$
$y = x + 2$

b) $y = 2x - 4$
$y = x + 1$

c) $y = 3x + 2$
$y = 3x - 5$

3. How many solutions does each linear system have? Justify your answers.

a) $x + 3y = 6$
$y = -\frac{1}{3}x + 6$

b) $3x - y = 12$
$4x - y = 12$

c) $x - 4y = 8$
$x + 4y = 20$

4. Describe the graph and the equations of a linear system that has

a) no solution

b) one solution

c) an infinite number of solutions

5. Describe a strategy for predicting how many solutions each system of linear equations has just by looking at it. Test your strategy by graphing each linear system.

a) $2x + 7y = 28$
$2x + 7y = 15$

b) $x + 2y = 12$
$2x + 4y = 24$

Apply

6. One of the equations in a linear system is $2x - y + 5 = 0$. What might the other equation be if the system has

a) no solutions?

b) one solution?

c) infinitely many solutions?

7. A provincial magazine employs sales people who earn money for every subscription they sell. Several employees are comparing their earnings so far this month.

	Current Earnings ($)	Earnings Per Subscription ($)
Alyssa	472	7.00
Brian	360	8.25
Charlie	360	8.25
Dena	413	8.25

Write a system of equations to represent the earnings for each pair of employees. Identify the solution to each system. Explain how the solution relates to the employees' earnings.

a) Brian and Charlie

b) Alyssa and Brian

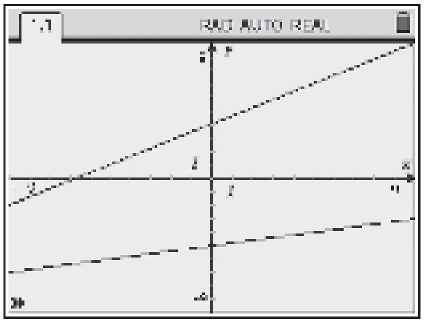
c) Charlie and Dena

8. Sal and Jeff graph the system $y = \frac{5}{8}x + \frac{11}{7}$ and $y = \frac{7}{11}x + \frac{8}{5}$ using technology. They use dimensions of −10 to 10 on the x-axis and −10 to 10 on the y-axis. Sal thinks there is one solution. Jeff claims that there are an infinite number of solutions.

   a) Describe how to predict the number of solutions.

   b) Use technology to recreate their graph. Explain why each person might have come to his conclusion.

9. Stephanie graphs a system of equations using technology. She obtains the following graph and concludes that the system has no solutions.



   a) Explain why Stephanie might have made her conclusion.

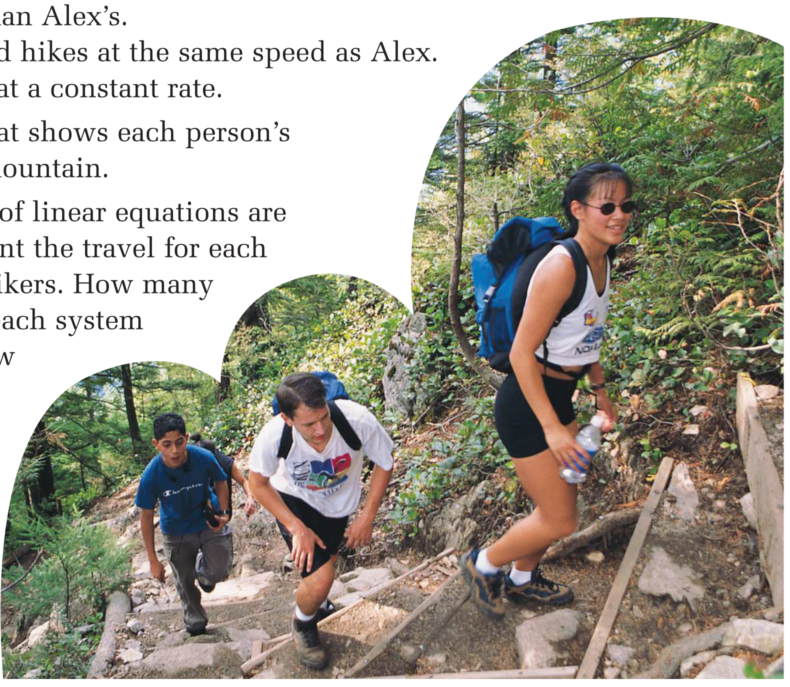
   b) Is Stephanie correct? Explain.

