CHAPTER

7

Linear Equations and Graphs

We make sense of our world by understanding how things are related. One common relationship is the linear relation. This is often expressed as an equation. Linear equations are involved in almost every career and field of study. Archaeologists and paleontologists use linear equations to learn about past civilizations and forms of life. Police detectives and forensic analysts use linear equations to solve crimes. Developers and civil engineers use linear equations to predict future trends.

Big Ideas

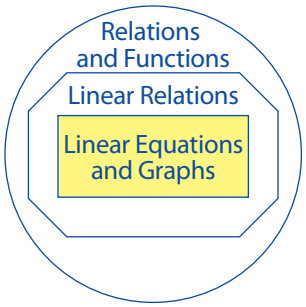
When you have completed this chapter, you will be able to ...

- graph a line using a variety of methods
- determine the equation of a line using characteristics of the line
- convert the equation of a line from one form to another
- model and solve real-life problems using a linear equation

Key Terms

y-intercept
slope-intercept form
parameter
general form (of a linear equation)
x-intercept
slope-point form
parallel lines
perpendicular lines

Your Relations and Functions Organizer

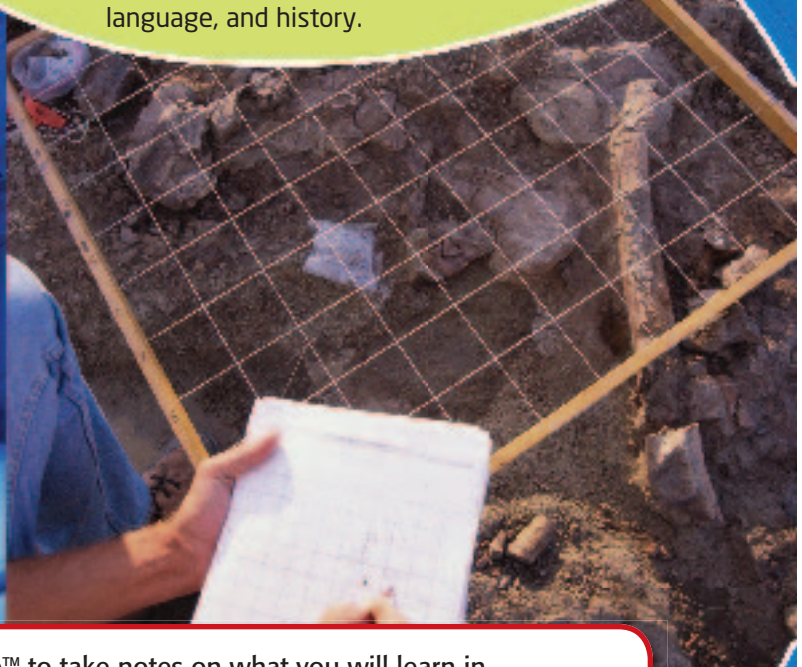


Archaeologist

Archaeologists recover and analyse materials and artifacts from the past. They examine tools, debris, architecture, artwork, physical remains, and landscape features. The information they gather provides clues about traditions, beliefs, customs, languages, and practices. They use these clues to reconstruct human cultures. Archaeologists use skills from many different fields, such as mathematics, science, language, and history.

WWW Web Link

To learn more about archaeology, go to www.mhrmath10.ca and follow the links.

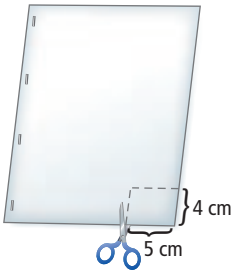


FOLDABLES Study Tool

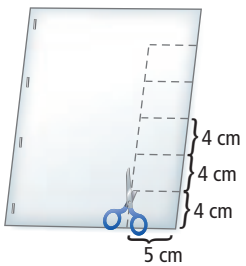
Make the following Foldable™ to take notes on what you will learn in Chapter 7.

1. Stack six sheets of 0.5-cm grid paper with the grid sides down. Staple them together along the left edge.

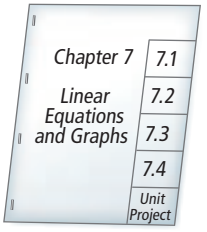
2. Make a mark 5 cm in from the bottom right corner of the top page. Cut through the top five sheets from this mark 4 cm up and then across to the right edge. Do not cut the last page because this is your back cover.



3. Cut through the top four sheets of paper, up 4 cm more and across to the right edge. As you do this, you will form tabs along the right side of the foldable. Continue to cut tabs until you have six tabs.



4. Label the tabs as shown.



7.1 Slope-Intercept Form

Focus on ...

- identifying the slope and *y*-intercept of a straight-line graph
- determining a linear equation using slope and *y*-intercept
- rewriting a linear relation in slope-intercept form
- graphing equations in slope-intercept form
- solving problems using equations in slope-intercept form

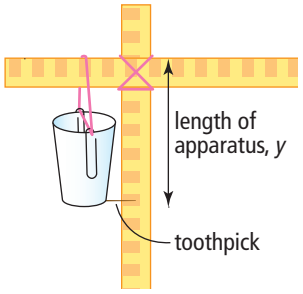
Many relationships can be modelled by the graph of a straight line. For example, a farmer purchases chicken feed pellets by mass, yet dispenses the feed by volume. When the farmer analyses the feeding of chickens, the relationship between the volume of feed pellets in a dispenser and the mass of the feed may be modelled by a linear equation. Linear equations can be written in different forms. Each form can provide specific information about the graph.

Investigate the Graph of a Linear Equation

Materials

- two metre sticks
- elastic band
- foam cup
- paper clips, string, or tape
- toothpick or straightened paper clip
- six identical marbles or other items of equal mass
- ruler
- grid paper

1. a) Suspend a metre stick between two chairs or desks. Attach an elastic band across the diameter of a foam cup using paper clips. Loop the other end of the elastic band around the metre stick. Poke a toothpick horizontally through the bottom edge of the cup.



b) Position another metre stick vertically beside the suspended cup. Measure the length of the apparatus, *y*, in centimetres, from the top of the elastic band to the bottom of the cup. Use the toothpick to help you locate the reading on the metre stick.

2. a) Place one marble in the cup. Let the cup come to rest. Measure and record the length of the apparatus. One at a time, place the other marbles in the cup. After adding each marble, measure the length of the apparatus again.

b) On grid paper, plot the individual data points. Plot the number of marbles along the x-axis. Using a ruler, draw a straight line that represents the data.

3. Identify the independent and dependent variables.

4. Determine the slope of the line. What are the units of the slope? What does the slope of the line represent in this situation?

5. a) Identify the point where the line intersects the y-axis. What does this point represent in this situation?

b) Starting at this point, explain how to use the slope to determine the length of the apparatus when one, two, or three marbles are in the cup.

6. Use the ***y*-intercept** and slope to determine what the length of the apparatus would be if ten marbles were in the cup. Does your graph support your answer?

7. a) Write the equation of the line in **slope-intercept form**. Let y represent the length of the apparatus, in centimetres. Let x represent the number of marbles in the cup.

b) Use the equation to estimate the length of the apparatus if the cup contains 15 marbles.

8. Reflect and Respond Suppose you use a cup that is 4 cm taller but has the same mass as the foam cup used.

a) How would this change the data you collected?

b) Describe how the graph would change.

c) Write an equation in slope-intercept form that represents this situation.

9. Suppose you use items that are double the mass of the marbles used.

a) How would this change your data?

b) Describe how the graph would change.

c) Represent this situation using an equation written in slope-intercept form.

y-intercept

- the y-coordinate of the point where a line or curve crosses the y-axis
- the value of y when $x = 0$

slope-intercept form

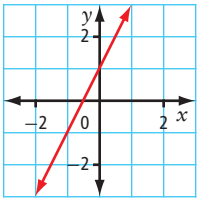
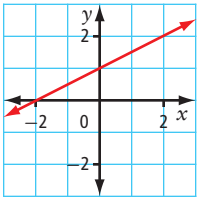
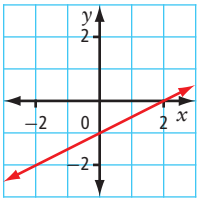
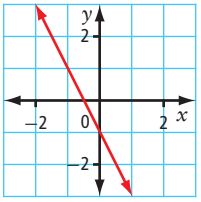
- the equation of a line in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept

Link the Ideas

To write the equation of a straight-line graph, you can use the following two constants:

- the rate of change or slope, m
- the y-intercept. If $(0, b)$ is the point where the line crosses the y-axis, then b is the y-intercept.

The equation of a non-vertical straight-line graph can be written in slope-intercept form. The equation is $y = mx + b$, where m represents the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ and b represents the y-intercept.

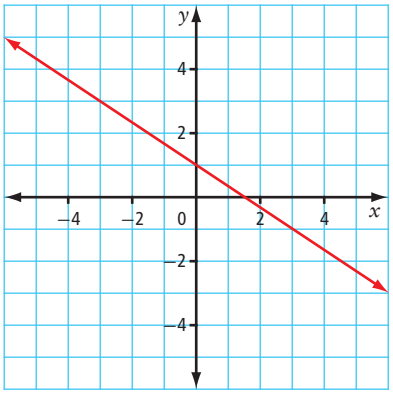
Table of Values	Graph	Slope, m	y-intercept, b	Equation, $y = mx + b$
x | y 0 | 1 1 | 3 2 | 5		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{2}{1}$ $m = 2$	1	$y = 2x + 1$
x | y 0 | 1 2 | 2 4 | 3		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{1}{2}$	1	$y = \frac{1}{2}x + 1$
x | y 0 | −1 2 | 0 4 | 1		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{1}{2}$	−1	$y = \frac{1}{2}x + (-1)$
x | y 0 | −1 1 | −3 2 | −5		$m = \frac{\Delta y}{\Delta x}$ $m = \frac{-2}{1}$ $m = -2$	−1	$y = -2x + (-1)$

Example 1 Write the Equation of a Line in Slope-Intercept Form

a) What are the slope and y-intercept of the line shown in the graph?

b) Write the equation of the line in slope-intercept form, $y = mx + b$.

c) Use graphing technology to check your equation.



Solution

a) The y-intercept is 1. Therefore, $b = 1$.
Using the points (0, 1) and (3, −1), the slope is

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 1}{3 - 0}$$

$$m = -\frac{2}{3}$$

The slope is $-\frac{2}{3}$ and the y-intercept is 1.

What do you know about the slope if the line falls from left to right?

How else could you determine the slope?

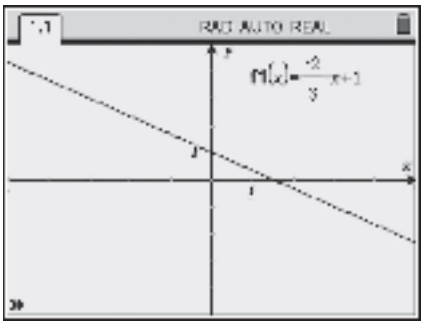
b) Substitute the values of m and b into the slope-intercept form of an equation.

$y = mx + b$

$y = -\frac{2}{3}x + 1$

The equation of the line in slope-intercept form is $y = -\frac{2}{3}x + 1$.

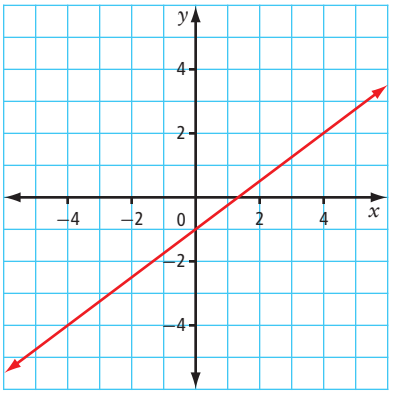
c) Create the graph using technology.



How can you confirm that this is the equation of the line that passes through the points (0, 1) and (3, −1)?

Your Turn

a) What are the slope and y-intercept of the line shown in the graph?



b) What is the equation of the line in slope-intercept form, $y = mx + b$?

c) Use graphing technology to check your equation.

Example 2 Convert an Equation to Slope-Intercept Form

A students' council rents a portable dunk tank as a fund-raising activity. Students pay for the chance to hit a target with a ball and dunk a teacher into a tank of water.

The relationship between the number of balls thrown, x, and the profit, y, in dollars, may be represented by the equation $3x - 2y - 600 = 0$.

a) Rewrite the equation in slope-intercept form.

b) State the slope of the line. What does the slope represent?

c) Identify the y-intercept. What does it represent?

d) The break-even point is the point at which the money raised equals the money spent. How many balls must the students sell to reach the break-even point?

Solution

a) Rearrange the equation into the form $y = mx + b$. To do this, isolate the variable y.

$$\begin{aligned} 3x - 2y - 600 &= 0 \\ 3x - 2y - 600 + 2y &= 0 + 2y \\ 3x - 600 &= 2y \\ \frac{3}{2}x - 300 &= y \end{aligned}$$

The equation written in slope-intercept form is $y = \frac{3}{2}x - 300$.

$3x - 600 = 2y$
Dividing all terms by 2 gives $\frac{3x}{2} - \frac{600}{2} = \frac{2y}{2}$.
So, $y = \frac{3}{2}x - 300$.

b) The slope of the line is $\frac{3}{2}$ or 1.5. It represents income of \$1.50 per ball. The slope is positive because the money is coming in.

c) The y-intercept is -300. It represents a cost of \$300 to rent the portable dunk tank. It is negative because the money is paid out as an expense.

d) At the break-even point, students do not make or lose money. So, the profit is zero.

Substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} y &= \frac{3}{2}x - 300 \\ 0 &= \frac{3}{2}x - 300 \\ 300 &= \frac{3}{2}x - 300 + 300 \\ 2(300) &= \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}x\right) \\ 600 &= 3x \\ 200 &= x \end{aligned}$$

To reach the break-even point, they must sell 200 balls, at a rate of \$1.50 per ball. They will make money if they sell more than 200 balls. They lose money if they sell fewer than 200 balls.

Your Turn

Parents of members of the cheerleading squad rent a hall. They arrange a talent show as a fundraiser. The relationship between the number of tickets sold, x, and the profit, y, in dollars, may be represented by the equation $12x - y - 840 = 0$.

a) What is the slope of the line? What does the slope represent?

b) Identify the y-intercept. What does it represent?

c) How many tickets must the parents sell to reach the break-even point?

Did You Know?

In 1998, the Canadian Armed Forces purchased four submarines from Britain. The submarines were named after Canadian port cities. The *Victoria* is based at Esquimalt, BC. It is part of the Maritime Forces Pacific fleet.

parameter

- a variable that has a constant value in a particular equation

Example 3 Model and Solve a Problem Using an Equation in Slope-Intercept Form

Submarines must withstand tremendous pressure exerted on all sides by the water. The table shows the linear relationship between pressure and water depth.

Depth (m)	Pressure (kPa)
0	100
25	350
50	600
75	850

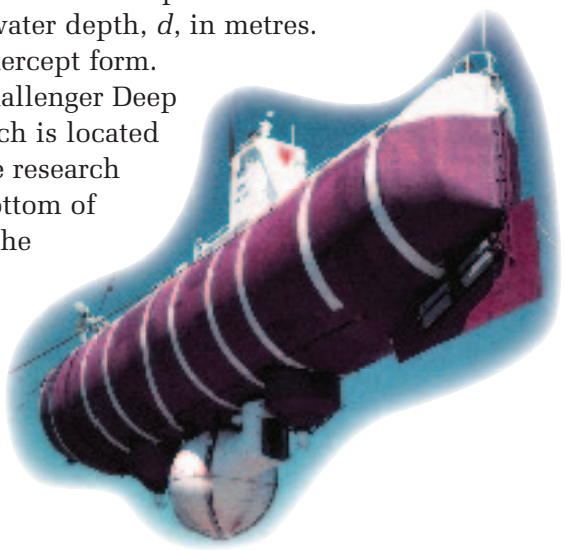
a) Sketch a graph of the data.

b) What is the slope of the line? What does it represent?

c) Determine the value of the **parameter** b. What does this value represent?

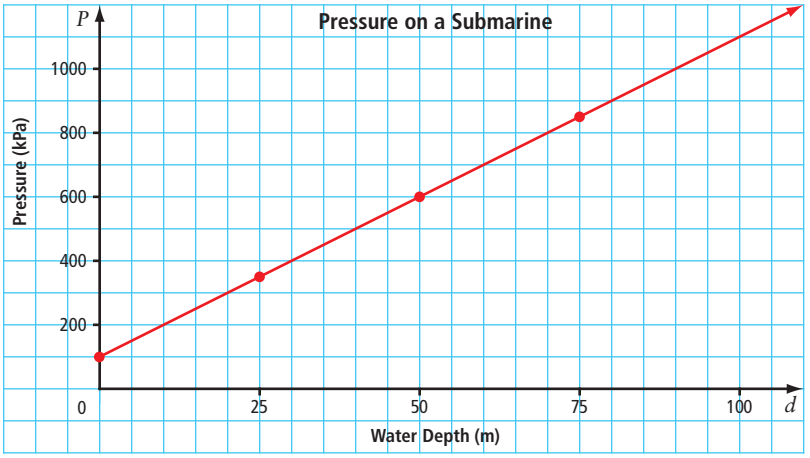
d) Write an equation that models the relationship between pressure, P, in kilopascals, and water depth, d, in metres. Express the equation in slope-intercept form.

e) The deepest point on Earth is Challenger Deep in the Mariana Trench. This trench is located in the Pacific Ocean. In 1960, the research submarine *Trieste* reached the bottom of Challenger Deep. At this depth, the walls protecting the two crew members had to withstand a pressure of 109 300 kPa. What is the approximate depth of Challenger Deep?



Solution

a) The independent variable is depth. The dependent variable is pressure. Plot the coordinate pairs (0, 100), (25, 350), (50, 600), and (75, 850). Both depth and pressure can be any real number, so join the points with a straight line.



b) Determine the ratio of vertical change to horizontal change.

$$m = \frac{\Delta y}{\Delta x}$$

What values could you use to calculate Δy and Δx?

$$m = \frac{250}{25}$$

$$m = 10$$

The slope of the line is 10. This means that for every metre you descend, the pressure increases by 10 kPa.

c) The parameter, b, represents the y-intercept, which is equal to 100. The air pressure is 100 kPa at the surface of the water, where water depth is 0 m.

d) Substitute $m = 10$ and $b = 100$ into $y = mx + b$.