10. Alex, Sandra, Christine, and Jared are hiking the Grouse Grind® near Vancouver, BC.
    - Alex starts first.
    - Sandra and Christine start a short time later. They hike together at a faster speed than Alex's.
    - Jared starts last and hikes at the same speed as Alex.

    Each person travels at a constant rate.

    a) Sketch a graph that shows each person's progress up the mountain.

    b) Suppose systems of linear equations are created to represent the travel for each possible pair of hikers. How many solutions would each system have? Explain how the number of solutions relates to the hike.



Did You Know?

The Grouse Grind® is the most used hiking trail in the Vancouver area. The trail is 2.9 km long and rises 853 m. Over 100 000 people walk or run this steep climb every year. Maintenance workers have built steps up most of the trail to protect against erosion and for the safety of the hikers.

11. Is the following statement true or false? Explain and provide an example to support your answer.

"If a system of linear equations has an infinite number of solutions, then any pair of numbers is a solution to the system."

12. Suppose you are given only the following pieces of information about a system of linear equations. Would you be able to predict the number of solutions to the system? Explain.

a) The slopes of the lines are the same.

b) The y-intercepts of the lines are the same.

c) The x-intercepts are the same, and the y-intercepts are the same.

13. PaperWest has produced 5000 kg of napkins. It continues to manufacture 350 kg of napkins per week. Northern Paper manufactures napkins at a rate of 1400 kg per month and has already produced 28 000 kg. Assume one month has exactly four weeks.

a) Write a system of linear equations to represent the manufacturing of the napkins.

b) Explain how the number of solutions to the system relates to this situation.

Extend

14. For the linear system $2x + 3y = 12$ and $4x + 6y = C$, what value(s) of C will give the system

a) an infinite number of solutions?

b) no solution?

15. Two taxis travel the same route from the airport. One taxi is 6 km from the airport and has a fuel economy of 20 km/L. The other taxi is just leaving the airport and uses 5 L of fuel for every 100 km travelled.

a) Create a system of linear equations relating the distance travelled (y kilometres) to the amount of fuel used (x litres) for each taxi.

b) Explain how the number of solutions to the system relates to the travel of the taxis.

16. Consider the system $y = 56 - 2x$ and $y = 10 + x$.

a) Suppose the domain for the system is restricted to $0 \leq x \leq 8$. How many solutions does the system have? Explain.

b) Suppose there are no restrictions on the domain. How many solutions does the system have?

c) What effects do restrictions on the domain of the equations have when you are predicting the number of solutions? Explain.

17. Consider the system $Wx + 3y = 2W$ and $12x + Wy = 24$.

a) What value(s) of W give the system an infinite number of solutions?

b) How many solutions will the system have for value(s) of W other than those you found in part a)? Justify your answer.

Create Connections

18. Two plants are growing according to the equations $h = 60 + 3t$ and $h = 55 + 2t$. In the equations, h represents height, in centimetres, and t represents time, in weeks. Wendy reasons that one plant is shorter and is growing slower, so it will never catch up to the other plant. Wendy concludes that the system has no solutions. Harriet states that the two lines have different slopes, so there must be an intersection point. Who do you agree with? Create a graph and use it to explain your answer.

19. Can a system of two linear equations have exactly two solutions? Explain your answer, using words and diagrams.

20. Do you think it is always possible to tell how many solutions a system of linear equations has just by looking at the equations? Explain your thinking.

21. Describe a real-life situation that could be modelled by a system of linear equations having each number of solutions. Include examples of possible systems of linear equations.

a) no solution

b) one solution

c) an infinite number of solutions

22. **MINI LAB** Investigate relationships between equations in a linear system and the number of solutions to the system.

Materials
- graphing calculator or computer with graphing software

Step 1 Consider $2x + y = 4$ as equation #1. Create four linear systems (A, B, C, and D) by writing equation #2 according to each of the following instructions.

System A: Multiply (both sides of) equation #1 by a number of your choice.

System B: Add a number of your choice to only the right side of equation #1.

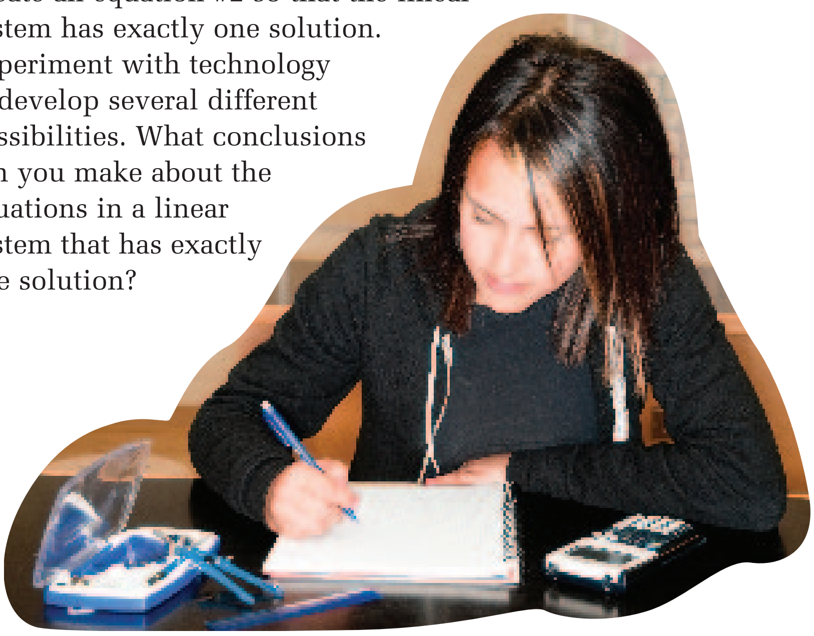
System C: Perform an operation of your choice on both sides of equation #1.

System D: Perform an operation of your choice on only one side of equation #1.

Step 2 Use a graph to analyse the number of solutions for each system of linear equations.

Step 3 What can you conclude about the number of solutions for a system of linear equations if one equation is a multiple of the other? What if two *different* equations in general form have identical coefficients of x-terms and identical coefficients of y-terms?

Step 4 Create an equation #2 so that the linear system has exactly one solution. Experiment with technology to develop several different possibilities. What conclusions can you make about the equations in a linear system that has exactly one solution?



8 Review

8.1 Systems of Linear Equations and Graphs, pages 416–431

1. For each system of linear equations, explain how you could verify whether the given point is a solution. Is the given point a solution?

a) $y = x + 11$
$y = -2x - 10$
$(-7, 4)$

b) $11x + 3y = 18$
$9x - 4y = 7$
$(3, -5)$

2. Solve each system of linear equations by graphing.

a) $y = -2x - 6$
$y = \frac{3}{2}x + 8$

b) $3x + y = 2$
$x + y = 3$

c) $x - y = 1$
$5x - 4y = 12$

3. Use technology to solve each system of linear equations graphically.

a) $y = \frac{2}{5}x - 7$
$y = -\frac{5}{8}x + 2$

b) $6x + 5y = -45$
$2x + 5y = 40$

c) $9x - 8y = -24$
$4x - 3y = -3$

4. The travel of two boats can be represented by the system $d = 4t + 20$ and $d = 5t + 12$. In the equations, t represents time, in seconds, and d represents distance, in metres.

a) Represent the linear system numerically and then graphically.

b) Describe how to identify the solution in each representation.

c) What is the solution to this linear system? What does the solution represent?