$y = 10x + 100$

If P represents pressure and d represents depth, then the equation of the line is $P = 10d + 100$.

e) Substitute 109 300 for P.

$$P = 10d + 100$$
$$109\,300 = 10d + 100$$
$$109\,200 = 10d$$
$$10\,920 = d$$

The approximate depth of Challenger Deep is 10 920 m.

Your Turn

Asha has selected a hotel for her wedding reception. The cost involves a fee for the deluxe ballroom and a buffet charge that depends on the number of guests. This is shown in the table.

Number of Guests	Cost ($)
0	425
25	1800
50	3175
100	5925

a) Sketch a graph of the data in the table.

b) What are the slope and y-intercept of the line? What does each parameter represent?

c) Write an equation that describes the relationship between the cost and the number of guests. Express the equation in slope-intercept form.

d) What is the cost for 140 guests?

e) Asha would like the total cost to be no more than $15 000. What is the maximum number of guests that can attend?

Example 4 Determine an Unknown Parameter

An archaeologist simulates a First Nations method of boiling water by adding hot rocks to an earthen pit filled with water. As the rocks cool and lose their heat, the archaeologist replaces them with new rocks from the fire. Suppose the water temperature rises at a constant rate. The temperature of the water at the start of the experiment is 10 °C.

Cooking in a Fire Pit by Shayne Tolman
Painting on display at Head-Smashed-In Buffalo Jump Interpretive Centre likely represents 2500-year-old Besant culture.

The equation $W = mt + 10$ models how the temperature of the water, W, in degrees Celsius, increases at a constant rate of m degrees Celsius per minute for t minutes.

a) After 5 min, the water temperature is 19 °C. Determine the value of the parameter m. What does m represent?

b) How long will it take for the water to boil?

Solution

a) Represent a temperature of 19 °C after 5 min as the point (5, 19). Substitute the coordinates of the point (5, 19) into the given equation. Solve for m.

$$W = mt + 10$$
$$19 = m(5) + 10$$
$$19 - 10 = 5m$$
$$9 = 5m$$
$$\frac{9}{5} = m \text{ or } m = 1.8$$

The parameter m represents the rate at which the temperature of the water increases per minute. The water temperature increases at a rate of 1.8 °C/min.

b) Solve the equation for t when W is 100 °C.

What does $W = 100$ °C represent?

$$W = 1.8t + 10$$
$$100 = 1.8t + 10$$
$$90 = 1.8t$$
$$50 = t$$

The water boils after 50 min.

Your Turn

A decorator's fee can be modelled by the equation $F = 75t + b$. In the equation, F represents the fee, in dollars, t represents time, in hours, and b represents the cost of the initial consultation, in dollars.

a) Suppose the decorator spends 4 h working for a client and charges the client $450. Determine the value of the parameter b.

b) How many hours does the decorator work if a client is charged $975?

Did You Know?

To boil water, some First Nations people used to dig a bowl-shaped pit in the ground. Then, they lined the bottom with buffalo hide. They added water and red-hot rocks to the pit, until the water boiled.

Did You Know?

The coordinates of a point on a graph satisfy the equation of the graph. Conversely, a point whose coordinates satisfy an equation is a point on a graph of the equation.

For example,
The point (1, 8) is a point on the graph of $y = 3x + 5$.
Substituting $x = 1$ and $y = 8$.
$8 = 3(1) + 5$
$8 = 8$ True.

Conversely, since the point (1, 8) satisfies the equation, (1, 8) is a point on the graph of the equation $y = 3x + 5$.

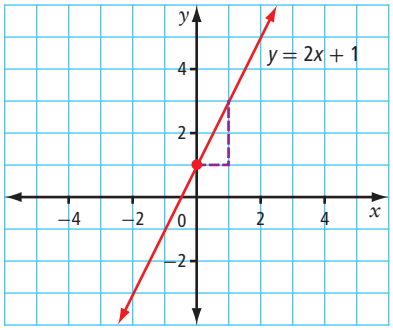
Key Ideas

- The slope-intercept form of a linear equation is $y = mx + b$, where m represents the slope and b represents the y-intercept.

$y = 2x + 1$

slope $= 2$ $\qquad$ y-intercept $= 1$

$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ $\qquad$ The graph passes through $(0, 1)$.



Check Your Understanding

Practise

1. What are the slope and y-intercept of each line?

a) $y = -5x + 4$ $\qquad$ **b)** $y = \frac{3}{4}x + 1$

c) $y = x - 7$ $\qquad$ **d)** $y = -4x$

e) $y = -3$ $\qquad$ **f)** $y = 0.5x - 1.25$

2. Consider the line $y = -3x + 2$.

a) What are the slope and y-intercept of the line?

b) Explain how you could sketch the graph of this line using the slope and y-intercept.

3. Sketch the graph of each line using the slope and y-intercept. Use graphing technology to check your graphs.

a) $y = 2x - 3$ $\qquad$ **b)** $y = -4x + 8$

c) $y = -x + 1$ $\qquad$ **d)** $y = \frac{5}{2}x - 4$

e) $y = -\frac{3}{4}x + 2$ $\qquad$ **f)** $y = 5$

4. Omar determines the slope and y-intercept of the line $3x - 2y - 8 = 0$. His work is shown.

$3x - 2y = 8$ Step 1

$-2y = -3x + 8$ Step 2

$y = \frac{3}{2}x + 4$ Step 3

The slope of the line is $\frac{3}{2}$ and the y-intercept is 4. Step 4

a) In which line did Omar first make an error?

b) Correct Omar's work.

5. Express each equation in slope-intercept form. Write the slope and y-intercept of each line.

a) $2x + y = 6$

b) $3x + y + 9 = 0$

c) $5x + 6y = 8$

d) $6x - y = 4$

e) $7x - y + 9 = 0$

f) $8x - 4y = 3$

6. Write the equation of each line in the form $y = mx + b$.

a) slope $= -3$, y-intercept $= 2$

b) slope $= \frac{5}{6}$, y-intercept $= -4$

c) slope $= -0.75$, y-intercept $= -5$

d) slope $= 1$, y-intercept $= -7$

e) slope $= -1$, y-intercept $= 0$

f) slope $= 0$, y-intercept $= \frac{1}{3}$

7. Use the following equations to answer each question. Justify your answers.

Equation A: $y = 3x - 5$
Equation B: $y = -4x + 1$
Equation C: $y = x + 2$
Equation D: $y = -\frac{1}{2}x$

a) Which lines slope *up* from left to right?

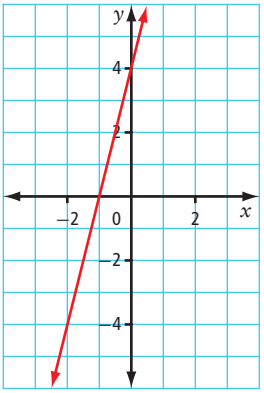
b) Which lines slope *down* from left to right?

c) Arrange the lines from greatest to least y-intercepts.

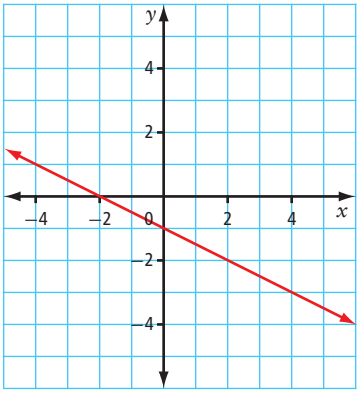
d) Which lines pass through the origin?

8. What are the slope and y-intercept of each line? Write the equation of each line in slope-intercept form.

a)



b)



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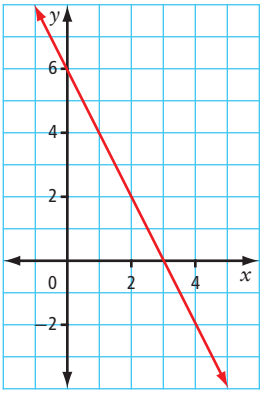
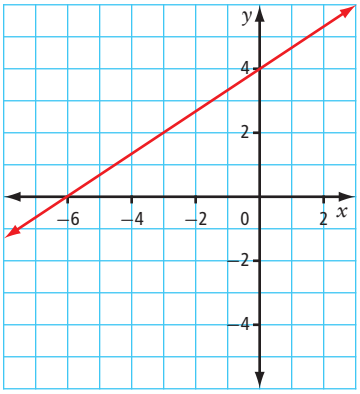
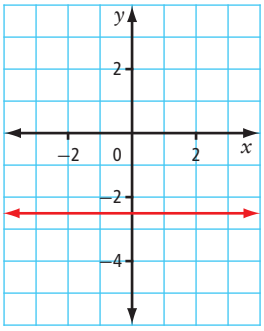


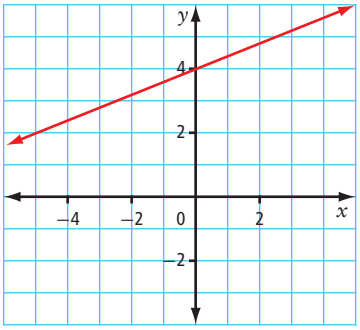
d)



e)



f)



9. Consider the equation $y = 3x + b$. What is each value of b if a graph of the line passes through each point?

a) $(4, 9)$ **b)** $(-4, 6)$

c) $(3, -2)$ **d)** $(-1, -8)$

10. For the equation $y = mx - 2$, what is each value of m if the line passes through each point?

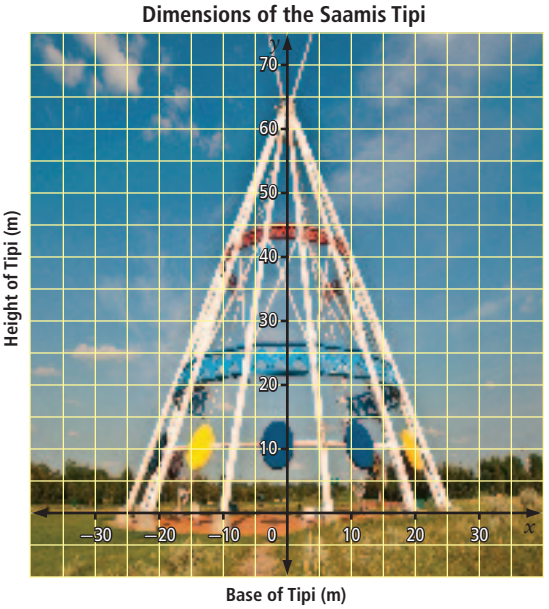
a) $(3, 1)$ **b)** $(-2, 8)$

c) $(4, -8)$ **d)** $(-6, -1)$

Did You Know?

The Saamis Tipi, a stylized structure, is located in Medicine Hat, AB. It was originally constructed for the 1988 Olympics as a tribute to First Nations peoples.

11. A photograph of the Saamis Tipi is shown on a metre grid. Write the equation of each line representing the extreme left post and the extreme right post. Express the equations in slope-intercept form.



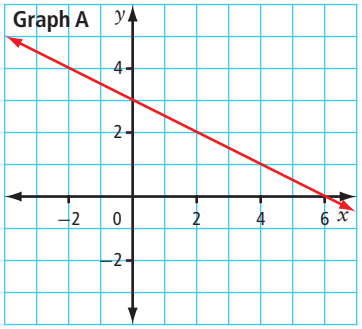
12. State the slope and y-intercept of each equation. Then, identify the graph that matches the equation. Use graphing technology to check your answers.

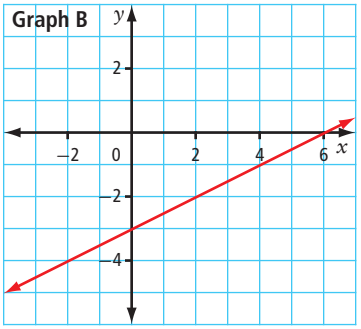
a) $y = -2x + 3$

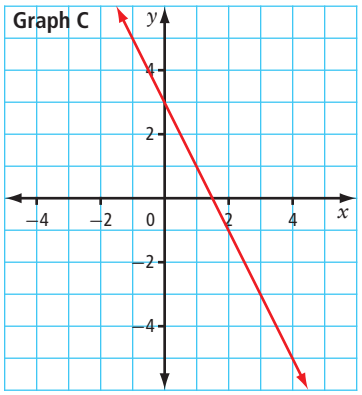
b) $y = 2x - 3$

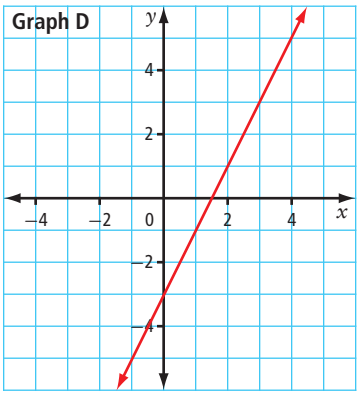
c) $y = \frac{1}{2}x - 3$

d) $y = -\frac{1}{2}x + 3$









13. Write an equation to represent each situation.

a) The cost, C, to take n students to the theatre is \$300 to rent a bus and \$6.25 per ticket.

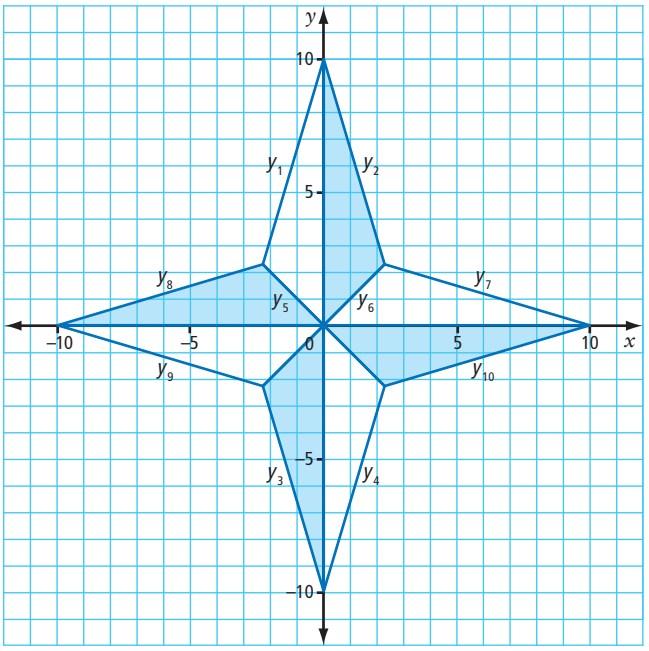
b) The taxi fee, T, is \$3.60 to start plus \$1.48 for each kilometre travelled, x.

c) A rewritable Blu-ray disc has 1024 MB of data stored on it. When new data is added to the disc, the total data, D, in megabytes, stored on the disc at time t seconds increases at a rate of 54 MB/s.

d) A water delivery truck is filling the water tank in Simeonie's house. The truck arrived with 2500 L of water. The number of litres of water, L, remaining in the truck at time t minutes decreases at a rate of 120 L/min.

Apply

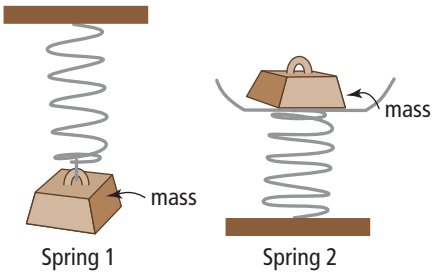
14. At the centre of the North Atlantic Treaty Organization (NATO) emblem is a compass that may be created using the ten line segments, labelled y_1 to y_{10}, in the figure. Work with a partner to determine the equation of each line segment. Express your equations in the form $y = mx + b$. For each equation, do not consider the boundaries of the line segment.



Did You Know?

Canada has been a member of NATO since NATO's formation in 1949. The main goal of NATO is to safeguard the freedom and security of its member countries.

15. A group of students tested how different masses changed the lengths of two different coil springs. The results of their experiments are summarized in the table.



Mass (g)	Spring 1 Length (cm)	Spring 2 Length (cm)
0	8	24
4	14	18
8	20	12
12	26	6

a) For each spring, write an equation to model how spring length, L, in centimetres, changes with mass, x, in grams. Express each equation in slope-intercept form.

b) What does a negative slope represent in the experiment?

Did You Know?

Many animals that roamed Earth at the time of the last glacial period are now extinct. These include the sabre-toothed cat, mammoth, mastodon, and giant beaver, bison, and bear. Other animals that roamed the area that is now known as western Canada during this time include the horse, camel, antelope, and ground sloth.

16. About 12 000 years ago during the last glacial period, giant bison roamed the plains of North America. Using fossil bones, paleontologists can estimate the size of these huge animals. The equation $y = 2.4x - 7.9$ approximates the relationship between an adult male bison's front limb length, y, in centimetres, and the length of its humerus bone, x, in centimetres.

Painting of giant bison by Ludo Beogaet

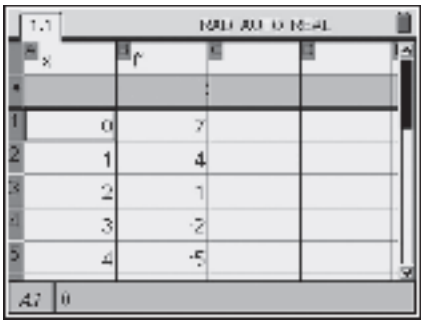
a) Sketch a graph of the equation $y = 2.4x - 7.9$.

b) Use graphing technology to check your graph. Then, estimate, to the nearest centimetre, the front limb length of each bison:

- an extinct giant bison with a fossil humerus bone length of 40.2 cm
- a modern North American bison with a humerus bone length of 32.6 cm

c) By what percent was the giant bison taller than the modern bison?

17. Marge uses graphing technology to graph a line. A table of values for the line is shown. What is the equation of the line? Express the equation in slope-intercept form.



	x	y
1	0	7
2	1	4
3	2	1
4	3	−2
5	4	−5

18. **Unit Project** Exposure to cold weather can cause frostbite and hypothermia. Mountain climbers, sky divers, and other high-altitude enthusiasts must protect their skin because air temperature decreases as altitude increases. This rate of decrease in temperature is nearly constant, up to about 11 000 m.