8.2 Modelling and Solving Linear Systems, pages 432–445

5. Write a system of linear equations to model each situation.

a) One gym membership is \$85 for the first year plus \$30 per month. Another gym membership is \$35 per month.

b) Two grain bins start filling at the same time. The amount of grain in one bin increases from 5 m^3 to 30 m^3 in 10 min. The other bin is empty and increases to 40 m^3 in 8 min.

c) Two crews begin resurfacing side-by-side roads at the same time. One crew has 3 km left to resurface and covers 100 m every 30 min. The other crew has 4 km to resurface and covers 250 m every 60 min.

6. A flyer advertises two cell phone plans.

a) Write an equation to represent each plan.

b) Compare the plans using a graph.

c) Explain how the graph can help someone make a choice.

7. Atmospheric pressure in two cities is monitored at the same time. The pressure in one city drops from 102.6 kPa to 100.6 kPa in 4 h. In the other city, the atmospheric pressure rises from 99.8 kPa to 101.3 kPa in 6 h.

a) Write a system of linear equations to model the change in atmospheric pressure.

b) Graph the system. What does the point of intersection represent?

8. Karen and Andrew ski down the same run at constant speeds. They begin skiing at the same time. Karen skis from an altitude of 2000 m for 3 min and stops at 1700 m. Andrew skis from 2100 m to 1500 m in 4 min. Will Andrew pass Karen before she stops? If so, when and at what altitude? Verify your answer.

8.3 Number of Solutions for Systems of Linear Equations, pages 446–459

9. Describe the graph of a system of linear equations that has

a) no solution

b) one solution

c) an infinite number of solutions

10. Predict the number of solutions for each linear system. Explain your prediction and justify by graphing.

a) $y = 2x - 3$
$y = 2x + 1$

b) $y = 3x + 10$
$y = 6x + 20$

c) $2x + 3y - 6 = 0$
$14x + 21y - 42 = 0$

d) $2x - y - 10 = 0$
$4x - 2y - 30 = 0$

11. Pam, Myk, Luisa, and Carl are each holding a helium balloon. Pam and Myk let go of their balloons. The balloons rise into the air. A few seconds later, Luisa lets her balloon go, and it rises more quickly than the first two. After a few more seconds, Carl lets his balloon go. It rises at the same rate as Pam's balloon. Each balloon rises at a constant rate.

a) Sketch a graph to represent the ascent of the four balloons.

b) Identify pairs of balloons whose ascent can be represented by a linear system with each of the following numbers of solutions. Explain why you chose each pair.
- zero
- one
- infinite

12. Companies A and B produce sports beverages. Company A has made 150 L and is producing more at a rate of 75 L every 15 min. Company B makes the same beverage at a rate of 300 L per hour and has already made 600 L.

a) Write a system of linear equations to represent the production of the sports beverages.

b) How many solutions are there? How does the number of solutions relate to the situation?

13. Suppose two investments earn the same rate of interest.

a) Which of the following is *not* a possible number of solutions for the linear system representing the investments over time?
- zero
- one
- infinite

b) Explain your answer.

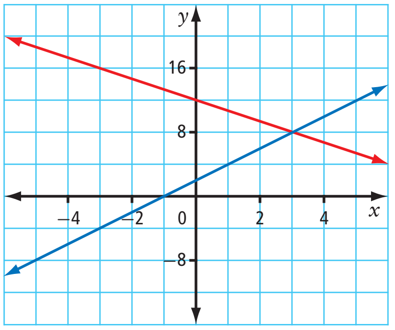
8 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. What is the solution to the linear system shown in the graph?