An airplane taking off in the Yukon Territory recorded the following temperatures.

Altitude (m)	Temperature (°C)
0	12
4000	−13.6
8000	−39.2

Did You Know?

The coldest recorded temperature in North America was −63 °C on February 3, 1947. It was recorded in the village of Snag, YT. Temperatures at higher altitudes can get even colder.

a) Sketch a straight-line graph of the data.

b) What is the slope of the line expressed as a fraction in lowest terms? What does the slope represent?

c) What is the y-intercept? What does it represent?

d) Write an equation that describes the relationship between temperature, T, in degrees Celsius, and altitude, A, in metres.

e) Mount Logan is in Kluane National Park and Reserve, YT. At a height of 5959 m, Mount Logan is the highest mountain in Canada. It is the second highest mountain in North America. Predict the temperature a climber would experience at the peak of Mount Logan on the day that the airplane collected the data. Assume minimal wind.

f) Most people are at risk of frostbite within 10–20 min of exposure to temperatures below −20 °C. Predict the altitudes at which the temperature will be below −20 °C.

19. Write the equation of a line with the same slope as the line $x + 4y + 8 = 0$ and the same y-intercept as the line $2x - 3y + 12 = 0$.

Extend

20. When graphed, the equations $x + 4y = n$ and $5x - 2y = 10$ have the same y-intercept. What is the value of n?

21. A graph of $6x - ny = 8$ and $2x + 3y = 12$ shows two lines that have the same slope. What is the value of n?

22. A line has a y-intercept of 3 and forms an angle of 45° with the x-axis. Write the equation of this line in the form $y = mx + b$.

23. Consider the equation $2x + y = 200$. When graphed, how many points (x, y) on the line will have natural numbers for both the x-coordinate and y-coordinate?

Create Connections

24. Explain how to determine the slope and y-intercept of a line given each representation.

a) a table of (x, y) values

b) a graph of the line

c) an equation of the line in the form $y = mx + b$

25. Explain how you could use the slope and y-intercept of a line to

a) write the equation of the line

b) create a graph of the line

Materials

- graphing technology or grid paper and ruler

26. MINI LAB

Step 1 Use technology to graph each set of equations on the same display. Then, sketch a graph of each set.

a) $y_1 = x + 5$
$y_2 = -2x + 5$
$y_3 = 3x + 5$

b) $y_1 = x - 1$
$y_2 = 2x - 2$
$y_3 = -3x + 3$

c) $y_1 = -x + 2$
$y_2 = -x + 4$
$y_3 = -x + 6$

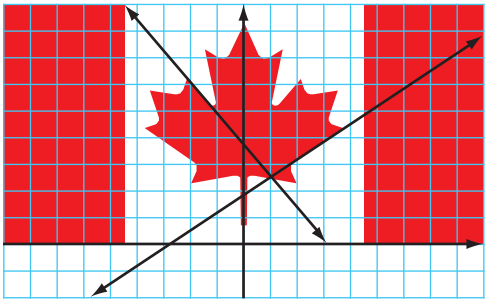
Step 2 Describe the similarities and differences between each set of lines. Lines that share at least one characteristic are called a *family of lines*. Why might each set be considered a family of lines?

Step 3 Write the equation of another line that belongs to each family. Use graphing technology to check your answers.

Step 4 Create your own family of lines. Write the equations of the lines in slope-intercept form. Share with a partner and describe each other's family of lines.

7.2 General Form

The slope-intercept form of an equation is one of the most common ways to write the equation of a line. Suppose you place a transparent grid on the Canadian flag. Most, but not all, of the line segments in the flag could be expressed as equations in slope-intercept form. Can you think of a type of line that cannot be expressed in the form $y = mx + b$? In this section, you will explore a general way of writing the equation of any line.



Focus on ...

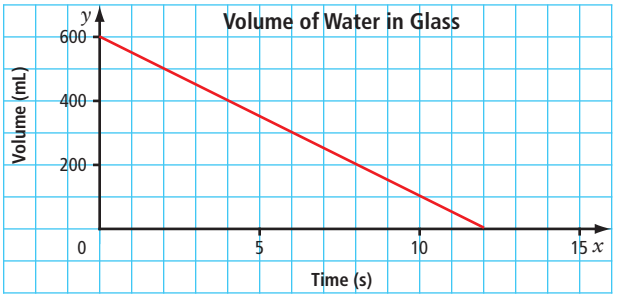
- converting a linear equation to general form
- using intercepts to graph a line
- relating the intercepts of a graph to the situation
- solving problems using equations in general form

Materials

- bottle of water
- stopwatch
- grid paper and ruler or graphing technology

Investigate Intercepts and General Form

Leora quenches her thirst after a soccer game by drinking a large glass of water at a constant rate. The straight-line graph on the following page shows how the volume of water in the glass changes with time.



1. Identify the domain and range in this situation.
2. Identify the slope of the line segment. What does the sign of the slope mean? What does the slope represent?
3. What is the y-intercept of the line segment? What does the y-intercept represent?
4. **a)** What is the equation of the line in slope-intercept form?
   b) Rearrange the terms in the equation so that the right side is zero. This is the **general form** of an equation.

general form
- the equation of a line in the form $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number. This means that A will always be positive.

5. **a)** Identify the **x-intercept** of the line. What does the x-intercept represent?
   b) Write the coordinates of the x-intercept and the y-intercept.

x-intercept
- the x-coordinate of the point where a line or curve crosses the x-axis
- the value of x when $y = 0$

6. Time yourself while drinking a 500-mL bottle of water slowly at a constant rate.
   a) Sketch a straight-line graph to show how the volume of water remaining in the bottle changes with time.
   b) Compare your graph to Leora's graph. How can you tell who drank more, who finished first, and who drank at a faster rate?
   c) Write the equation of your graph in both slope-intercept form and general form.
7. **Reflect and Respond**
   a) Recall that a point on a line satisfies the equation of the line. Are the x-intercept and y-intercept points on the line?
   b) When asked to determine the x-intercept, what coordinate value do you always know? When asked to determine the y-intercept, what coordinate value do you always know? How can you use this information to help you determine the x-intercept or y-intercept?
   c) Describe strategies for determining the y-intercept and x-intercept of a line if the equation is given in general form.
   d) Create two linear equations. Then, identify the y-intercept and x-intercept of each line. Use graphing technology to check your answers.
8. What form is the equation $x - 5 = 0$ expressed in? Can you write the equation in another form? Explain.

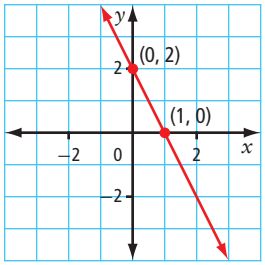
Link the Ideas

The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.

You can convert a linear equation from one form to another by applying the rules of algebra.

The x-intercept of a line is the x-coordinate of the point where the line crosses the x-axis. The y-intercept is the y-coordinate of the point where the line crosses the y-axis. To sketch a linear equation, you can draw a line joining the intercepts, $(x, 0)$ and $(0, y)$.

The line in the graph below has an x-intercept of 1 and a y-intercept of 2.



Example 1 Convert an Equation to General Form

Rewrite the equation $y = -\frac{2}{3}x + 6$ in general form, $Ax + By + C = 0$.

Solution

$$y = -\frac{2}{3}x + 6$$

$$3(y) = 3\left(-\frac{2}{3}x + 6\right)$$ Why are both sides multiplied by 3?

$$3y = \overset{1}{\cancel{3}}\left(-\frac{2}{\underset{1}{\cancel{3}}}x\right) + 3(6)$$

$$3y = -2x + 18$$

$$2x + 3y - 18 = 0$$

The equation written in general form is $2x + 3y - 18 = 0$.

Your Turn

Rewrite the equation $y = \frac{3}{4}x - 2$ in general form.

Example 2 Sketch a Graph Using Intercepts

For the linear equation $2x - 3y - 6 = 0$,

a) state the x-intercept of a graph of the equation

b) state the y-intercept

c) use the intercepts to graph the line

Solution

a) To determine the x-intercept, substitute $y = 0$. Then, solve for x.

$$\begin{aligned} 2x - 3y - 6 &= 0 \\ 2x - 3(0) - 6 &= 0 \\ 2x - 6 &= 0 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Why do you substitute $x = 0$ to find the y-intercept?

The x-intercept is 3. So, the line crosses the x-axis at the point $(3, 0)$.

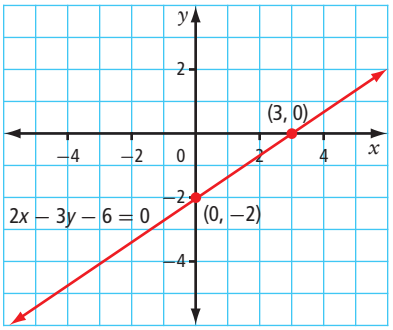
b) To determine the y-intercept, substitute $x = 0$. Solve for y.

$$\begin{aligned} 2x - 3y - 6 &= 0 \\ 2(0) - 3y - 6 &= 0 \\ -3y - 6 &= 0 \\ -6 &= 0 + 3y \\ \frac{-6}{3} &= y \\ -2 &= y \end{aligned}$$

Why do you substitute $y = 0$ to find the x-intercept?

The y-intercept is -2. The line crosses the y-axis at the point $(0, -2)$.

c) Locate the points $(3, 0)$ and $(0, -2)$ on the grid. Then, draw a line passing through these points.



How else could you graph an equation given in general form?

Your Turn

Consider the linear equation $4x + 5y - 20 = 0$.

a) What is the x-intercept of a graph of the equation?

b) What is the y-intercept?

c) Use the intercepts to graph the line.

Example 3 Identify Intercepts of Horizontal or Vertical Lines

Sketch each linear relation and identify the intercepts.
Then, state the domain and range.

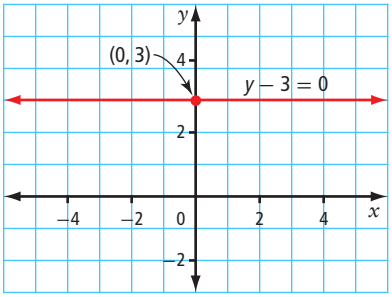
a) $y - 3 = 0$

b) $x + 4.5 = 0$

c) $y = 0$

Solution

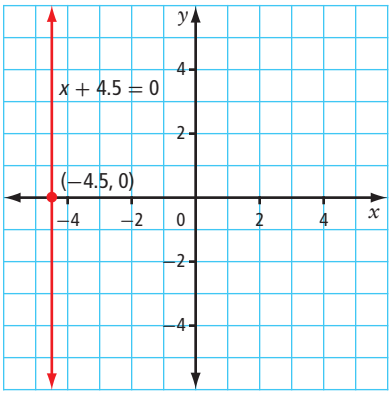
a) The equation $y - 3 = 0$ can be written in slope-intercept form as $y = 0x + 3$.
The graph is a horizontal line with slope zero.
The line crosses the y-axis at the point $(0, 3)$.



What would the values in a table representing this equation show?

The y-intercept is 3. There is no x-intercept.
The domain of the line $y - 3 = 0$ is $\{x \in \mathrm{R}\}$.
The range of the line is $\{3\}$.

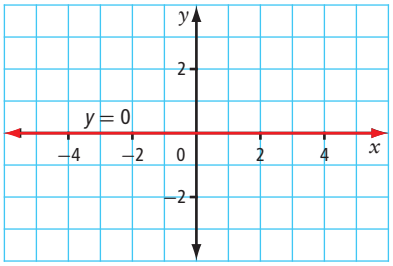
b) The equation $x + 4.5 = 0$ expressed in general form is $x + 0y + 4.5 = 0$. The coefficient of y is zero.
The value of x is always -4.5.
The graph is a vertical line and crosses the x-axis at $(-4.5, 0)$.



The x-intercept is -4.5. There is no y-intercept.
The domain of the line $x + 4.5 = 0$ is $\{-4.5\}$.
The range of the line is $\{y \in \mathrm{R}\}$.

c) The equation $y = 0$ is a horizontal line, which represents the entire x-axis. The graph always intersects the x-axis. Therefore, there are an infinite number of x-intercepts. The y-intercept is 0.

The domain of the line $y = 0$ is $\{x \in \mathrm{R}\}$.
The range of the line is $\{0\}$.



What do the coordinates of points on this line have in common?

Your Turn

Sketch each linear relation and identify the intercepts. What are the domain and range for each relation?

a) $x - 3 = 0$

b) $x = 0$

c) $y + 2 = 0$

Example 4 Interpret Intercepts

Spencer has 66 GB of disk space left on his laptop to fill with television shows and movies that he purchases on-line.

a) Suppose a one-hour show uses 1.1 GB of disk space and a movie uses 4.4 GB. Write a linear equation that represents the number of television shows, T, and movies, M, that Spencer can store on his laptop.

b) Determine the T-intercept of a graph of the linear equation. What does the T-intercept represent?

c) What would the M-intercept be? What does the M-intercept represent?

d) If Spencer stores 16 television shows, how many movies does he have space for?

Solution

a) The equation is $1.1T + 4.4M = 66$.

What does each term represent?

Simplify the equation.

Write the coefficient of T as a whole number.

$11T + 44M = 660$

How else could you reduce the original equation to lowest terms with the coefficient of T as a whole number?

Reduce the equation to lowest terms.

$T + 4M = 60$

The equation $T + 4M = 60$ represents the number of television shows and movies that Spencer can store on his laptop.

b) To determine the T-intercept, substitute $M = 0$. Solve for T.

$$T + 4M = 60$$
$$T + 4(0) = 60$$
$$T = 60$$

The T-intercept is 60. So, if Spencer stores no movies, he can store 60 television shows.

c) To determine the M-intercept, substitute $T = 0$. Solve for M.

$$T + 4M = 60$$
$$0 + 4M = 60$$
$$M = 15$$

The M-intercept is 15. So, Spencer can store 15 movies if he does not store any television shows.

d) Substitute $T = 16$ and solve for M.

$$T + 4M = 60$$
$$16 + 4M = 60$$
$$4M = 44$$
$$M = 11$$

Spencer has space on his laptop for 11 movies.

Your Turn

Brooke wants to save \$336 to decorate her bedroom. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns \$12 per hour. On weeknights, she earns \$16 per hour working as a high-school tutor.

a) Write an equation to represent the number of hours Brooke needs to work as a snowboard instructor, S, and as a tutor, T.

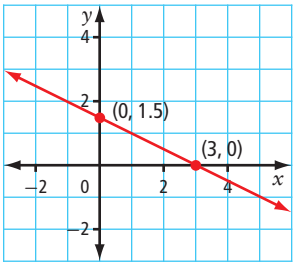
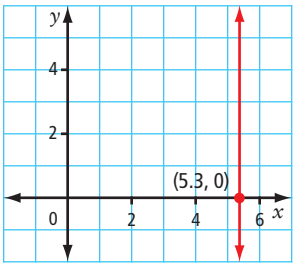
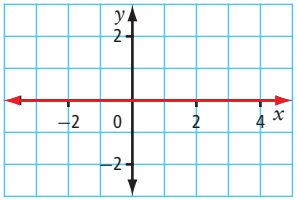
b) What is the S-intercept of a graph of the equation? What does the S-intercept represent?

c) What would the T-intercept be? What does it represent?

d) Suppose Brooke works 8 h as a snowboard instructor. How many hours will she need to work as a tutor?

Key Ideas

- The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, then draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x-intercept, substitute $y = 0$ and solve. To determine the y-intercept, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two, or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.

Equation	x-Intercept(s)	y-Intercept(s)	Graph
$x + 2y - 3 = 0$	$x + 2y - 3 = 0$ $x + 2(0) - 3 = 0$ $x = 3$	$x + 2y - 3 = 0$ $(0) + 2y - 3 = 0$ $2y = 3$ $y = 1.5$	
$x = 5.3$	$x = 5.3$	no y-intercept	
$3y = 0$	infinite number of x-intercepts	$3y = 0$ $y = 0$	

Check Your Understanding

Practise

1. Jasmine was asked to convert the equation $y = -\frac{3}{2}x + 4$ to general form. Her work is shown.

$$y = -\frac{3}{2}x + 4$$
$$2y = -3x + 4$$
$$3x + 2y - 4 = 0$$

Identify Jasmine's error. Then, correct her work.

2. Express each equation in general form, $Ax + By + C = 0$.

a) $y = 7x - 5$

b) $y = -x + 8$

c) $y = \frac{3}{2}x + 4$

d) $y = -\frac{3}{5}x - 2$

e) $y = 0.25x - 0.3$

f) $y = -\frac{5}{2}x + \frac{1}{8}$

3. Determine the intercepts of each line. Then, graph the line.

a) $2x + y - 9 = 0$

b) $4x - y - 8 = 0$

c) $x - 2y + 10 = 0$

d) $3x - 8y - 24 = 0$

e) $4x + 5y + 6 = 0$

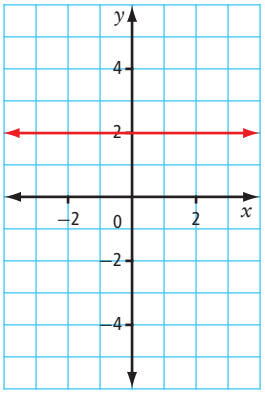
f) $x = 4$

g) $y = 0$

h) $4x - 12 = 0$

4. For each line, state the domain, range, intercepts, and slope. What is the equation of each line, in general form?

a)



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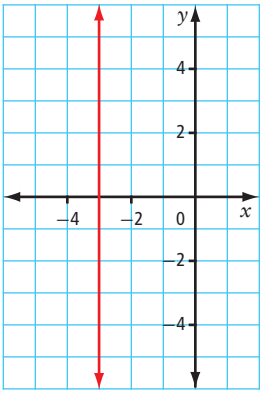


5. Graph each line using the given intercepts. What is the equation of each line?

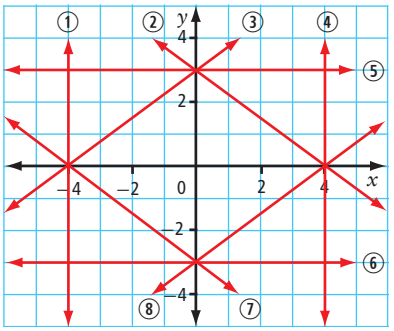
a) an x-intercept of 3 and no y-intercept

b) a y-intercept of -5 and no x-intercept

c) an infinite number of x-intercepts

d) an infinite number of y-intercepts

6. Match each equation with a line labelled in the figure.



a) $3x + 4y = 12$

b) $x = 4$

c) $3x - 4y = 12$

d) $y - 3 = 0$

e) $3x - 4y + 12 = 0$

f) $y = -3$

g) $3x + 4y + 12 = 0$

h) $x + 4 = 0$

Apply

7. Write an equation, in general form, for each line described.

a) a vertical line passing through the point (3, 5)

b) a horizontal line passing through the point $(-2, 6)$

c) the x-axis

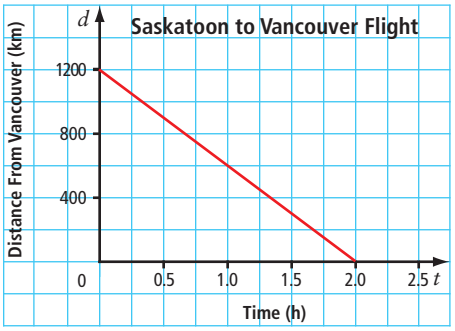
d) the y-axis

8. Write an equation, in general form, of a line that does not have a y-intercept and passes through the point (3, 6).

9. Having fibre in your diet helps with digestion, heart health, and maintaining a healthy weight. Courtney wants to increase her fibre intake by 21 g per day. She plans to do this by adding bran buds and one vegetable to her diet each day. Write an equation in general form that describes this situation. Courtney mixes 125 mL of bran buds in her cereal each day. What volume of green peas would she need to eat during the day? What volume of baked beans would she need to eat? Express your answers in whole millilitres.

Food	Grams of Fibre per 125 mL
Green peas	4
Baked beans	7
Bran buds	16

10. An airplane flies directly from Saskatoon, SK, to Vancouver, BC. The graph shows the relationship between the distance from Vancouver, d, in kilometres, and the flying time, t, in hours.



a) State the intercepts of the line segment. What does each intercept represent?

b) State a suitable domain and range of the graph.

c) Determine the slope of the line. What does the slope represent?

d) Write the equation of the line in general form.

e) For how many hours has the plane been flying when it is 200 km from Vancouver?

f) What is the distance from Vancouver when the plane has been flying for 45 min?

11. Luc swims as part of an active and healthy lifestyle. The number of calories burned by a swimmer of Luc's body weight is shown in the table.

Swimming Style	Calories Burned Per Minute
Backstroke	8
Butterfly	11

a) Write a linear equation to describe the number of minutes Luc would need to swim backstroke, x, and butterfly, y, to burn 440 cal.

b) What are the intercepts of the line? What does each intercept represent?

c) Suppose Luc swims butterfly for 16 min. How long will he need to swim backstroke in order to burn 440 cal in total?

12. Sanding trucks spread a mixture of sand and salt on roads to improve traction in winter. The density of the salt is 1200 kg/m^3. The density of the sand is 1800 kg/m^3.

a) Write a linear equation to represent the volume, in cubic metres, of salt, x, and of sand, y, in a mixture with a mass of 10 000 kg.

b) For temperatures below -12 °C, the volume of sand in 10 000 kg of the mixture is 5.22 m^3. What is the volume of salt in the mixture?

c) By mass, what percent of the mixture is salt?

13. Advance tickets for a local concert sold for $8 each. Tickets at the door were $12 each. The revenue from ticket sales was $1120.

a) Write a linear equation relating the number of advance tickets, a, to the number of tickets sold at the door, d. Express your equation in general form.

b) Describe the steps you would follow to graph this equation. Do not graph.

c) Suppose twice as many advance tickets were sold as tickets at the door. How many of each type of ticket were sold?

14. What is the value of the unknown parameter in each equation?

a) $Ax + 5y - 6 = 0$, passing through $(-3, 2)$

b) $2x + By + 7 = 0$, passing through $(4, -5)$

c) $4x - 3y + C = 0$, passing through $(-2, -6)$

Extend

15. The equation of a line is $x + 3y - 24 = 0$. Write the coordinates of a point on the line for each of the following conditions.

a) The x-coordinate is equal to the y-coordinate.

b) The x-coordinate is three times as great as the y-coordinate.

c) The y-coordinate is four greater than the x-coordinate.

16. The equation $6x + By + 5 = 0$ describes a line with a slope of $\frac{3}{2}$. What is the value of B?

17. What is the area of the triangle bounded by each set of lines?

a) the line $x + 2y = 10$, a line with an infinite number of x-intercepts, and a line with an infinite number of y-intercepts

b) a line with an infinite number of x-intercepts, the line $2x - y = 6$, and the line $x = 10$

Create Connections

18. **a)** Which form of a linear equation do you prefer to graph: $y = mx + b$ or $Ax + By + C = 0$? Why?

b) Describe a situation when you might work with a linear equation in the other form.

19. The equation of a line is $2x + y - 8 = 0$.

a) Explain how you could determine the x-intercept.

b) Explain two different ways to determine the y-intercept.

20. The general form of an equation is $Ax + By + C = 0$. Identify each line as horizontal, vertical, or oblique.

a) $A \neq 0$, $B = 0$, and C is a real number.

b) $A = 0$, $B \neq 0$, and C is a real number.

c) $A \neq 0$, $B \neq 0$, and C is a real number.

21. **MINI LAB** Explore the effects of changing parameters on a graph of $Ax + By + C = 0$.

Materials
- graphing technology or grid paper and ruler

Step 1 For each group of equations, graph the three linear relations on the same axes. Use graphing technology or sketch the graphs by hand.

a) $x + y - 2 = 0$
$x + y - 6 = 0$
$x + y - 8 = 0$

b) $x + 2y - 6 = 0$
$2x + 2y - 6 = 0$
$3x + 2y - 6 = 0$

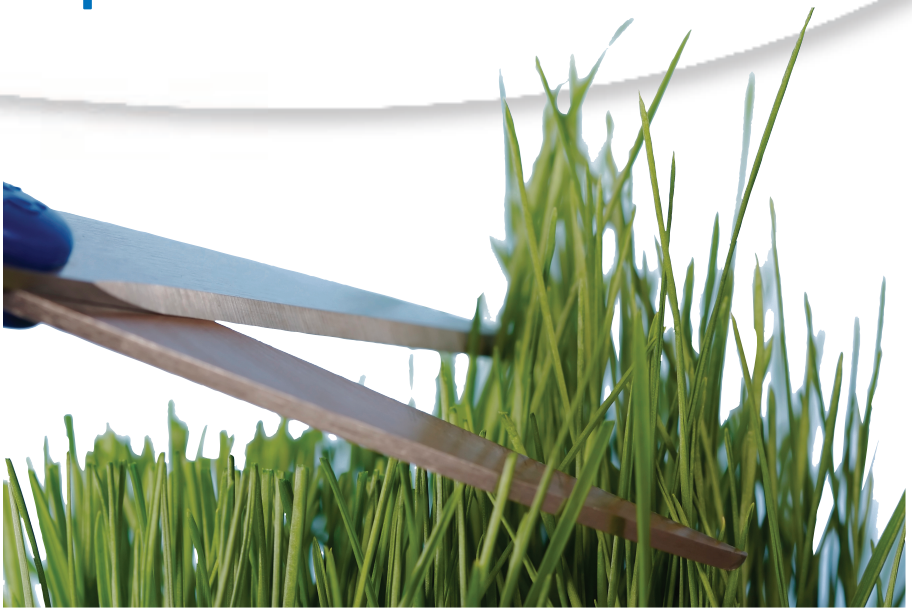
c) $2x + y - 6 = 0$
$2x + 2y - 6 = 0$
$2x + 3y - 6 = 0$

d) $x + y - 2 = 0$
$2x + 2y - 4 = 0$
$3x + 3y - 6 = 0$

Step 2 Compare the three graphs in each group of equations. What parameter(s) have changed? Explain how changing one or two parameters affects the slope, x-intercept, and y-intercept of the graph.

Step 3 How does changing the parameters A, B, and C affect the graph of a linear equation in general form?

7.3 Slope-Point Form



Focus on ...

- writing the equation of a line from its slope and a point on the line
- converting equations among the various forms
- writing the equation of a line from two points on the line
- solving problems involving equations in slope-point form

You can measure the length of an arena with a ruler, use scissors to cut your grass, or loosen a screw with a paperclip. All of these are possible, but are they using the best tool for the job? In mathematics and in life, using an inappropriate tool may prevent you from completing your task or make it take longer to finish. You have explored tools for writing linear relations in two forms. This section introduces a third form, slope-point form. Each form is best suited to certain situations.

Materials

- grid paper
- ruler

Investigate Equations in Slope-Point Form

1. Square ABCD in Figure 1 is a composite of four different polygons. The lengths of the sides are shown. What is the area of square ABCD?

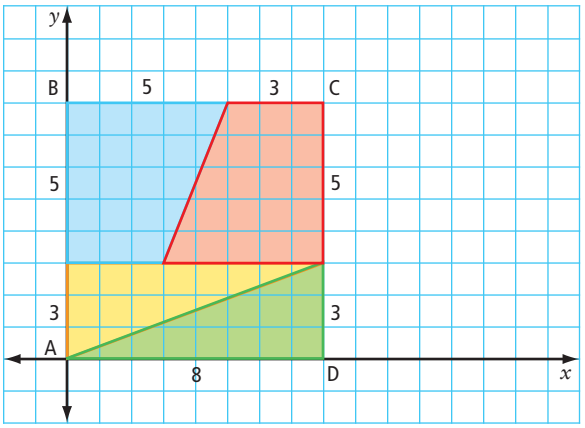


Figure 1

2. Square ABCD is reassembled to form rectangle EFGH, shown in Figure 2. What is the area of rectangle EFGH?

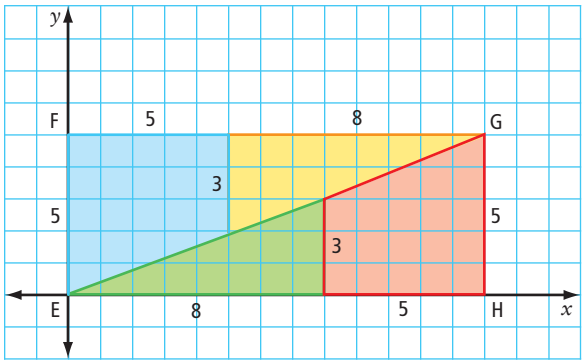


Figure 2

3. There is a discrepancy between the areas of the quadrilaterals shown in Figures 1 and 2. How is Figure 2 deceiving? Justify your answer.

4. On grid paper, draw a line that does not pass through the origin. Label points J, K, and L on the line. Determine the slope of your line.

a) Determine the equation of your line using point J and the slope-intercept form, $y = mx + b$.

b) Use points K and L to determine equations of your line. Compare your equations.

5. Let $P(x_1, y_1)$ represent a point on a line. Develop an equation of the line with slope m using point P.

If you know one point on a line, how can you use the slope to determine a second point?

6. Work with a partner. Have your partner test the equation you developed using his or her line from step 4.

7. **Reflect and Respond** Describe how to determine the equation of a line using the slope and a point on the line.

8. Show how the **slope-point form** of a linear equation can be developed by using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope-point form

- the equation of a non-vertical line in the form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a point on the line

9. What type of line cannot be written in slope-point form? Why?

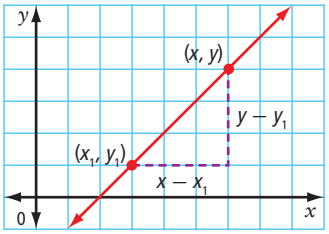
10. Is the following statement always true, sometimes true, or never true? Explain. "To determine the equation of a non-vertical line in slope-point form, you can use the coordinates of any point on the line."

Link the Ideas

The slope of a non-vertical line can be determined using $m = \frac{\Delta y}{\Delta x}$.

If (x_1, y_1) is one point on the line, then (x, y) could represent any other point on the line. Substitute the coordinates of these two points into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line could be written as $m = \frac{y - y_1}{x - x_1}$.



Multiplying both sides of the above equation by $(x - x_1)$ gives

$$(x - x_1)m = (x - x_1)\left(\frac{y - y_1}{x - x_1}\right)$$

$$(x - x_1)m = \cancel{(x - x_1)}^{1}\left(\frac{y - y_1}{\cancel{x - x_1}_{1}}\right)$$

$$m(x - x_1) = y - y_1$$

This equation is called the slope-point form of a non-vertical line through point (x_1, y_1) with slope m.

The slope-point form is commonly written as $y - y_1 = m(x - x_1)$.

Example 1 Write the Equation of a Line Using a Point and the Slope

a) Use slope-point form to write an equation of the line through $(-2, 5)$ with slope -3.

b) Express the equation in slope-intercept form, $y = mx + b$.

c) Graph the linear relation using technology.

Solution

a) Substitute -3 for m and the coordinates of the point $(-2, 5)$ for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$
$$y - (5) = -3(x - (-2))$$
$$y - 5 = -3(x + 2)$$

The equation in slope-point form is $y - 5 = -3(x + 2)$.

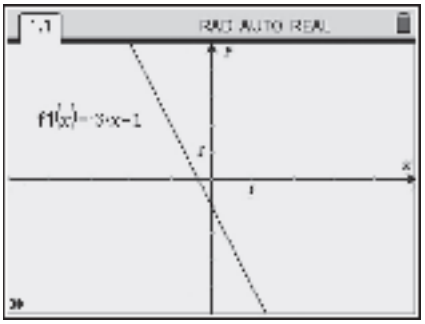
b) To express the equation in slope-intercept form, isolate y.

$$y - 5 = -3(x + 2)$$
$$y = -3(x + 2) + 5$$
$$y = -3x - 6 + 5$$
$$y = -3x - 1$$

In slope-intercept form, the equation is $y = -3x - 1$.

The equation $y = -3x - 1$ is written in the form $y = mx + b$. So, the slope is -3. This is consistent with the value given in the question.

c)



What strategies could you use to sketch the graph?

Your Turn

a) Use slope-point form to write an equation of the line through $(3, -4)$ with slope 2. Sketch a graph of the line.

b) Express the equation in slope-intercept form, $y = mx + b$. Sketch a graph of this line.

c) Compare your graphs.

Example 2 Determine the Equation of a Line Using Two Points

a) Use slope-point form to write an equation of the line through $(3, -4)$ and $(5, -1)$.

b) Sketch a graph of the line.

c) Rewrite the equation in general form, $Ax + By + C = 0$.

Solution

a) Points on the line are given. So, you need to determine the slope. Use the two given points, $(3, -4)$ and $(5, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-1 - (-4)}{5 - 3}$$
$$m = \frac{-1 + 4}{5 - 3}$$
$$m = \frac{3}{2}$$

In slope-point form, substitute $\frac{3}{2}$ for m and the coordinates of either point $(3, -4)$ or $(5, -1)$ for (x_1, y_1).

Using $(3, -4)$ for (x_1, y_1),

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{3}{2}(x - 3)$$
$$y + 4 = \frac{3}{2}(x - 3)$$

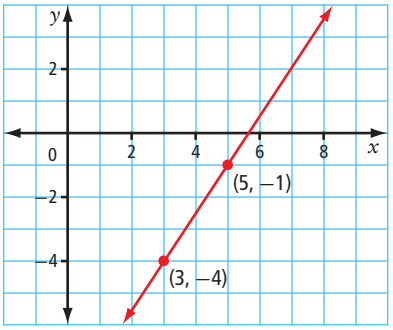
Using $(5, -1)$ for (x_1, y_1),

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{3}{2}(x - 5)$$
$$y + 1 = \frac{3}{2}(x - 5)$$

How can you verify that these equations are equivalent?

Both $y + 4 = \frac{3}{2}(x - 3)$ and $y + 1 = \frac{3}{2}(x - 5)$ are slope-point forms of the equation of the line through $(3, -4)$ and $(5, -1)$.

b)



c) Express $y + 4 = \frac{3}{2}(x - 3)$ in general form.

$$y + 4 = \frac{3}{2}(x - 3)$$
$$2(y + 4) = \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}(x - 3)\right)$$
$$2y + 8 = 3(x - 3)$$
$$2y + 8 = 3x - 9$$
$$0 = 3x - 9 - 2y - 8$$
$$0 = 3x - 2y - 17$$

Express $y + 1 = \frac{3}{2}(x - 5)$ in general form.

$$y + 1 = \frac{3}{2}(x - 5)$$
$$2(y + 1) = \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}(x - 5)\right)$$
$$2(y + 1) = 3(x - 5)$$
$$2y + 2 = 3x - 15$$
$$0 = 3x - 2y - 17$$

The equation, in general form, for the line through $(3, -4)$ and $(5, -1)$ is $3x - 2y - 17 = 0$.

Your Turn

Use slope-point form to write an equation of the line through $(-5, 2)$ and $(-2, 1)$. Explain your steps. Then, write the equation in general form, $Ax + By + C = 0$.

Example 3 Model a Real-Life Situation

Brad Zdanivsky is enthusiastic about mountain climbing. He is a quadriplegic and used custom gear as he climbed the Stawamus Chief in Squamish, BC, on July 31, 2005.

Supposed he moved at a constant rate and climbed the 660-m summit in 11 pitches (sections). Each pitch was approximately 60 m in height. At 5:45 a.m., Brad started his climb 60 m below the top of his first pitch. By 5:55 a.m., he was 40 m below the top of the first pitch.

a) Write an equation that describes Brad's distance, d, in metres, below the top of the first pitch in terms of t minutes past 5:45 a.m. Express the equation in $y = mx + b$ form.

b) How long did it take Brad to reach the top of the first pitch?

c) In total, Brad spent 8.5 h changing ropes between pitches. How long did it take Brad to climb the Stawamus Chief?