A $(-1, 12)$ **B** $(0, 0)$
C $(0, 2)$ **D** $(3, 8)$



2. Which number of solutions is *not* possible for a system of two linear equations?

A zero **B** one
C two **D** infinite

3. What is the point of intersection in a system of linear equations?

A the origin
B the ordered pair written as x-intercept, y-intercept of a line
C the point where a line starts
D the ordered pair representing the location where the lines cross

4. How many solutions does the linear system $2x + 3y = 6$ and $3x + 2y = 24$ have?

A zero **B** one
C two **D** infinite

5. The volume of liquid in two different containers begins decreasing at the same time. The volume in one container changes from 22 L to 10 L in 6 s. The volume in the other container changes from 30 L to 15 L in 5 s. Which system of linear equations can be used to model the situation?

A $V = 22 - 10t$
$V = 30 - 15t$

B $V = 10 - 2t$
$V = 15 - 3t$

C $V = 10 - 3t$
$V = 15 - 2t$

D $V = 22 - 2t$
$V = 30 - 3t$

Short Answer

6. For each system of linear equations, verify whether the given point is a solution. Explain what the results would show on a graph of the system.

a) $x + y = 10$
$2x - y = 3$
$(5, 2)$

b) $3x + y = 1$
$7y = 13 - 15x$
$(-1, 4)$

c) $x + 4y + 4 = 0$
$2x - 3y = 27$
$(8, -3)$

7. Consider the linear system $y = 5x + 1$ and $y = \frac{5}{2}x - 4$. Explain how you would solve this system.

8. **a)** Graph the linear system $5x - 4y = 32$ and $3x - 7y = -63$.

b) State the solution to this system of linear equations. Express your answer to the nearest hundredth.

c) Verify your solution. Describe your steps.

9. Explain how the number of solutions for each linear system can be determined without graphing. Briefly describe the relationship between the lines on a graph of each system.

a) $3x + 4y - 12 = 0$
$3x + 4y - 24 = 0$

b) $y = \frac{1}{2}x - 5$
$x - 2y = 10$

10. Consider the system of linear equations $14x + 35y = 7$ and $4x + By = 2$. If a graph of this system shows coincident lines, what is the value of B?

Extended Response

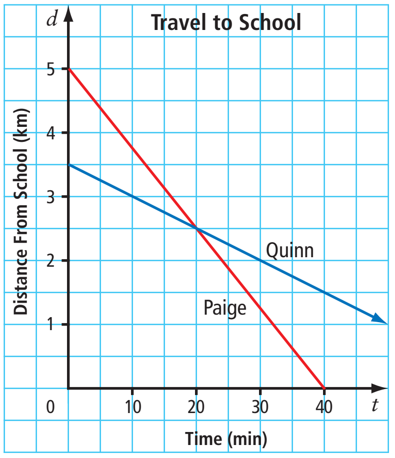
11. The grass on one sod farm is 6 cm high. The grass grows at a rate of 1.5 cm/week. The grass on another farm is 4 cm high. It grows at a rate of 2 cm/week.

a) Write a system of linear equations to model the growth of the grass on each farm.

b) Represent the linear system graphically.

c) Use the graph to describe the growth of the grass on each farm. What does the point of intersection represent?

12. The graph represents the travel of two students walking to school. Write a story that incorporates all the information shown in the graph.



13. Two feathers are released at the same time and begin falling steadily. One feather falls from 5.5 m and comes to rest on a 4-m-high ledge after 8 s. The other feather falls from 6 m to 3 m in 12 s. Are the feathers ever at the same height during their fall? Verify your answer.

14. The Opaskwayak Canoe Classic is a 60-mi canoe race held annually by the Opaskwayak Cree Nation. The race takes place on the Saskatchewan River near The Pas, MB. Information on four racers part way through the race is shown in the table of values.

	Current Distance (mi)	Current Speed (mph)
Taj	12.6	7.0
Donna	11.8	7.2
Marcus	11.8	7.2
Rose	11.2	7.2

Represent the data for each pair of racers using a system of linear equations, assuming each racer continues at his or her current speed. What is the solution to each linear system? What does the solution represent?

a) Donna and Marcus

b) Taj and Donna

c) Marcus and Rose