WWW Web Link

To learn more about mountain climbing, go to www.mhrmath10.ca and follow the links.

Solution

a) Brad was 60 m below the top of his first pitch at 0 min past 5:45 a.m. After 10 min, he was 40 m below the top of his first pitch. As coordinate pairs (t, d), the data may be represented as (0, 60) and (10, 40). Use these points to determine the slope of the line.

$$m = \frac{d_2 - d_1}{t_2 - t_1}$$

$$m = \frac{60 - 40}{0 - 10}$$

$$m = -2$$

$$m = \frac{60 - 40}{0 - 10}$$

$$m = \frac{20}{-10}$$

$$m = -2$$

Brad's distance to the top of the pitch was decreasing at a rate of 2 m/min.

Substitute the slope, -2, and the coordinates of either point (0, 60) or (10, 40) into the slope-point form of an equation.

Using point (0, 60),

$$d - d_1 = m(t - t_1)$$

$$d - 60 = -2(t - 0)$$

$$d - 60 = -2t$$

$$d = -2t + 60$$

How could you verify your equation?

In slope-intercept form, the equation $d = -2t + 60$ represents Brad's distance below the first pitch.

b) At the top of the first pitch, $d = 0$. Determine t.

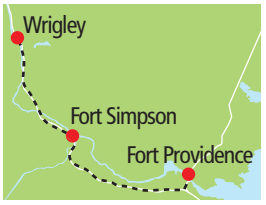
$$d = -2t + 60$$
$$0 = -2t + 60$$
$$2t = 60$$
$$t = 30$$

It took Brad 30 min or 0.5 h to reach the top of the first pitch.

c) To climb the 11 pitches, it took Brad 11(0.5 h) = 5.5 h. Adding 8.5 h to change ropes, it took Brad 14 h to climb the Stawamus Chief.

Your Turn

A family drives at a constant speed from Wrigley, NT, to visit relatives in Fort Providence, NT. When they start driving at 9:00 a.m., they are 540 km from Fort Providence. At 12:30 p.m., they reach Fort Simpson, located 225 km from Fort Providence.



a) Write an equation that describes their distance, d, in kilometres, from Fort Providence in terms of t hours past 9:00 a.m.

b) What time will the family reach Fort Providence?

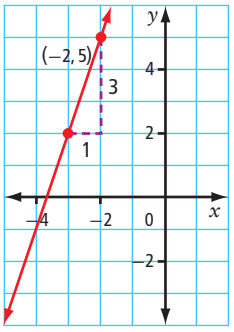
Key Ideas

- For a non-vertical line through the point (x_1, y_1) with slope m, the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$.

A line through $(-2, 5)$ has a slope of 3.

The slope-point form of the equation of this line is

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x - (-2))$$
$$y - 5 = 3(x + 2)$$



- An equation written in slope-point form can be converted to either slope-intercept form or general form.
- Any point on a line can be used when determining the equation of the line in slope-point form.

Check Your Understanding

Practise

1. Rewrite each equation from slope-point form to slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$.

a) $y + 3 = x - 5$

b) $y + 4 = 2(x + 3)$

c) $y - 6 = 4(x + 1)$

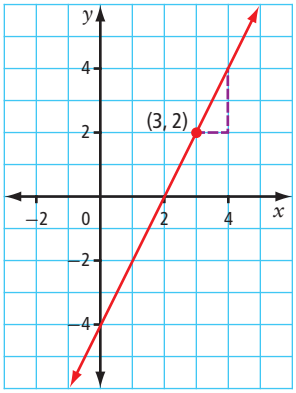
d) $y + 2 = -5(x + 3)$

e) $y - 3 = -\frac{1}{2}(x + 8)$

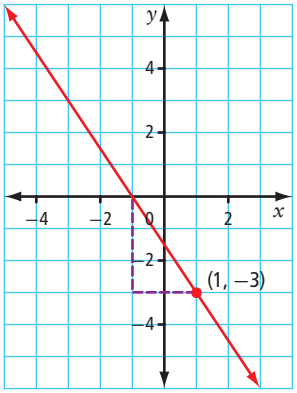
f) $y + 9 = -\frac{2}{3}(x - 6)$

2. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of each line passing through the given point.

a)



b)



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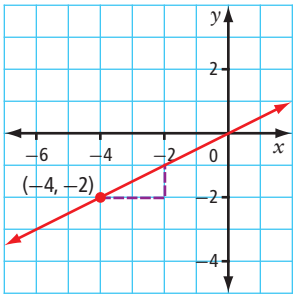

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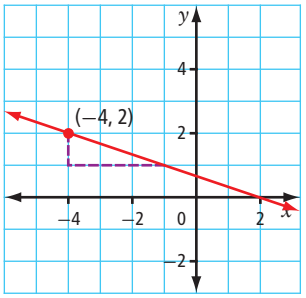


3. Determine the equation of each line using slope-point form. Then, express each equation in slope-intercept form and in general form.

a) $(5, -2)$, $m = 6$

b) $(-3, -5)$, $m = -2$

c) $(-8, 3)$, $m = \frac{1}{2}$

d) $(12, -6)$, $m = -\frac{2}{3}$

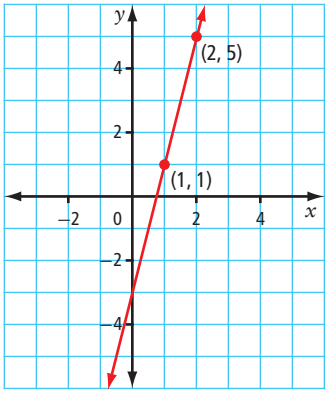
4. Consider the line represented by $y - 1 = \frac{2}{3}(x - 6)$.

a) Identify the slope and a point on the line.

b) Explain how you could sketch the graph of the line using the slope and a point on the line.

5. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of the line passing through the given points.

a)



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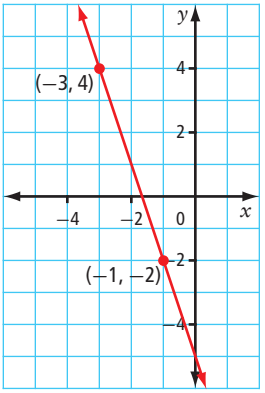

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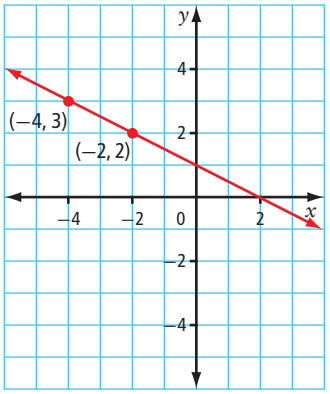

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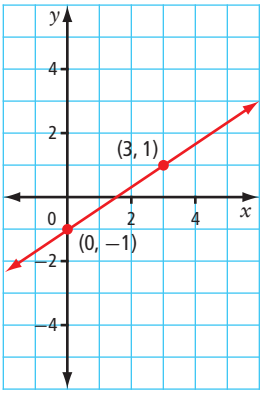


6. Use slope-point form to write an equation of a line through each pair of points. Express each equation in the form $y = mx + b$ and in the form $Ax + By + C = 0$.

a) (5, 1) and (3, −7)

b) (5, −8) and (1, 4)

c) (4, 5) and (2, 6)

d) (8, −3) and (4, −6)

e) (5, −1) and (3, −4)

f) (3, 6) and (−1, 0)

7. Terry's teacher writes the following on the board:

The four equations listed represent only two different lines.
Which equations represent the same line?

① $y - 2 = 3(x + 1)$
② $y - 10 = 3(x - 4)$
③ $y + 5 = 3(x + 1)$
④ $y - 11 = 3(x - 2)$

a) Describe possible strategies students could use to answer the question.

b) Which equations represent the same line? Justify your answers.

Apply

8. Identify the slope and a point on each line. Sketch a graph of each line. Use graphing technology to check your graphs.

a) $y - 3 = 2(x - 1)$

b) $y + 2 = -3x$

c) $y - 4 = \frac{1}{2}(x + 1)$

d) $y + 6 = -\frac{4}{5}(x - 2)$

9. Consider the line passing through the points $(-4, 2)$ and $(-2, 6)$.

a) Work with a partner to develop at least two different strategies for determining the y-intercept of the line.

b) What is the y-intercept of the line?

10. A line passes through $(0, 1)$ and $(3, 7)$.

a) Using only slope-intercept form, $y = mx + b$, write the equation of this line.

b) Determine the equation of the line using only slope-point form.

c) Compare the two equations graphically.

11. Write the equation of each line using slope-point form. Then, convert to slope-intercept form.

a) slope of 0 and through $(4, -5)$

b) same slope as $3x + y = 5$ and through $(-2, 4)$

c) same slope as the line $x - 2y + 6 = 0$ and the same x-intercept as the line $3x - 2y = 24$

d) same y-intercept as $x + 4y = 8$ and through $(3, -4)$

12. What is the equation of each line in slope-point form? Convert each equation to general form.

a) slope of 3 and x-intercept of 4

b) same slope as $y = -4x + 5$ and through $(2, -1)$

c) same x-intercept as the line $3x + y = 12$ and through $(0, 2)$

d) x-intercept of 2 and y-intercept of -6

13. An "iron horse" pumpjack starts to pump crude oil into a tank at a constant rate of 1.2 m^3/h. After 24 h, the tank contains 29 m^3 of oil.

a) Write an equation that describes the volume, V, in cubic metres, of oil in the tank after t hours.

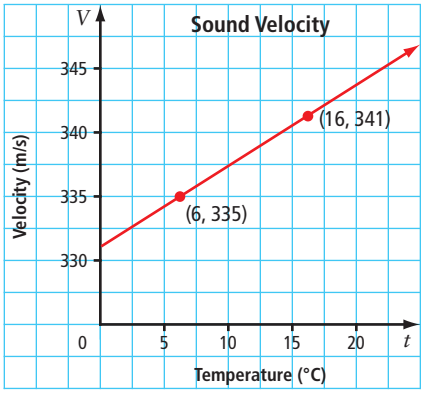
b) The tank can hold a maximum of 155 m^3 of oil. How long will it take to fill the tank?

c) Was the tank empty before it started filling? Explain.

Did You Know?

Oil pumpjacks are common in western Canada. They are a traditional method of oil recovery. The surface deposits of the Athabasca Oil Sands in present day northern Alberta were once used by the Cree and Dene peoples to waterproof their canoes and other items.

14. The graph shows the linear relationship between the velocity of sound, V, in metres per second, and the temperature, t, in degrees Celsius, of dry air. At 6 °C, the velocity of sound is 335 m/s. At 16 °C, it is 341 m/s.



a) What is the slope of the line?

b) What rate of change does the slope represent?

c) What is the equation of the line?

d) Determine the velocity of sound at 35 °C.

e) What is the air temperature when the velocity of sound is 348 m/s?

15. What is the y-intercept of a line with a slope of $\frac{1}{2}$ and an x-intercept of 4?

16. Determine the x-intercept of a line through (3, 4) having a y-intercept of 2.

17. Suppose Canada's population has grown steadily since 2000. In 2001, the population was 30.0 million. In 2009, it was 33.7 million.

a) Let t represent the number of years since 2000. Let p represent the population of Canada in millions. Write the coordinates of two points in the form (t, p).

b) Determine the slope of the line through the points.

c) What rate does the slope represent?

d) Write an equation to represent population growth in Canada since 2000.

e) Predict Canada's population in 2017.

WWW Web Link

To learn more about the components of population growth in parts of Canada, go to www.mhrmath10.ca and follow the links.

18. Suppose your friend's dinner tonight consists of one steak and mini potatoes. The steak has approximately 30 g of protein. The nutrition facts label shows the number of grams of protein per number of mini potatoes.

a) Write an equation relating the protein, p, in the meal to the number of potatoes, n, eaten. Use the data in the nutrition facts label.

b) What is the slope of the line? What does the slope represent?

c) What is the p-intercept of the line? Why is the p-intercept not zero?

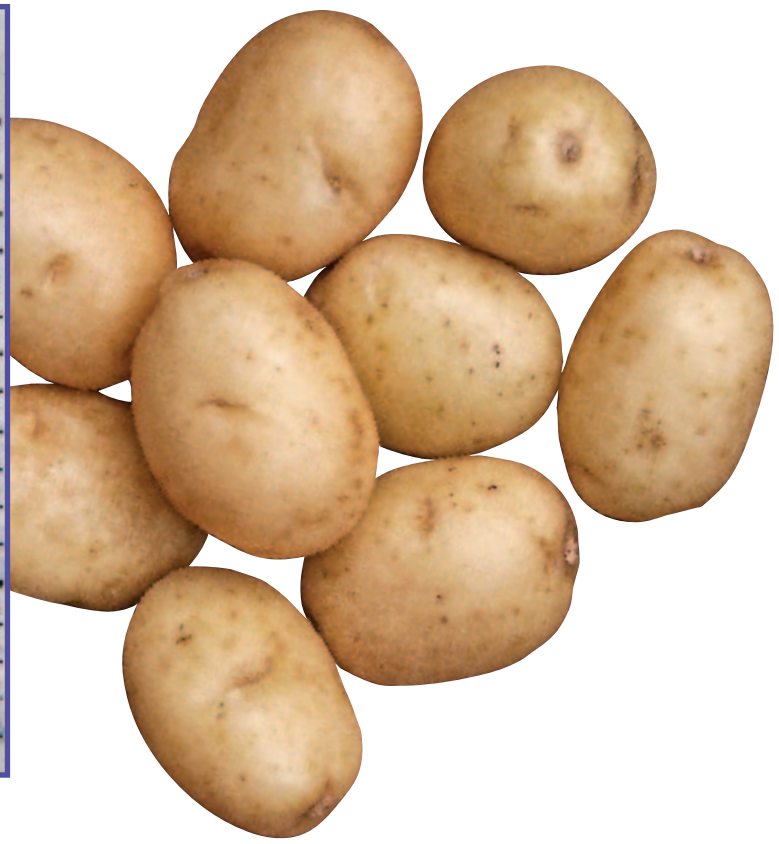
d) Suggest a reasonable domain and range for the graph.

Did You Know?

In Canada, potato production is a multi-million-dollar industry. Manitoba produces the second greatest amount of potatoes in Canada.

Nutrition Facts / Valeur nutritive		
Serving Size 5 potatoes Portions 5 pomme de terre	Yellow Potatoes Pommes de terre jaunes (110 g)	
	Amount Teneur	% DV* % VQ*
Calories / Calories	80	
Fat / Lipides	0 g	0 %
Saturated / saturés + Trans / trans	0 g 0 g	0 %
Cholesterol / Cholestérol	0 mg	0 %
Sodium / Sodium	2 mg	0 %
Potassium / Potassium	480 mg	14 %
Carbohydrate / Glucides	18 g	6 %
Fibre / Fibres	2 g	8 %
Sugars / Sucres	1 g	
Protein / Protéines	3 g	
Vitamin A / Vitamine A		0 %
Vitamin C / Vitamine C		4 %
Calcium / Calcium		0 %
Iron / Fer		6 %

* DV = Daily Value / VQ = valeur quotidienne



Extend

19. Write the equation of a line with an x-intercept of n and a slope of m.

20. A line passes through the point of intersection of the lines $y = -\frac{1}{2}x - 6$ and $y = 2x + 4$. Determine the equation of the line if it has a slope of $\frac{1}{2}$.

Create Connections

21. How can you develop the slope-intercept form, $y = mx + b$, by substituting a point into $y - y_1 = m(x - x_1)$?

22. To determine the equation of a line in slope-point form, you need to know two pieces of information about the line. List three sets of information that would allow you to determine the equation of a line.

23. To solve a particular problem you may want to write a linear equation in one of the three forms. You may wish to use slope-intercept form, $y = mx + b$; general form, $Ax + By + C = 0$; or slope-point form, $y - y_1 = m(x - x_1)$. Create a visual that helps you decide which form you should start with.

Materials
- SI measuring tape
- grid paper
- ruler

24. **MINI LAB** **Unit Project** Paleontologists can predict the anatomy of humans and animals based on skeletal remains.

Step 1 Work with a partner of the same gender as yourself. Measure and record the length of each other's humerus bone. It runs from the shoulder to the elbow. Measure and record each other's height without shoes.

Step 2 Collect and share your data with other students of the same gender. Record all data. Use grid paper to plot the data as coordinate pairs. Label the axes and scale used.

Step 3 Draw a straight line that represents the data. What is the equation of this line?

Step 4 Measure the humerus bone of a teacher of the same gender as you. Use your equation to predict the height of the teacher. Compare the teacher's actual height with your predicted height.

7.4 Parallel and Perpendicular Lines

Controlling the movement of your body is important when you play sports or exercise. The words *parallel* and *perpendicular* describe the position of one thing relative to another. Athletes may need to visualize parallel and perpendicular lines to help them improve their performance. For example,

- A football coach may instruct quarterbacks to position their shoulders perpendicular to the target at which they are throwing the ball.
- The gymnast in the photo has trained hard to keep her legs parallel to her arms.

Focus on ...

- identifying whether two lines are parallel, perpendicular, or neither
- writing the equation of a line using the coordinates of a point on the line and the equation of a parallel or perpendicular line
- solving problems involving parallel and perpendicular lines

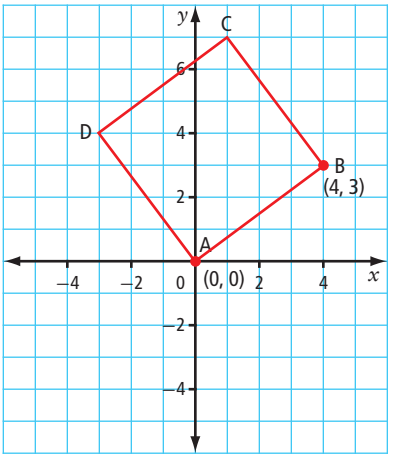
Materials

- two sheets of identical grid paper
- scissors
- ruler

Investigate Slopes of Parallel and Perpendicular Lines

1. On a sheet of grid paper, create a coordinate system by drawing and labelling an x-axis and a y-axis. From another sheet of grid paper, cut out the following three shapes:

- a square with side length of 5 units
- a square with side length of 13 units
- a rectangle with side lengths that are whole units. Draw a diagonal across the rectangle.

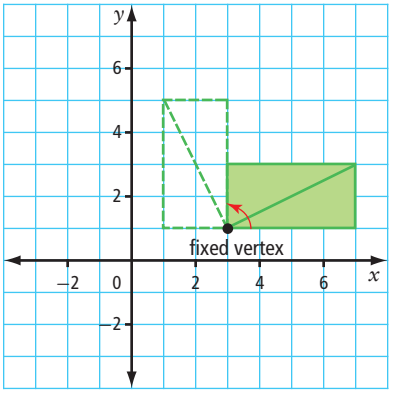


2. For the smaller square, label the vertices A, B, C, and D in a counterclockwise direction. Position the square so that point A is at (0, 0) and point B is at (4, 3). What are the coordinates of vertices C and D? Determine the slope of each side. Compare the slopes of the opposite sides and the adjacent sides.

3. Repeat step 2 using the larger square. Position the square with point A at (0, 0) and B at (12, 5).

4. Position the rectangle on your coordinate system. Determine the slope of each side. Hint: Line up the vertices with the integer coordinates of the grid. Compare the slopes.

5. Compare the slopes of the diagonals formed by rotating the rectangle 90° about one vertex. Discuss your results with a classmate.



6. **Reflect and Respond** How are the slopes of parallel sides related? Explain using an example.

7. How are the slopes of perpendicular sides related? Explain using an example.

8. **a)** How are the slopes of vertical sides related?

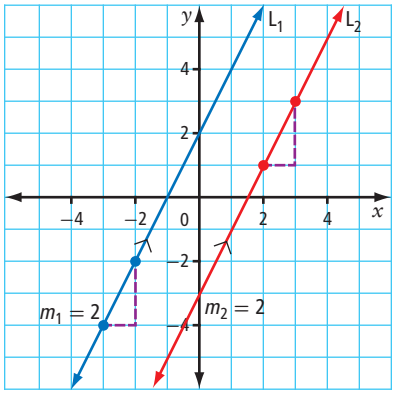
b) How are the slopes of a vertical and a horizontal side related?

Link the Ideas

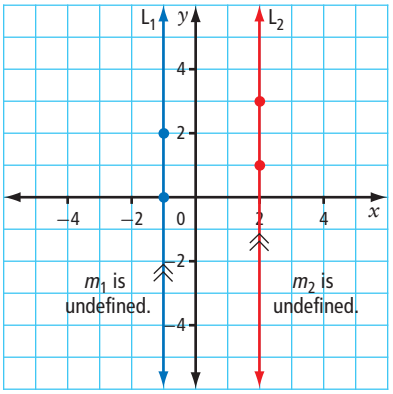
Parallel lines have the same slope but different intercepts. This includes horizontal lines, which have a slope of zero. Vertical lines, which have an undefined slope, are also parallel.

parallel lines

- lines in the same plane that do not intersect
- lines that have the same slope but different intercepts



$m_1 = m_2$

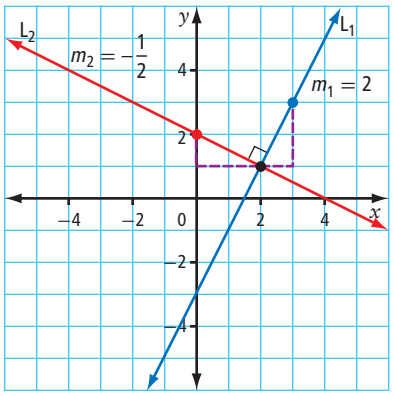


m_1 and m_2 are undefined.

The slopes of **perpendicular lines** are negative reciprocals of each other. The product of negative reciprocals is -1. A vertical line, which has an undefined slope, and a horizontal line, which has a slope of 0, are perpendicular to each other.

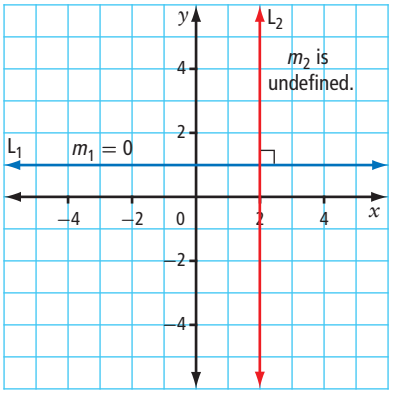
perpendicular lines

- two lines that intersect at right angles (90°)
- lines that have slopes that are negative reciprocals of each other.



$m_1(m_2) = 2\left(-\frac{1}{2}\right)$

$m_1(m_2) = -1$



$m_1 = 0$ and m_2 is undefined.

Example 1 Identify Parallel and Perpendicular Lines

State whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = 3x - 6$
$y = -\frac{1}{3}x + 4$

b) $y = 4x + 3$
$y = 4x - 5$

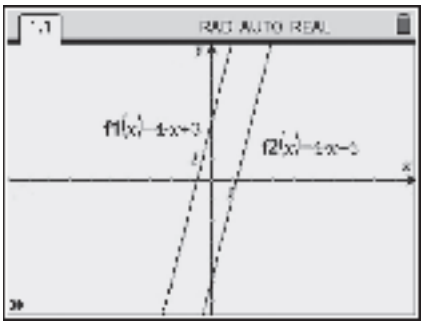
c) $y = 2x + 6$
$6x + 3y + 3 = 0$

Solution

a) The slope of the line $y = 3x - 6$ is 3.
The slope of the line $y = -\frac{1}{3}x + 4$ is $-\frac{1}{3}$.
Since the slopes, 3 and $-\frac{1}{3}$, are negative reciprocals, the lines are perpendicular.

How can you verify that two values are negative reciprocals of each other?

b) The slope of the line $y = 4x + 3$ is 4. The slope of the line $y = 4x - 5$ is also 4. The slopes are equal. So, the lines are parallel.

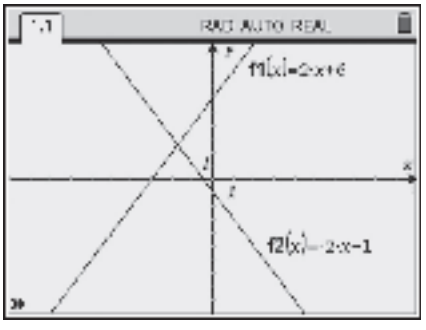


How do you know the lines are not equivalent?

c) The slope of line $y = 2x + 6$ is 2. To determine the slope of the line $6x + 3y + 3 = 0$, rewrite the equation in slope-intercept form, $y = mx + b$.

$$6x + 3y + 3 = 0$$
$$3y + 3 = -6x$$
$$\frac{3y}{3} = \frac{-6x - 3}{3}$$
$$y = -2x - 1$$

The slope of the line $y = -2x - 1$ is -2. The slopes 2 and -2 are not equal and they are not negative reciprocals. Therefore, the two lines are neither parallel nor perpendicular.



Your Turn

Determine whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = \frac{1}{2}x - 7$
$y = 2x - 7$

b) $y = 3x - 4$
$y = 3x + \frac{1}{4}$

c) $y = \frac{2}{5}x - 6$
$5x + 2y = 8$

Example 2 Write an Equation Involving a Parallel Line

a) Write the equation of a line that is parallel to $2x - y + 4 = 0$ and through $(1, -6)$. Express the equation in slope-intercept form.

b) Write the equation in general form.

c) Use technology to verify that the lines are parallel.

Solution

a) The slope of the line will be equal to the slope of $2x - y + 4 = 0$. To find the slope, convert $2x - y + 4 = 0$ to slope-intercept form, $y = mx + b$.

$$2x - y + 4 = 0$$
$$2x - y + 4 + y = 0 + y$$
$$2x + 4 = y \text{ or } y = 2x + 4$$

The slope of the line $y = 2x + 4$ is 2.

Method 1: Use Slope-Point Form

Substitute 2 for m and the coordinates of the point $(1, -6)$ for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$
$$y - (-6) = 2(x - 1)$$
$$y + 6 = 2(x - 1)$$

To convert to slope-intercept form, isolate y.

$$y + 6 = 2(x - 1)$$
$$y + 6 = 2x - 2$$
$$y = 2x - 2 - 6$$
$$y = 2x - 8$$

The equation of the line, in slope-intercept form, is $y = 2x - 8$.

Method 2: Use Slope-Intercept Form

The point $(1, -6)$ lies on the line, so the coordinates must satisfy the equation of the line. Substitute 2 for m and the coordinates $(1, -6)$ for (x, y). Then, determine the y-intercept and rewrite the equation.

$$\begin{aligned} y &= mx + b \\ -6 &= 2(1) + b \\ -6 - 2 &= b \\ -8 &= b \end{aligned}$$

Substitute the values for m and b into $y = mx + b$.

$$\begin{aligned} y &= 2x + (-8) \\ y &= 2x - 8 \end{aligned}$$

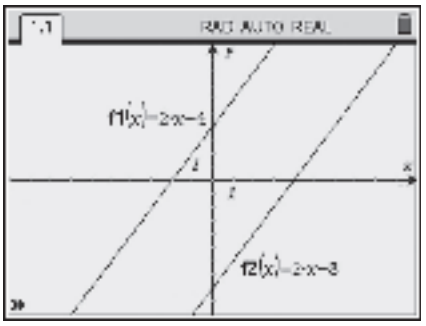
The slope-intercept form of the equation is $y = 2x - 8$.

b) Convert the slope-point equation to general form.

$$\begin{aligned} y + 6 &= 2(x - 1) \\ y + 6 - (y + 6) &= 2(x - 1) - (y + 6) \\ 0 &= 2x - 2 - y - 6 \\ 0 &= 2x - y - 8 \end{aligned}$$

The equation of the line, in general form, is $2x - y - 8 = 0$.

c) The graph to the right shows the original line and the new line, represented by their equations in slope-interecept form.



The equation of original line is $y = 2x + 4$

The equation of original line is $y = 2x - 8$.

The slope-intercept form of the lines and the graph both show that the slopes of the lines are the same, but the y-intercepts are different. Therefore the lines are parallel.

Your Turn

Write the equation of a line that is parallel to $3x + y + 3 = 0$ and passes through $(5, -6)$. Express the equation in slope-intercept form and in general form. Use technology to verify that the lines are parallel.

Example 3 Write an Equation Involving a Perpendicular Line

Write the equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9. Express the equation in slope-intercept form and in general form.

Solution

To determine the slope of $3x + 2y - 6 = 0$, rewrite the equation in slope-intercept form.

$$3x + 2y - 6 = 0$$
$$2y = -3x + 6$$
$$y = -\frac{3}{2}x + 3$$

The slope of the line $y = -\frac{3}{2}x + 3$ is $-\frac{3}{2}$.

The negative reciprocal of $-\frac{3}{2}$ is $\frac{2}{3}$.

Therefore, the slope of a line perpendicular to the given line is $\frac{2}{3}$. Substitute $\frac{2}{3}$ for m and the coordinates of the point (9, 0) for (x_1, y_1) into the slope-point form of an equation.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{2}{3}(x - 9)$$
$$y = \frac{2}{3}(x - 9)$$

How else could you find the equation of the line?

For slope-intercept form, $y = mx + b$, expand the slope-point equation.

$$y = \frac{2}{3}(x - 9)$$
$$y = \frac{2}{3}x - \cancel{9}^{3}\left(\frac{2}{\cancel{3}}\right)_{1}$$
$$y = \frac{2}{3}x - 6$$

For general form, $Ax + By + C = 0$, rearrange the slope-intercept equation.

$$y = \frac{2}{3}x - 6$$
$$3y = 2x - 18$$
$$0 = 2x - 3y - 18$$

What other equation could you use to convert to general form? Why might some people choose to use that equation in this solution?

The equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9 is $y = \frac{2}{3}x - 6$ in slope-intercept form. Written in general form, the equation is $2x - 3y - 18 = 0$.

ME

The reciprocal of $-\frac{3}{2}$ is $-\frac{2}{3}$. So, the negative reciprocal is $-\left(-\frac{2}{3}\right)$ or $\frac{2}{3}$.

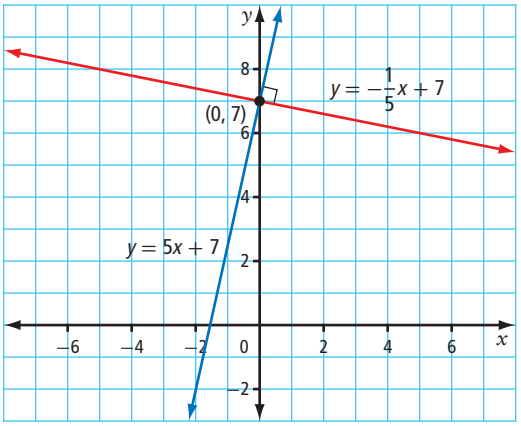
Your Turn

A line is perpendicular to $4x + y - 12 = 0$ and passes through $(8, -6)$. Write the equation of the line in either slope-intercept form or general form.

Key Ideas

- Parallel lines have the same slope and different intercepts. Vertical lines are parallel to each other, as are horizontal lines, if they have different intercepts.
- Perpendicular lines have slopes that are negative reciprocals of each other. A vertical line with an undefined slope and a horizontal line with a slope of zero are also perpendicular.
- The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.

  A line perpendicular to $y = 5x + 7$ has the same y-intercept. The line $y = 5x + 7$ has a slope of 5 and a y-intercept of 7. The perpendicular line has a slope of $-\frac{1}{5}$ and a y-intercept of 7. So, the equation of the perpendicular line is $y = -\frac{1}{5}x + 7$.



Check Your Understanding

Practise

1. For a line with each slope, state the slope of a line parallel to it. What is the slope of a line perpendicular to it?

a) $m = 5$

b) $m = -7$

c) $m = -\frac{1}{3}$

d) $m = \frac{6}{7}$

e) $m = 0.5$

f) $m = -0.75$

g) $m = 0$

h) m is undefined.

2. State the slopes of lines that are parallel and lines that are perpendicular to each linear equation.

a) $y = \frac{3}{7}x + 4$

b) $y = -x + 9$

c) $3x + y - 5 = 0$

d) $2x + y + 11 = 0$

e) $3x - 2y + 6 = 0$

f) $5x + 4y - 20 = 0$

g) $y = 7$

h) $x + 3 = 0$

3. Consider the line joining points P(−6, 9) and Q(−2, 1).

a) What is the slope of a line parallel to this line?

b) What is the slope of a line perpendicular to this one?

4. For each pair of slopes, what is the value of n if the lines are parallel? What is the value of n if the lines are perpendicular?

a) $\frac{n}{10}$, 2

b) $\frac{24}{n}$, $-\frac{1}{3}$

c) $\frac{3}{2}$, $\frac{n}{9}$

d) $\frac{3}{n}$, $-\frac{7}{2}$

5. Identify whether each pair of lines is parallel, perpendicular, or neither. Explain how you know.

a) $y = -6x$
$y = 6x + 1$

b) $y = \frac{1}{5}x + 3$
$y = -5x - 4$

c) $y = -x + 8$
$x + y = 2$

d) $y = \frac{3}{4}x + 5$
$4x + 3y = 6$

e) $5x + 2y - 10 = 0$
$2x + 5y + 10 = 0$

f) $3x - 4y - 24 = 0$
$4x + 3y - 12 = 0$

6. Write an equation of a line that is parallel to each line and passes through the given point.

a) $y = 2x + 5$, (1, −6)

b) $y = -3x + 7$, (−2, 5)

c) $5x + y - 1 = 0$, (3, −8)

d) $6x - 2y + 10 = 0$, (3, −5)

e) $y = 8$, (3, 4)

f) $x - 5 = 0$, (−1, −8)

7. Write an equation of a line that passes through each point and is perpendicular to each line.

a) $y = 3x + 5$, (9, 5)

b) $y = -4x + 7$, (−12, −7)

c) $x + 3y + 4 = 0$, (5, −9)

d) $4x - 3y - 6 = 0$, (−2, −1)

e) $x - 2 = 0$, (−3, 7)

f) $y = -5$, (4, −6)

Apply

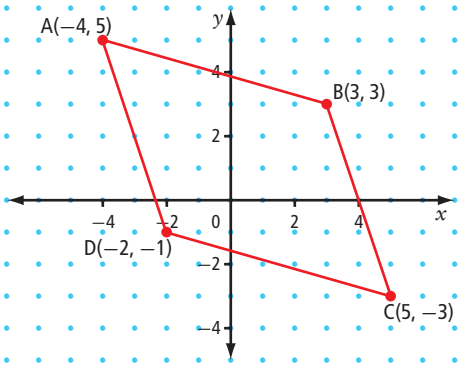
8. Sheldon was asked if line segment AB with A(−9, 2) and B(−3, 4) is parallel to line segment CD with C(−7, −7) and D(1, −3). He sketches a graph of the two line segments and concludes that they appear parallel.

a) Is it correct to assume from a sketch that the two line segments are parallel? Explain.

b) How could you prove that two lines segments are parallel?

c) Is line segment AB parallel to line segment CD? Justify your answer.

9. Is quadrilateral ABCD with vertices A(−4, 5), B(3, 3), C(5, −3), and D(−2, −1) a parallelogram? Justify your answer. Hint: A parallelogram is a quadrilateral with opposite sides parallel.



10. Write the general form equation, $Ax + By + C = 0$, of a line that passes through (7, 5) and is

a) parallel to the x-axis

b) perpendicular to the x-axis

11. Line L_1 passes through points P(n, 4) and Q(1, −2). Line L_2 passes through points R(4, 3) and S(1, 5).

a) What is the value of n if the lines are parallel?

b) If the lines are perpendicular, what is the value of n?

12. Prove that $\triangle ABC$ with vertices A(−3, 5), B(4, 7), and C(−1, −2) is a right triangle.

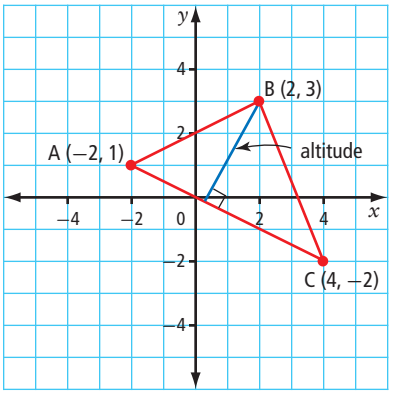
13. Determine an equation representing each line.

a) parallel to $5x + y + 4 = 0$ with a y-intercept of -6

b) perpendicular to $x + 5y - 10 = 0$ with the same y-intercept as $y = 4x - 3$

c) perpendicular to $5x + 4y - 2 = 0$ with the same x-intercept as $3x - 5y = 15$

14. Triangle ABC has vertices A(−2, 1), B(2, 3), and C(4, −2). Write the equation of the line containing the altitude from point B to side AC.

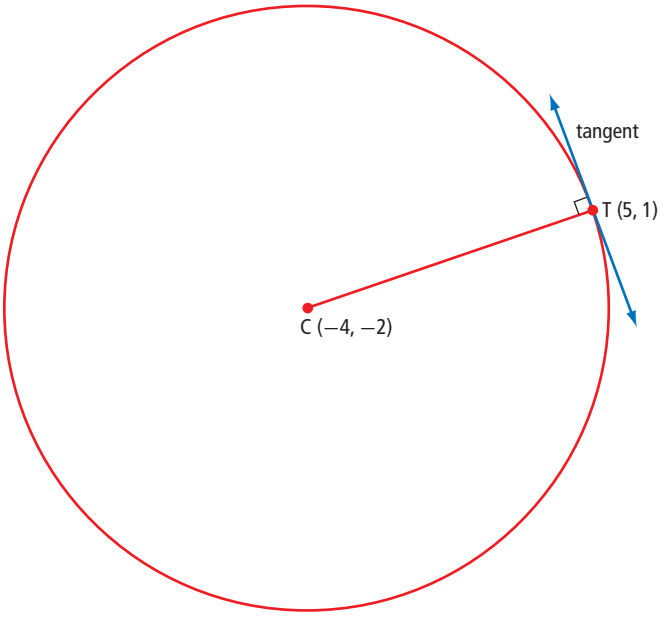


Did You Know?

An altitude is a line segment drawn from a vertex perpendicular to the opposite side.

15. The line through $(5, n)$ and $(1, -2)$ is parallel to the line $3x + 2y - 1 = 0$. What is the value of n?

16. The centre of a circle is located at C(−4, −2) on a coordinate grid. Write the equation of a tangent at point T(5, 1).

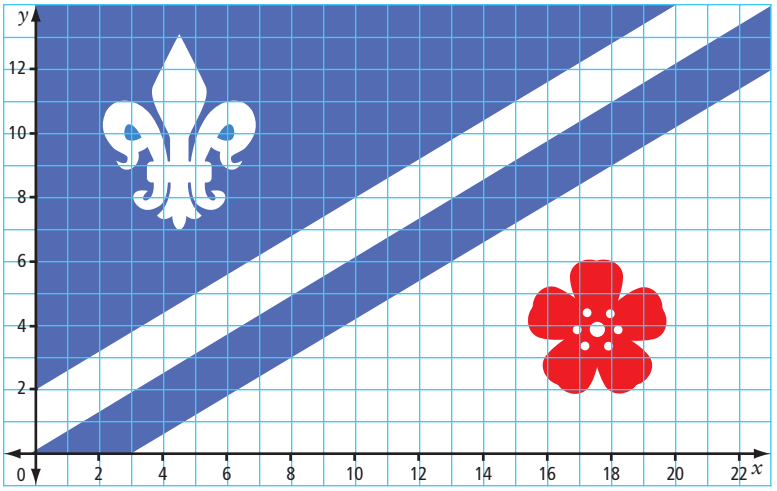


Did You Know?

A tangent is a line that touches a circle at exactly one point. It is perpendicular to the radius at that point.

17. You can monitor your heart rate while you exercise. The Karvonen formula states that the target heart rate during an aerobic workout should be between $H = 0.7(220 - A)$ and $H = 0.8(220 - A)$. In the equations, H represents your target heart rate and A represents your age. Predict whether a graph of these equations would show parallel lines. Justify your answer.

18. In 1982, the French-Canadian Association of Alberta adopted the Franco-Albertan flag. Suppose a coordinate grid is laid over a replica of the flag, with the base along the x-axis and the lower left corner of the flag at the origin. The three parallel lines in the flag pass through the points (20, 14), (23, 14), and (23, 12). Write an equation representing each line.

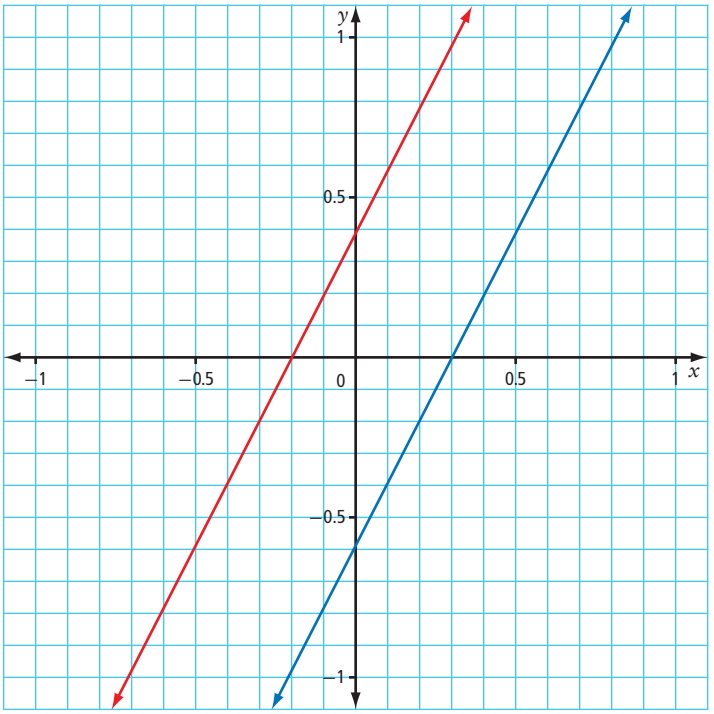


Extend

19. What is the value of n if the graphs of $nx + 4y + 3 = 0$ and $5x - 2y + 6 = 0$ are parallel?

20. The lines $6x - ny + 5 = 0$ and $x + 2y + 4 = 0$ are perpendicular. What is the value of n?

21. What is the shortest distance between the two lines in the graph? Explain your reasoning.



22. Two vertices of right triangle ABC are A(−2, 6) and C(7, 3). If the right angle is at vertex A and vertex B is on the x-axis, identify the coordinates of point B.

23. The lines $nx + 12y - 2 = 0$ and $3x + ny + 6 = 0$ are parallel. What are the possible values of n?

24. Determine the value of n if the lines $nx - 2y + 8 = 0$ and $3x + ny + 6 = 0$ are perpendicular.

Create Connections

25. Is the following statement always true, sometimes true, or never true? "The slopes of perpendicular lines are always negative reciprocals of each other." Explain your reasoning.

26. Suppose you want to determine whether two lines are parallel. Which form of an equation would you prefer to use? Why?

7 Review

7.1 Slope-Intercept Form, pages 340–356

1. What are the slope and y-intercept of each line?

a) $y = -5x + 6$ **b)** $5x - 6y + 12 = 0$

2. Write the equation of the line with each slope and y-intercept. Explain your steps.

a) slope $= -\frac{4}{5}$, y-intercept $= 6$ **b)** slope $= 0$, y-intercept $= -8$

3. Kayla is in grade 10. She wants to have longer hair for her graduation. Her hair grows at a constant rate. If it is 24.0 cm long today, in 30 days, it will be 25.2 cm long.

a) Sketch a straight-line graph representing the rate at which Kayla's hair grows.

b) What is the slope of the line? What does it represent?

c) What is the y-intercept? What does the y-intercept represent?

d) Write an equation in slope-intercept form that describes how Kayla's hair length, L, in centimetres, grows each day, d.

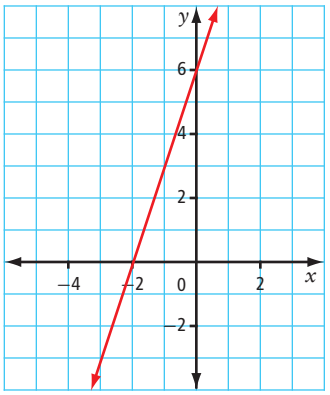
e) Predict the length of Kayla's hair at graduation if she does not get it trimmed over the next two years.

4. Explain how you could use the slope and y-intercept to graph the line $2x + 5y - 20 = 0$.

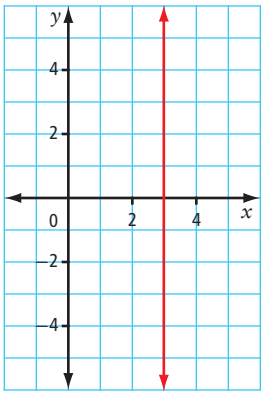
7.2 General Form, pages 357–369

5. State the intercepts of each line as ordered pairs. Then, write the equation of each line in general form.

a)



b)



6. What are the x-intercepts and y-intercepts of each line? Use the intercepts to sketch a graph of each line.

a) $5x - 2y - 20 = 0$

b) $y = 4$

7. At a community dance, students sell country food as a fundraiser. They sell bannock wheels for \$2 each and buffalo burgers for \$3 each. Sales, at the end of the day, total \$600.

a) Write an equation in general form that represents the food sales for the day.

b) What does each intercept represent?

c) What are the domain and range?

d) Suppose the students sell 135 bannock wheels. How many buffalo burgers would they sell?

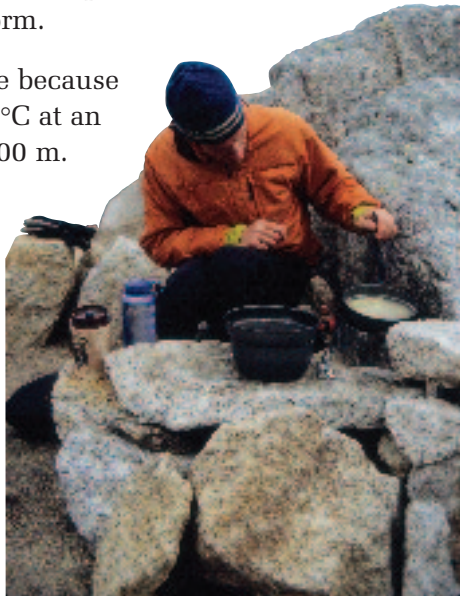
7.3 Slope-Point Form, pages 370–382

8. Use slope-point form to write an equation of a line through each point with the given slope. Express each answer in slope-intercept form and in general form.

a) $(-2, 1)$ and $m = -4$ **b)** $(8, -3)$ and $m = \frac{1}{2}$

9. Explain how you would use slope-point form to write an equation of a line through points (1, 10) and (3, 2). Then, express the equation in slope-intercept form and in general form.

10. At higher altitudes, water boils at a lower temperature because the air pressure is lower. Suppose water boils at 96.5 °C at an altitude of 1000 m, and at 93.0 °C at an altitude of 2000 m.



a) What is the slope of the line representing the data? What rate does the slope represent?

b) Write an equation of the line. Express your answer in slope-intercept form or in general form.

c) Mountain climbers need to adjust their ingredients and cooking techniques when cooking at higher altitudes. What is the boiling temperature of water at an elevation of 4000 m?

7.4 Parallel and Perpendicular Lines, pages 383–395

11. A line joins points R(5, −3) and S(7, 2). What is the slope of a line

a) parallel to line RS?

b) perpendicular to line RS?

12. The slopes of two lines are $-\frac{3}{2}$ and $\frac{18}{n}$. What is the value of n if the two lines are

a) parallel?

b) perpendicular?

13. The equations of five lines are given. Which pairs of lines meet each criterion? Justify your reasoning.

Ⓐ $x + 2y - 6 = 0$
Ⓑ $y = 2x - 3$
Ⓒ $2x + y - 3 = 0$
Ⓓ $y = -\frac{1}{2}x - 3$
Ⓔ $x + 2y + 6 = 0$

a) parallel lines

b) perpendicular lines

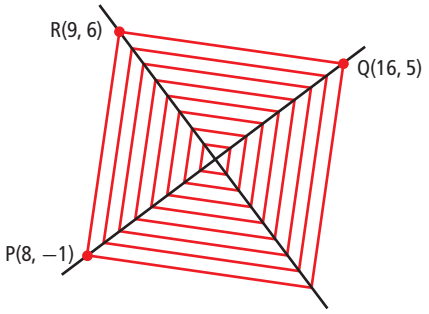
c) equivalent lines

14. Write an equation of a line through (−1, 2) and parallel to $y = -7$. Explain your reasoning.

15. Write an equation of a line through (−6, 7) and perpendicular to $3x + 4y - 12 = 0$. Explain your reasoning.

16. How could you determine the equation of a line perpendicular to $2x + 5y + 10 = 0$ with the same x-intercept as $3x - 2y = 12$?

17. Suppose a spider weaves sticky threads to catch its prey and non-sticky radial lines to allow it to cross its own web. A model of a spider web is shown. Determine an equation of the radial line through R(9, 6) and perpendicular to line PQ.

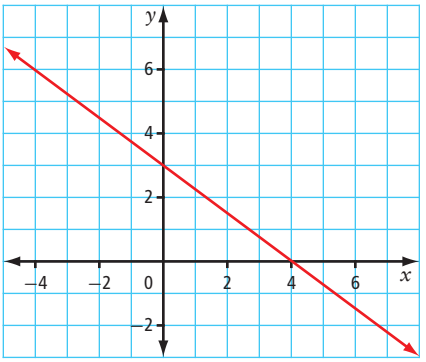


7 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. What are the slope and y-intercept of the graph shown?



A slope: $-\frac{4}{3}$, y-intercept: $(0, 4)$

B slope: $-\frac{3}{4}$, y-intercept: $(0, 4)$

C slope: $-\frac{3}{4}$, y-intercept: $(0, 3)$

D slope: $-\frac{4}{3}$, y-intercept: $(0, 3)$

2. What are the intercepts of the line $2x - 3y = -6$?

A x-intercept: $(3, 0)$, y-intercept: $(0, -2)$

B x-intercept: $(-3, 0)$, y-intercept: $(0, 2)$

C x-intercept: $(-3, 0)$, y-intercept: $(0, 3)$

D x-intercept: $(3, 0)$, y-intercept: $(0, -3)$

3. The slope and y-intercept of the line $7x + 2y - 10 = 0$ are

A slope: $\frac{7}{2}$, y-intercept: $(0, 5)$

B slope: $-\frac{7}{2}$, y-intercept: $(0, 5)$

C slope: $\frac{7}{2}$, y-intercept: $(0, -5)$

D slope: $-\frac{7}{2}$, y-intercept: $(0, -5)$

4. The equation $y = -\frac{3}{4}x + 2$ expressed in general form is

A $3x - 4y - 2 = 0$ **B** $3x + 4y - 2 = 0$

C $3x + 4y - 8 = 0$ **D** $3x - 4y + 8 = 0$

5. A line has a slope of -2 and passes through the point $(3, -1)$. When the equation of the line is written in the form $y = mx + b$, the value of b is

A 5 **B** 1 **C** -1 **D** -5

6. Which line is parallel to the line $2x + 4y - 8 = 0$?

A $y = -\frac{1}{2}x + 5$ **B** $y = \frac{1}{2}x - 1$

C $y = -2x + 3$ **D** $y = 2x - 7$

Short Answer

7. **a)** If two lines are perpendicular, what is the relationship between their slopes?

b) If two lines with slopes of $\frac{6}{n}$ and $-\frac{3}{2}$ are perpendicular, what is the value of n?

8. Express the equation of the line passing through the points $(2, 4)$ and $(2, -4)$ in general form.

9. Tickets for a school play were \$10 for adults and \$6 for students. Total ticket sales were \$2900.

a) Write an equation that represents the ticket sales.

b) How many adult tickets were sold if 275 students bought tickets?

10. Four equations are listed.

Ⓐ $y = -3x + 4$
Ⓑ $y + 10 = -3(x - 2)$
Ⓒ $6x + 2y - 8 = 0$
Ⓓ $y + 8 = -3(x - 4)$

Which equations represent the same line? Justify your answers.

11. Jacob rides his mountain bike at 15 km/h along a 30-km trail. His distance, D, in kilometres, from the end of the trail at time t hours may be modelled by the equation $D = 30 - 15t$.

a) What does the D-intercept of a graph of the equation represent?

b) Felicia rides at the same speed as Jacob. She starts 2 km behind Jacob on the same trail. Explain why she will never catch up to Jacob. Provide support for your explanation.

12. What is the equation of a line through $(3, -1)$ and parallel to the line $5x + y - 1 = 0$? Express your answer in general form, $Ax + By + C = 0$. Explain your steps.

Extended Response

13. Explain three different strategies you could use to graph the line $y - 4 = -2(x + 3)$, without creating a table of values.

14. When a railway car is unloaded at a grain elevator, the mass of wheat, W, in tonnes, remaining after t minutes decreases at a constant rate as shown in the table.

Time, t (min)	Mass of Wheat Remaining, W (tonnes)
0	88.0
6	61.6
12	35.2

a) Use the data in the table to sketch a straight-line graph.

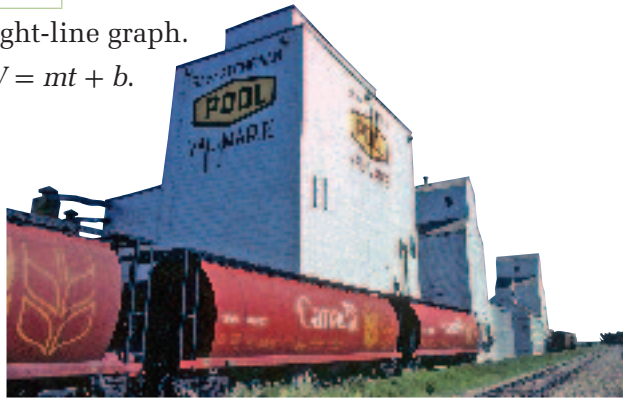
b) Write the equation of the line in the form $W = mt + b$.

c) Identify the intercepts. What does each intercept represent?

d) What is the slope of the line? What does the slope represent?

e) What are the domain and range?

f) How long would it take for half of the wheat to be emptied?



15. The relationship between air temperature and how fast a male cricket chirps is linear. A group of biology students conducted the following experiment. The students counted the number of chirps per minute by a cricket at various locations within the school. In a room where the air temperature was 14 °C, the cricket chirped 70 times per minute. In the cafeteria, the air temperature was 21 °C. The cricket chirped 119 chirps per minute.

a) Write a linear equation relating the number of cricket chirps per minute, n, to the air temperature, T, in degrees Celsius. Express the equation in slope-intercept form.

b) Sketch a graph of the linear equation. Explain your method.

c) In the boiler room, the cricket chirped 168 chirps per minute. What is the temperature in the boiler room?

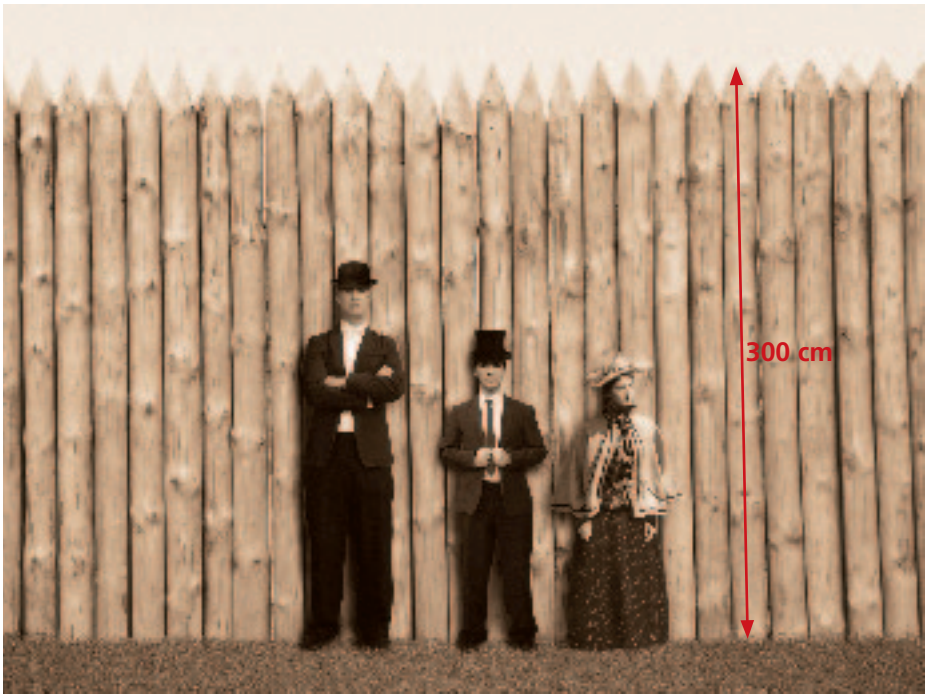
3 Unit Connections

Unit 3 Project

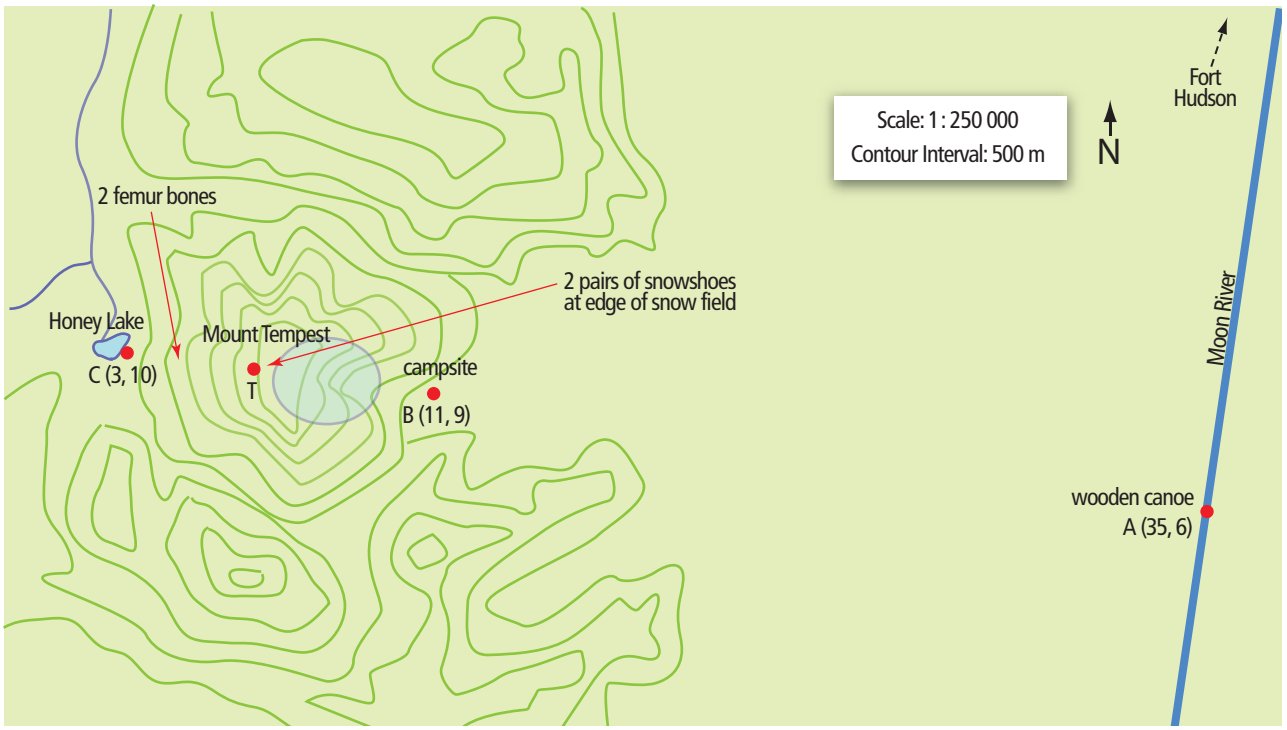
This project is fictitiously set in western Canada. The scenario, places, and people involved are entirely fictitious.

In 1899, Sam Mason, Billy Pratt, and Kittie Carmack left Fort Hudson and paddled by canoe down Moon River to retrieve a stash of gold they had left at Honey Lake. The trio vanished and the gold was never found. You are a forensic archaeologist newly hired to investigate their disappearance.

This photograph of the trio was taken outside Fort Hudson prior to their departure.



Part of a wooden canoe from that period was recently unearthed along the banks of Moon River. This new evidence allows you to retrace the route that Sam, Billy, and Kittie may have taken. A contour map of the region is shown on the following page. The coordinates of three points are labelled.



Part A: Collecting Evidence

The previous archaeologist on the case provided you with the following information.

- The three gold seekers left Fort Hudson on April 15, 1899. They had enough food and camping equipment for ten days.
- The trip by canoe from Fort Hudson to point A takes three days.
- There is evidence that the three gold seekers climbed over Mount Tempest. The east side of Mount Tempest has a snow field that has to be crossed using snowshoes. After climbing the snow field, the gold seekers would have left their snowshoes behind for the return journey. Two pairs of snowshoes are still at the top of Mount Tempest.
- Fort Hudson is at the same elevation as point A. According to fort records, the average daily temperature during April 1899 was 4 °C.
- On the west side of Mount Tempest, two femur (thigh) bones were found. They measure 40.9 cm and 56.1 cm in length.
- When the bones were found, 98.6% of the carbon-14 remained.

Part B: Analysing the Evidence

Use what you have learned in the unit and the information in the Math Toolkit to answer the questions at the bottom of the page.

Math Toolkit

- As altitude increases, air temperature decreases at a rate of 0.0064 °C/m.
- A linear approximation of the age of a bone less than 500 years old is $A(p) = -85.4p + 8541.1$, where $A(p)$ is the age, in years, at p percent of carbon-14 remaining.
- The formula for a male's height, $M(x)$, in centimetres, at a femur bone length x, in centimetres, is $M(x) = 2.38x + 61.41$.
- The formula for a female's height, $H(x)$, in centimetres, at a femur bone length x, in centimetres, is $H(x) = 2.47x + 54.10$.
- The table shows general hiking speeds for several climbing slopes.

Slope of Climb	Average Hiking Speed (km/h)
$0.0 \leq m < 0.1$	4.0
$0.1 \leq m < 0.2$	2.0
$0.2 \leq m < 0.3$	1.5
$0.3 \leq m < 0.4$	1.3
$0.4 \leq m < 0.5$	1.1
$0.5 \leq m < 0.6$	1.0
$0.6 \leq m < 0.7$	0.9

- The speed for downhill travel is about the same as the flat ground speed, 4.0 km/h.

1. How long would it take the gold seekers to travel from point A to point B?

2. Is the path from point A passing through point B to point C a straight line? Justify your answer.

3. Is the path from point A to point B level? Explain how you know.

4. a) How many metres above points B and C is the top of Mount Tempest, point T?

b) What was the mountain terrain like? Explain your answer in terms of the slope or angle of inclination.

5. a) How long would it take them to travel from point B to the top of Mount Tempest?

b) How long would the trip down to Honey Lake, point C, take?

6. At that time of year, what were the approximate temperatures at the base and top of Mount Tempest? How might the temperatures affect the travellers? Explain your reasoning.

7. Use the photograph taken at Fort Hudson to estimate the height of each gold seeker. Blueprints show that the wooden walls of the Fort were 300 cm tall. Express your answers to the nearest tenth of a metre.

8. a) How old were the bones when they were found? Verify that this age is within the range for the relation that you used.

b) Whose bones do you think they are? Why?

9. The portion of Moon River shown in the contour map is perpendicular to path AB.

a) What is the equation of the line from point A to point B?

b) Write an equation of a line in general form that contains this section of Moon River.

c) An additional clue is found along the same straight section of Moon River. The clue is located at the coordinates (n, 30). What is the value of n?

Part C: Making Conclusions

Create an explanation of what happened to the three gold seekers. Make sure your explanation is consistent with your answers to the questions in Part B. As part of your explanation, include an additional clue of your choice found along the banks of the Moon River. This clue may explain what happened to the gold treasure.

Your presentation may be in the form of a story, a digital presentation, a video, a song, or a play.

Unit Review

Chapter 6 Linear Relations and Functions

1. Sketch and label a graph that could represent each scenario. Describe any restrictions on the domain and state whether the relation is discrete or continuous.

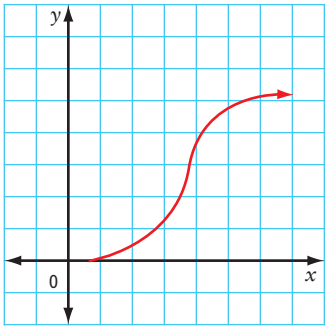
a) The speed versus time of a person riding a bike up a hill and then down the other side.

b) The cost of purchasing bags of peanuts at a baseball game.

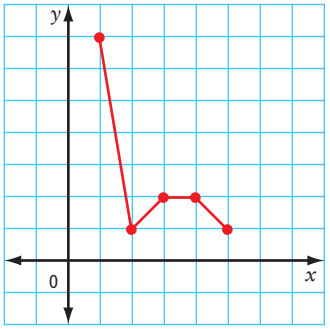
c) The height of the valve on your bicycle tire as you ride down the street.

2. Write a short story that could be represented by each graph.

a)



b)



3. Rachel was hired to work as a tree planter on the south coast of BC. She began with a wage of $125 per day.

a) Create a table of values of her earnings for at least five weeks.

b) Is the relation a function or a non-function? Explain your choice.

c) Is the relation considered discrete or continuous? Explain why.

d) Write an equation that relates the number of weeks worked to Rachel's earnings.

4. Determine whether each set of ordered pairs represents a function.

a) $\{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

b) $\{(1, 5), (3, 5), (5, 5), (7, 5), (9, 5)\}$

c) $\{(-3, 5), (0, 0), (3, 5), (6, 10), (9, 15)\}$

d) $\{(0, -1), (0, -2), (0, -3), (0, -4), (0, -5)\}$

5. A human heart can pump 18 L of blood in 3.6 min and 37.5 L of blood in 7.5 min.

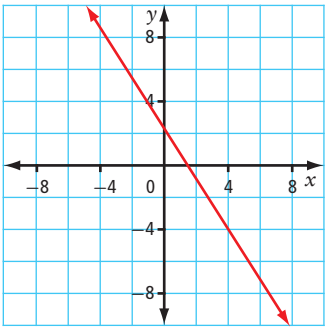
a) Identify the independent and the dependent variables in this situation.

b) Draw a graph, then determine the slope of the resulting line.

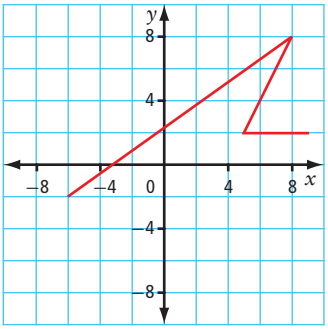
c) Explain the meaning of slope in terms of rate of change.

6. Classify each graph as a function or a non-function.

a)



b)



7. Determine the slope of each line segment in #6b).

Chapter 7 Linear Equations and Graphs

8. Determine the slope and y-intercept of each line.

a) $y = \frac{1}{2}x + 6$

b) $3x + 5y = 10$

c) $x - 3y = 0$

d) $y + 1 = 0$

9. For each line, write the equation of the line in slope-intercept form. Then, graph and label the line on a grid.

a) slope is -5; line passes through the point $(2, 4)$

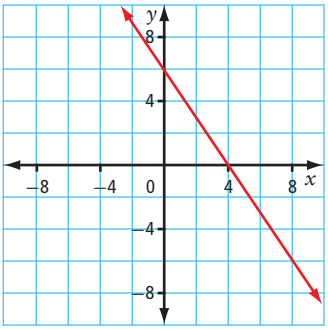
b) $y + 4 = 0$

c) x-intercept is 2; y-intercept is -5

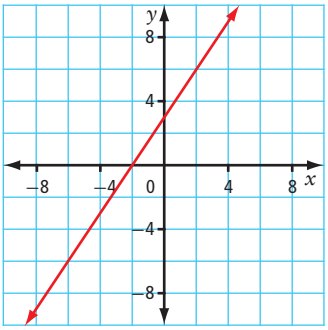
d) slope is $\frac{3}{2}$, line pass through the point $(-4, -3)$

10. Write the intercepts of each line as coordinate pairs. Then, determine the equation of the line using the intercepts. Express the equations in general form.

a)



b)



11. Determine the equation, in general form, given the characteristics for each linear relation.

a) passing through the point $(1, 6)$; parallel to the y-axis

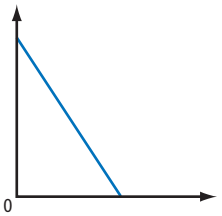
b) perpendicular to the line $5x - 3y + 2 = 0$; passing through $(-2, -3)$

3 Unit Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Which of the following is not a possible situation describing the graph.



A Temperature of water in an ice tray after being put in a freezer

B Volume of fuel remaining in the tank of a car driving on the highway

C Speed of a cyclist going down a hill

D Distance from home as you walk home from school

2. Which of the relations listed are functions?

i) $\{(3, 1), (6, 2), (3, 3)\}$

ii) $\{(-2, 7), (3, 5), (2, 5), (1, -7)\}$

iii) $\{(5, 1)\}$

iv) $\{(1, 4), (2, 4), (3, 4), (4, 4)\}$

A i, ii, iii **B** i, ii, iv **C** i, iii, iv **D** ii, iii, iv

3. The slope of a line passing through the points $(-3, 3)$ and $(11, 10)$ can be represented by the expression

A $\frac{1}{7}$ **B** $\frac{1}{2}$ **C** $\frac{7}{8}$ **D** $\frac{11}{8}$

4. If the lines $5x - 2y + 12 = 0$ and $y = \frac{k}{4}x - 3$ are parallel, the value of k is

A -10 **B** -10 **C** 5 **D** 10

5. A line is parallel to $y = 2x + \frac{5}{2}$. It passes through the point $(1, 5)$. The equation of the line is

A $2x - y + 3 = 0$ **B** $2x + y + 3 = 0$

C $2x - y - 3 = 0$ **D** $-2x + y - 3 = 0$

Numerical Response

Complete the statements in #6 and #7.

6. The slope of a graph of $6x - 2y + 11 = 0$ would be ■.

7. The equation of the line that passes through $(-3, -1)$ and $(7, -6)$ can be written in the form $Ax + By + C = 0$. The value of B is ■.

Written Response

8. The mark on an exam could be a function of the number of hours you study. Suppose you could achieve a mark of 32% without studying at all and this mark increases by 8% for each hour you study.

a) Identify the domain for this situation.

b) Create a table of values representing the hours studied and the predicted mark for the first 4 h of studying.

c) Write the linear equation that represents this function.

d) According to the function, how many hours must you study in order to achieve a mark of 100%?

e) Is a linear model a valid model for this situation? Explain your reasoning.

9. Determine the equation of a line that is perpendicular to $y = \frac{x}{2} + \frac{3}{2}$ and has the same y-intercept as the line $4x + 2y + 12 = 0$. Express the equation in the form $Ax + By + C = 0$.

10. A milk container has the instruction *Keep Refrigerated* printed on it. The function d gives the number of days the milk will last when stored at different temperatures. The temperatures are expressed in degrees Celsius. The output is $d(-7) = 24$, $d(-1) = 10$, $d(5) = 5$, $d(10) = 2$, $d(15) = 1$, $d(20) = 0.5$, and $d(25) = 0.5$.

a) Create a graph relating the number of days the milk will last to the temperature at which it is stored.

b) Estimate the number of days that milk will last if it is stored at 1 °C.

11. Joshua is taking golf lessons. The cost can be modelled by the relation $C = 30t + 25$, where C is the total cost, in dollars, and t is the number of hours of lessons he takes. Assume that lessons can end on the half hour.

a) Explain why this relation is a linear function.

b) Is this function discrete or continuous? Explain why.

c) List five ordered pairs of this function.

d) Suppose Joshua's lessons cost $130. How long were the lessons?